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Estimation and Optimal Plan in Step-Stress Partially Accelerated Life Test Model with Progressive Hybrid Censored Data from Pareto Distribution

Yimin Shi[1] and Xiaolin Shi[2]

1.Department of Applied Mathematics, Northwestern Polytechnical University
Xi'an, 710072, China. E-mail: ymshi@nwpu.edu.cn
2.School of Electronic Engineering, Xi'an University of Posts and Telecommunications, Xi'an, 710121, P.R. China

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ABSTRACT

This paper proposes a step-stress partially accelerated life test model from Pareto lifetime distribution under progressive type-I hybrid censoring. Maximum likelihood estimators (MLEs) of the distribution parameters and acceleration factor are derived by using Newton-Raphson algorithm. In addition, the approximate fisher information matrix is calculated for constructing the approximate confidence intervals of the parameters and acceleration factor. The approximate confidence intervals (ACIs) are derived based on normal approximation to the asymptotic distribution of MLEs. Optimal step-stress partially accelerated life test plan is developed by minimizing the generalized asymptotic variance (GAV) of the MLEs of the model parameters. Finally, a Monte-Carlo simulation study is carried out to illustrate the effectiveness of the proposed methods.

Keywords: Step-stress partially accelerated life test, progressive type-I hybrid censoring, parameters estimators, optimal plan, Pareto distribution, Monte-Carlo simulation

1. Introduction

With the continual improvement in manufacturing, it is more difficult to obtain failure data for high reliability items under normal use conditions. This makes the lifetime testing under these conditions very costly, take a long time. To get the information about the lifetime distribution of these items, the censoring schemes are preferred to be used in manufacturing industries and lifetime test to obtain failure data in a short period of time. The two most common censoring schemes are termed as type-I and type-II censoring schemes. One of the drawbacks of them is that they do not allow for removal of units at points other than the terminal point of the experiment. Sulabh Dube et al.[1] considered the parameters estimation of log-normal distribution with hybrid censoring. One censoring scheme known as the progressive censoring scheme had become very popular in the last few years. Fernandez [2] discussed the exponential based on progressive type II censored. Raqab et al. [3] presented the prediction for Pareto distribution based on progressively Type-II censored samples. Recently, a new type of censoring schemes: progressive hybrid censoring scheme (PHCS) has been proposed by Kundu and Joarder [4]. The PHCS have an advantage: it allows to continual removal of a prespecified number of un-failed test items at the end of testing time at each stage. The PHCS has become quite popular for analyzing highly reliable data. Kuang et al. discussed the reliability analysis for accelerated life-test with progressive hybrid censored data by using Geometric process [5].

Besides, accelerated life test (ALT) are analyzed in terms of a model to relate life length to stress for the product in reliability and survival analysis. Wang [6] derived the exact confidence intervals for the exponential step-stress ALT model. N. Balakrishnan, Q. Xie [7-8] applied exact inference for a simple step-stress model with type-I and type-II hybrid censored data from the exponential distribution. Li and Xu [9-10] discussed the parameters inferences and obtained optimal hold times on the simple step-stress model in ALT with progressive type-I hybrid censoring. Further more, the constant-stress ALT was studied by several authors. Kim & Bai [11] and Watkins & John [12] discussed the ALT with two failure models and type-II censoring, respectively. Other related studies see References [13-16].

Pareto distribution was originally introduced by Pareto as a model for the distribution of income, but is now used as a model in areas involving business, economics applications and reliability engineering. Its models, usually in two different forms known as Pareto distribution and Lomax distribution, have been studied in Ref.[17]-[20]. Ismail, Abdel-Ghaly[17] considered the case of constant-stress partially ALT when two stress levels are involved under type-I censoring. Hassan,Ghamdi[18] studied the optimum design in step stress accelerated life testing for Lomax distribution. Wang [19] studied Bayesian analysis of two-parameter Pareto distribution under progressively first-failure-censored data.

