

## **Heat and Mass Transfer Flow Past a Vertical Porous Plate in the Presence of Radiation**

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### **ABSTRACT**

An analysis is made on the three dimensional mixed convection and mass transfer flow past a vertical porous plate in the presence of radiation with uniform free stream velocity. The solutions have been obtained for the velocity, temperature and concentration fields, shear stresses, rate of heat transfer and mass transfer using perturbation technique. It is found that the primary velocity decrease with the increase of both radiation parameter and Schmidt