It may be mentioned that although the progressive hybrid censoring scheme seems to be an important censoring scheme, not mach work has been done on the inference for the ALT under Type-I progressive hybrid censoring. This paper investigates the SSPALT model that is subject to Type-I PHCS. The model and the basic assumptions are described In Section 2. The MLEs for the distribution parameters and acceleration factor are obtained in Section 3. In Section 4, the approximate confidence intervals are developed. The optimal plan for SSPALT in Section 5 . Section 6 contains the simulation results that demonstrate and evaluate the performance of the estimators based on the proposed censoring schemes. Conclusions are provided in Section 7.

2. Model description and basic assumptions

2.1 Step-stress PALT

The step-stress PALT under type-I PHCS can be described as follows: suppose n identical items are put on a test under use stress level S_0. Let τ, T_0 , m and $r_1,\cdots,r_m$ ($\tau < T_0$, $m < n$) be pre-fixed constants. At the first failure time Y_1, r_1 items are randomly removed from the remaining $n-1$ items. Similarly, at the second failure time Y_2 , r_2 items are randomly removed from the remaining $n-r_1-2$ items, and so on. At the i th failure time Y_i, r_i items are randomly removed from the remaining $n-i-r_1-r_2-...-r_{i-1}$ items ($i=1,2,\cdots,n_1$). The test is continued until time τ ($\tau \ge Y_{n_1}$), where n_1 is the number of failure items before time τ . At the time τ , all of the surviving items are put on accelerated stress level S_1 to continue the life test. At the k th failure time Y_k ($n_1 < k < m$), r_k items are randomly removed from the remaining items. If the mth failure time Y_m occurs before the predetermined time T_0, then the test stops at the time Y_m, and all the remaining $r_m = n - m - (r_1 + \cdots + r_{m-1})$ items are removed. On the other hand, if the mth failure does not occur before time τ and only d failure occurred ($0 < d < m$), then at the time T_0, all $r_* = n - d - (r_1 + \cdots + r_d)$ remaining items are removed and the test terminates at time T_0. Therefore, in step-stress PALT under Type-I PHCS, the observed failure time are.

$Y_1 < Y_2 < \cdots < Y_{n_1} \le \tau < Y_{n_1+1} < \cdots < Y_m \le T_0$, if $Y_m \le T_0$, or,

$Y_1 < Y_2 < \ldots < Y_{n_1} \le \tau < Y_{n_1+1} < \ldots \le Y_d$, if $Y_d < T_0 < Y_{d+1}$.

2.2 Basic assumptions

A1: Two stress levels S_0 and S_1 ($S_0 < S_1$) are used in step-stress PALT. The lifetime of the tested items follow a two-parameter Pareto distribution with probability density function (PDF)

$$f(t;\alpha,\theta) = \alpha\theta^{\alpha} t^{-(\alpha+1)}, \alpha > 0, t \ge \theta > 0,$$

where θ and α are the scale parameter and the shape parameter, respectively.

A2: There is at least one tested items failure under stress levels S_0 and S_1.

A3: The lifetime Y of the tested item in SSPALT follows a tampered random variable model (see [19]), that is

$$Y = \begin{cases} T, & T \le \tau, \\ \tau + \beta^{-1}(T - \tau), & T > \tau. \end{cases}$$

where T is the lifetime of the tested item under use stress level S_0, τ is the stress change time and $\beta > 1$ is the acceleration factor.

Based on this assumption, the PDF and reliability function of the Y can be written as

$$f(y) = \begin{cases} f_1(y) = \alpha\theta^{\alpha} y^{-(\alpha+1)}, & y \le \tau, \\ f_2(y) = \alpha\beta\theta^{\alpha}[\tau + \beta(y - \tau)]^{-(\alpha+1)}, & y > \tau. \end{cases}$$

$$R(y) = \begin{cases} R_1(y) = (y/\theta)^{-\alpha}, & y \le \tau, \\ R_2(y) = [(\tau + \beta(y - \tau))/\theta]^{-\alpha}, & y > \tau. \end{cases}$$

Let n_2 denote the failure number of tested items under accelerated stress level s_1, then under the Type-I progressive hybrid censoring scheme, one of the following data is observed.

Case I: $Y_1 < Y_2 < \cdots < Y_{n_1} \le \tau < Y_{n_1+1} < \cdots < Y_m \le T_0$, if $Y_m \le T_0$,

Case II: $Y_1 < Y_2 < \cdots < Y_{n_1} \le \tau < Y_{n_1+1} < \cdots < Y_{n_1+n_2} \le T_0$, if $Y_{n_1+n_2} \le T_0 < Y_{n_1+n_2+1}$.

Note that $n_1 + n_2 = m$ for Case I, and $Y_{n_1+n_2+1}, \cdots, Y_m$ are not observed for Case II.

3. Maximum likelihood estimation

Let $Y_1 < Y_2 < \cdots < Y_{n_1+n_2}$ denote Type-I progressive hybrid censored sample, then the likelihood function can be written as follows

$$L = L(y;\alpha,\theta,\beta) = \prod_{i=1}^{n_1} f_1(y_i)[R_1(y_i)]^{r_i} \prod_{i=n_1+1}^{n_1+n_2} f_2(y_i)[R_2(y_i)]^{r_i}[R_2(T_0)]^{r}$$

$$= \alpha^{n_1+n_2}\theta^{n\alpha}\beta^{n_2} \prod_{i=1}^{n_1} y_i^{-(1+\alpha+\alpha r_i)} \prod_{i=n_1+1}^{n_1+n_2} [\tau + \beta(y_i - \tau)]^{-(1+\alpha+\alpha r_i)}[\tau + \beta(T_0 - \tau)]^{-\alpha r}.$$

where r=0 for Case I, r=r_* for Case II. The logarithm of likelihood function is

$$\ln L = (n_1+n_2)\ln\alpha + n\alpha\ln\theta + n_2\ln\beta - \sum_{i=1}^{n_1}(1+\alpha+\alpha r_i)\ln y_i$$
$$-\sum_{i=n_1+1}^{n_1+n_2}(1+\alpha+\alpha r_i)\ln[\tau+\beta(y_i-\tau)] - \alpha r\ln[\tau+\beta(T_0-\tau)]\,.$$

Maximum likelihood estimators of α , θ and β are solutions to the system of equations obtained by letting the first partial derivatives of the total log likelihood to be zero with respect to α , θ and β , respectively. Therefore, the system of equations is as follows:

$$\frac{\partial \ln L}{\partial \alpha} = (n_1+n_2)/\alpha + n\ln\theta - \sum_{i=1}^{n_1}(1+r_i)\ln y_i - \sum_{i=n_1+1}^{n_1+n_2}(1+r_i)\ln[\tau+\beta(y_i-\tau)] - r\ln[\tau+\beta(T_0-\tau)]\,. \quad (1)$$

$$\frac{\partial \ln L}{\partial \theta} = n\alpha/\theta \quad (2)$$

$$\frac{\partial \ln L}{\partial \beta} = n_2/\beta - \sum_{i=n_1+1}^{n_1+n_2}(1+\alpha+\alpha r_i)(y_i-\tau)/[\tau+\beta(y_i-\tau)] - \alpha r(T_0-\tau)/[\tau+\beta(T_0-\tau)]\,. \quad (3)$$

From Eq.(2), the MLE of θ is easily given as $\hat{\theta}=Y_1$. By substituting $\hat{\theta}=Y_1$ into Eq. (1), and letting Eq.(1) and (3) to be zero, the system of equations is reduced to the following two equations

$$\hat{\alpha} = (n_1+n_2)[\sum_{i=1}^{n_1}(1+r_i)\ln y_i + \sum_{i=n_1+1}^{n_1+n_2}(1+r_i)\ln[\tau+\hat{\beta}(y_i-\tau)] + r\ln[\tau+\hat{\beta}(T_0-\tau)] - n\ln\hat{\theta}]^{-1}, \quad (4)$$

$$\hat{\beta} = n_2\{\sum_{i=n_1+1}^{n_1+n_2}(1+\hat{\alpha}+\hat{\alpha} r_i)(y_i-\tau)/[\tau+\hat{\beta}(y_i-\tau)] + \hat{\alpha} r(T_0-\tau)/[\tau+\hat{\beta}(T_0-\tau)]\}^{-1}\,. \quad (5)$$

By substituting Eq.(4) into (5), the MLE of β can be calculated by adopting such an iterative procedure as Newton-Raphson algorithm, numerically. Once the value of $\hat{\beta}$ is determined, $\hat{\alpha}$ is easily obtained from Eq. (4).

4. Approximate confidence intervals

The approximate confidence intervals of distribution parameters and accelerated factor are derived based on the approximate Fisher information matrix. Let the elements of the Fisher information matrix be $I_{ij}(\lambda)=E\{-\partial^2\ln L/\partial\lambda_i\partial\lambda_j\}, i,j=1,2,3$, where $\lambda=(\lambda_1,\lambda_2,\lambda_3)=(\alpha,\theta,\beta)$. Since the exact expression of the above expectation is very difficult to obtain, the approximate Fisher information matrix is thus given by $I=[I_{ij}(\lambda)]=[-\partial^2\ln L/\partial\lambda_i\partial\lambda_j]$, $i,j=1,2,3$. The elements of matrix I can be expressed as follows:

$$\frac{\partial^2 \ln L}{\partial\alpha^2} = -\frac{n_1+n_2}{\alpha^2}, \quad \frac{\partial^2 \ln L}{\partial\alpha\partial\theta} = n/\theta, \quad \frac{\partial^2 \ln L}{\partial\alpha\partial\beta} = -\sum_{i=n_1+1}^{n_1+n_2}\frac{(1+r_i)(y_i-\tau)}{\tau+\beta(y_i-\tau)} - \frac{r(T_0-\tau)}{\tau+\beta(T_0-\tau)},$$

$$\frac{\partial^2 \ln L}{\partial\theta^2} = -n\alpha/\theta^2, \quad \frac{\partial^2 \ln L}{\partial\theta\partial\beta} = 0, \quad \frac{\partial^2 \ln L}{\partial\beta^2} = -\frac{n_2}{\beta^2} + \sum_{i=n_1+1}^{n_1+n_2}\frac{(1+\alpha+\alpha r_i)(y_i-\tau)^2}{[\tau+\beta(y_i-\tau)]^2} + \frac{\alpha r(T_0-\tau)^2}{[\tau+\beta(T_0-\tau)]^2}\,.$$

Note that $I_{ij}(\lambda)=I_{ji}(\lambda)$. We know that the asymptotic distribution of MLEs of λ is given by

$$((\hat\alpha-\alpha),(\hat\theta-\theta),(\hat\beta-\beta)) \to N_3(0,I^{-1}(\alpha,\theta,\beta)),$$

where $I^{-1}(\alpha,\theta,\beta)$ is the inverse matrix of the Fisher information matrix I, $I_{11}^{-1}=-de/\Delta$, $I_{22}^{-1}=-(ae-c^2)/\Delta$, $I_{33}^{-1}=-(ad-b^2)/\Delta$, $a=I_{11}$, $b=I_{12}$, $c=I_{13}$, $d=I_{22}$, $e=I_{33}$, $\Delta=eb^2+dc^2-ade$. Thus, the $100(1-\gamma)\%$ approximate confidence intervals for α,θ and β are respectively given by

$$(\hat\alpha-Z_{\gamma/2}\sqrt{\hat{I}_{11}^{-1}},\hat\alpha+Z_{\gamma/2}\sqrt{\hat{I}_{11}^{-1}}),\ (\hat\theta-Z_{\gamma/2}\sqrt{\hat{I}_{22}^{-1}},\hat\theta+Z_{\gamma/2}\sqrt{\hat{I}_{22}^{-1}}),\ (\hat\beta-Z_{\gamma/2}\sqrt{\hat{I}_{33}^{-1}},\hat\beta+Z_{\gamma/2}\sqrt{\hat{I}_{33}^{-1}}),$$

where $Z_{\gamma/2}$ is the upper $(\gamma/2)$ th percentile of a standard normal distribution.

5. Optimal SSPALT plan

To determine the optimal plan for SSPALT, the GAV of the MLEs of the parameters is considered as the optimality criterion, which is the reciprocal of the determinant of the Fisher information matrix I, namely, $GAV(\hat\alpha,\hat\theta,\hat\beta)=|I|^{-1}$, where $|I|$ is the determinant of Fisher information matrix I. Taking into account the overall parameter space, this optimality criterion is relatively better than others which only take into account a subset from the parameter space in that it helps enhance the estimation accuracy.

The optimal plan for SSPALT is to find the optimum stress change time τ^* such that the GAV of the MLEs of the parameters is minimized by solving equation $\partial GAV/\partial\tau=0$, which is reduced to $\partial|I|/\partial\tau=0$. The $|I|$ can be calculated by $|I|=I_{11}I_{22}I_{33}-I_{22}I_{13}{}^2-I_{12}{}^2I_{33}$. We can get

$$\frac{\partial|I|}{\partial\tau}=I_{33}{}'(I_{11}I_{22}+I_{22}{}^2)+2I_{13}{}'I_{13}I_{22},$$

where $I_{11}^{-1}=-de/\Delta$, $I_{22}^{-1}=-(ae-c^2)/\Delta$, $I_{33}{}'=\sum_{i=n_1+1}^{n_1+n_2}\frac{2(1+\hat\alpha+\hat\alpha r_i)y_i(y_i-\tau)}{[\tau+\hat\beta(y_i-\tau)]^3}+\frac{2\hat\alpha rT_0(T_0-\tau)}{[\tau+\hat\beta(T_0-\tau)]^3}$,

$I_{13}{}'=-\sum_{i=n_1+1}^{n_1+n_2}\frac{y_i(1+r_i)}{[\tau+\hat\beta(y_i-\tau)]^2}-\frac{rT_0}{[\tau+\hat\beta(T_0-\tau)]^2}$, $a=I_{11}$, $c=I_{13}$, $d=I_{22}$, $e=I_{33}$, $\Delta=eb^2+dc^2-ade$

Stress change time τ^* can be derived from the following equation.

$$\hat\alpha\Big[\sum_{i=n_1+1}^{n_1+n_2}\frac{y_i(1+r_i)}{(\tau+\hat\beta(y_i-\tau))^2}+\frac{rT_0}{(\tau+\hat\beta(T_0-\tau))^2}\Big]\Big[\sum_{i=n_1+1}^{n_1+n_2}\frac{(1+r_i)(y_i-\tau)}{\tau+\hat\beta(y_i-\tau)}+\frac{r(T_0-\tau)}{\tau+\hat\beta(T_0-\tau)}\Big]$$
$$-\frac{n_1+n_2+n\hat\alpha}{\hat\alpha\hat\theta}\Big[\sum_{i=n_1+1}^{n_1+n_2}\frac{(1+\hat\alpha+\hat\alpha r_i)y_i(y_i-\tau)}{(\tau+\hat\beta(y_i-\tau))^3}+\frac{\hat\alpha rT_0(T_0-\tau)}{(\tau+\hat\beta(T_0-\tau))^3}\Big]=0. \qquad (6)$$

By employing an iterative method such as Newton-Raphson algorithm, stress change time τ^* can be obtained numerically.

6. Simulation study

In this section, simulation studies are carried out for illustrating the theoretical results of the estimation problem. The performance of the MLEs and confidence intervals of the parameters has been considered in terms of absolute bias (AB) and mean square error (MSE), and covering percentage (CP) for confidence intervals of the parameters, respectively. Furthermore, the optimal stress change time and optimal numbers of failure items are obtained under use stress level as well as accelerated stress level. The simulation procedure can be described as follows:

Step 1: Random samples of different sizes are generated from Pareto distribution, which can be achieved by using the transformation $y_i = \theta(1-u_i)^{-1/\alpha}$, if $y_i \le \tau$. But if $y_i > \tau$, then using the transformation $y_i = \tau - \theta\tau/\beta + \theta(1-u_i)^{-1/\alpha}/\beta$, $i = 1,\cdots,n$ .where $u_1,\cdots,u_n$ are a random sample from uniform distribution $\mathrm{U}(0,1)$.

Step 2: Under progressive type-I hybrid censoring, the censoring time T_0 and the stress change time τ are chosen with different combinations of parameter values for α,θ and β. The censoring schemes are selected in the following forms:

CS1: $r_1 = \cdots = r_{m-1} = 1, r_m = n - 2m + 1$;

CS2: $r_1 = r_3 = \cdots = r_{m-1} = 1, r_2 = r_4 = \cdots = r_{m-2} = 0, r_m = n - 1.5m$;

CS3: $r_1 = n - 1.5m + 1, r_2 = r_4 = \cdots = r_m = 0, r_3 = r_5 = \cdots = r_{m-1} = 1$.

Step 3: The estimations of α,θ and β are obtained from equations $\hat{\theta} = Y_1$, and Eq. (4) and (5) based on the observed data.

Step 4: Repeat the previous steps 2000 times and the average of these estimations are taken as the MLEs of the parameters.

Step 5: The AB, MSE and covering percentage with confidence level $1-\gamma = 0.95$ of the estimations are computed.

Step 6: The optimal stress change time τ^* is obtained by numerically solving equation (6) using the results of step 4. In addition, the value of n_1^* (optimal number of failure items under use stress level S_0) and n_2^* (optimal number of failure items under accelerated stress level S_1) are calculated.

Tables 1-5 represent the AB, MSE for parameters α,θ and β, and covering percentage for the confidence intervals with different combinations of initial values of α,θ,β,T_0 and τ. According to the values of AB and MSE for MLE and CP of the confidence intervals for parameters in Tables 1-5, it can be see that the performance of the MLEs under CS1, CS2 and CS3 is better. Also, as the sample size and failure proportion increase, the accuracy of the estimations increases. Table 6 represents the optimal stress change time τ^* and the corresponding values of n_1^* and n_2^* under CS1, where choices of initial values are selected as follows:

$IV1$: $\beta = 1.5, \theta = 6, \alpha = 1.8, T_0 = 30, \tau = 8$;

$IV2: \beta = 1.2, \theta = 5, \alpha = 1.5, T_0 = 40, \tau = 7$;

$IV3: \beta = 1.3, \theta = 7, \alpha = 2, T_0 = 40, \tau = 9$.

The result indicates that the optimal stress change time τ^* and the corresponding proportion of n_2^* in total numbers of failure items tend to increase as the sample size and failure proportion increase.

Table 1. The AB, MSE of MLE and CP of the confidence intervals for parameters β,θ,α
$(\beta = 1.2, \theta = 14, \alpha = 2, T_0 = 30, \tau = 15, 1-\gamma = 0.95)$

(n, m)	CS	CP (%)			AB (MSE)		
		β	θ	α	β	θ	α
	CS1	90.50	93.58	92.06	0.0371(0.1321)	0.0925(0.0165)	0.3299(0.1319)
(80,40)	CS2	90.10	93.65	92.32	0.0180(0.1458)	0.0867(0.0175)	0.2530 (0.1638)
	CS3	91.10	92.60	91.94	0.0173(0.1643)	0.0884(0.0198)	0.2423(0.1461)
	CS1	92.51	96.06	95.06	0.0166(0.1253)	0.0820(0.0153)	0.2360(0.1152)
(80,50)	CS2	91.28	95.84	92.84	0.0123(0.1309)	0.0812(0.0162)	0.1977(0.1315)
	CS3	90.80	94.93	91.29	0.0155(0.1598)	0.0834(0.0151)	0.1601(0.1257)
	CS1	91.57	93.50	93.02	0.0352(0.1156)	0.0696(0.0094)	0.2704(0.1757)
(100,40)	CS2	90.68	93.31	92.14	0.0155(0.1417)	0.0708(0.0115)	0.2077(0.1682)
	CS3	90.40	93.89	91.03	0.0142(0.109)	0.0703(0.0127)	0.1618(0.1451)
	CS1	95.36	98.10	97.28	0.0121(0.0914)	0.0614(0.0087)	0.2223(0.1381)
(100,60)	CS2	93.97	97.26	95.61	0.0134(0.1048)	0.0709(0.0102)	0.2008(0.1595)
	CS3	92.45	95.21	93.90	0.0129(0.0978)	0.0695(0.0098)	0.1523(0.1319)

Table 2: The AB, MSE and CP of the confidence intervals for parameters β,θ,α
$(\beta = 1.5, \theta = 14, \alpha = 2, T_0 = 30, \tau = 15, 1-\gamma = 0.95)$

(n, m)	CS	CP (%)			AB (MSE)		
		β	θ	α	β	θ	α
	CS1	89.23	93.02	92.61	0.0801(0.1097)	0.0779(0.0175)	0.6025(0.1317)
(80,40)	CS2	92.20	95.01	96.68	0.0144(0.1687)	0.0898(0.0169)	0.3373(0.1551)
	CS3	92.55	94.96	93.43	0.0204(0.1882)	0.0791(0.0185)	0.0544(0.1714)
	CS1	91.60	95.58	94.83	0.0317(0.0952)	0.0774(0.0153)	0.2164(0.0915)
(80,50)	CS2	94.39	97.31	96.59	0.0108(0.1273)	0.0862(0.0156)	0.2685(0.1229)
	CS3	93.13	96.30	95.79	0.0197(0.1755)	0.0792(0.0160)	0.0375(0.1451)
	CS1	93.79	95.65	93.12	0.0208(0.2417)	0.0698(0.0093)	0.2298(0.0856)
(100,40)	CS2	94.06	97.04	96.71	0.0425(0.2456)	0.0724(0.0116)	0.2126(0.1073)
	CS3	92.87	96.15	94.89	0.0461(0.2799)	0.0655(0.0145)	0.3121(0.1213)
	CS1	95.14	97.49	98.10	0.0201(0.1724)	0.0624(0.0098)	0.3071(0.0769)
(100,60)	CS2	96.19	98.06	97.62	0.0339(0.2779)	0.0892(0.0108)	0.2076(0.0943)
	CS3	94.92	97.12	96.56	0.0346(0.2044)	0.0690(0.0137)	0.1389(0.1192)

Table 3: The AB, MSE and CP of the confidence intervals for parameters β,θ,α
$(\beta = 1.2, \theta = 12, \alpha = 1.8, T_0 = 30, \tau = 14, 1-\gamma = 0.95)$

(n, m)	CS	CP (%)			AB (MSE)		
		β	θ	α	β	θ	α
	CS1	91.74	95.34	93.07	0.0154(0.1909)	0.0807(0.0158)	0.2063(0.1639)
(80,40)	CS2	93.06	96.35	94.81	0.0189(0.1797)	0.0837(0.0152)	0.1317(0.1571)
	CS3	91.44	95.95	95.18	0.0174(0.1695)	0.0818(0.0148)	0.1175(0.1537)
(80,50)	CS1	92.98	96.65	94.94	0.0121(0.1806)	0.0846(0.0137)	0.1056(0.1417)

	CS2	94.26	97.20	95.78	0.0143(0.1671)	0.0731(0.0146)	0.1030(0.1550)
	CS3	93.14	96.50	95.53	0.0167(0.1576)	0.0801(0.0141)	0.1135(0.1601)
	CS1	91.89	96.58	94.25	0.0158(0.1598)	0.0652(0.0098)	0.0530(0.1796)
(100,40)	CS2	93.15	96.22	95.45	0.0162(0.1120)	0.0872(0.0137)	0.1020(0.1567)
	CS3	92.27	95.47	94.98	0.0085(0.1016)	0.0690(0.0089)	0.0652(0.1609)
	CS1	93.51	97.80	95.89	0.0112(0.1439)	0.0568(0.0090)	0.0461(0.1614)
(100,60)	CS2	94.01	98.36	96.56	0.0129(0.1325)	0.0714(0.0105)	0.0983(0.1349)
	CS3	95.84	97.96	96.23	0.0078(0.1218)	0.0614(0.0056)	0.0584(0.1421)

Table 4: The AB, MSE and CP of the confidence intervals for parameters β,θ,α

$(\beta=1.2,\theta=12,\alpha=1.5,T_0=30,\tau=14,1-\gamma=0.95)$

(n,m)	CS	CP (%)			AB (MSE)		
		β	θ	α	β	θ	α
	CS1	89.42	92.26	93.25	0.0146(0.2022)	0.1003(0.0202)	0.2196(0.2307)
(80,40)	CS2	91.27	94.05	94.04	0.0187(0.2153)	0.1016(0.0195)	0.1146(0.1804)
	CS3	90.56	91.69	92.84	0.0103(0.2152)	0.0979(0.0190)	0.1058(0.1749)
	CS1	90.53	93.56	94.96	0.0121(0.1769)	0.0990(0.0191)	0.1294(0.1873)
(80,50)	CS2	92.19	96.87	95.41	0.0165(0.2043)	0.0981(0.0176)	0.1024(0.1612)
	CS3	91.79	94.82	96.30	0.0097(0.1938)	0.0979(0.0180)	0.0983(0.1571)
	CS1	90.71	93.58	95.02	0.0153(0.1914)	0.0807(0.0130)	0.1725(0.2363)
(100,40)	CS2	92.70	95.82	93.46	0.0154(0.2075)	0.0789(0.0126)	0.1039(0.1334)
	CS3	91.36	93.23	95.87	0.0138(0.2035)	0.0816(0.0136)	0.1604(0.2751)
	CS1	91.61	94.94	96.15	0.0172(0.1522)	0.0712(0.0112)	0.1065(0.1843)
(100,60)	CS2	93.18	97.15	96.51	0.0161(0.1980)	0.0808(0.0104)	0.0994(0.1297)
	CS3	92.46	95.87	97.04	0.0048(0.2010)	0.0804(0.0125)	0.0709(0.2316)

Table 5: The AB, MSE and CP of the confidence intervals for parameters β,θ,α

$(\beta=1.2,\theta=13,\alpha=1.8,T_0=30,\tau=14,1-\gamma=0.95)$

(n,m)	CS	CP (%)			AB (MSE)		
		β	θ	α	β	θ	α
	CS1	90.12	93.07	92.15	0.0152(0.1473)	0.0901(0.0162)	0.3457(0.1846)
(80,40)	CS2	91.25	94.37	92.21	0.0082(0.2001)	0.0924(0.0168)	0.1265(0.1949)
	CS3	91.97	93.32	91.06	0.0248(0.2117)	0.0895(0.0159)	0.1976(0.1715)
	CS1	91.27	94.57	93.01	0.0142(0.1135)	0.0942(0.0158)	0.2217(0.1659)
(80,50)	CS2	92.07	96.07	95.78	0.0064(0.1917)	0.0900(0.0141)	0.0876(0.1529)
	CS3	92.75	96.55	95.10	0.0143(0.1745)	0.0897(0.0147)	0.0142(0.1603)
	CS1	91.05	94.63	93.05	0.0145(0.0961)	0.0720(0.0112)	0.2282(0.1524)
(100,40)	CS2	92.44	95.95	94.95	0.0074(0.0862)	0.0729(0.0106)	0.1772(0.1712)
	CS3	91.27	93.59	92.67	0.3874(0.0695)	0.0731(0.0107)	0.1319(0.1456)
	CS1	92.42	95.78	94.08	0.0129(0.0461)	0.0761(0.0102)	0.1934(0.1387)
(100,60)	CS2	93.81	97.17	96.73	0.0086(0.0408)	0.0729(0.0096)	0.1569(0.1416)
	CS3	93.10	95.45	96.39	0.0220(0.0315)	0.0718(0.0089)	0.1181(0.1245)

Table 6: The results of optimal plans with different sample sizes, failure proportions, and choices of initial values in SSPALT

(n, m)	Initial values	τ^*	n_1^*	n_2^*
	$IV1$	7.2466	19	6
(50,25)	$IV2$	7.7290	18	7
	$IV3$	9.0607	22	3
(50,30)	$IV1$	20.9838	16	14

	$IV2$	12.1384	20	10
	$IV3$	37.2389	18	12
	$IV1$	8.8571	32	8
(80,40)	$IV2$	6.9054	29	11
	$IV3$	8.6607	32	8
	$IV1$	19.8794	30	30
(80,60)	$IV2$	35.6349	30	30
	$IV3$	21.0704	28	32

7. Conclusions

This paper studies the parameter estimation and optimal plan for the step-stress partially accelerated life test with progressive type-I hybrid censoring. The test items are assumed to follow Pareto distribution. The MLEs together with the asymptotic confidence intervals of the parameters are obtained, and their performances are evaluated by AB, MSE and CP, respectively. The optimal plan was constructed via the stress change time τ^* from normal use condition to accelerated condition. As is shown in the numerical results, appropriate censoring times and schemes as well as relatively large sample sizes and failure proportions contribute much to a high level of estimation accuracy.

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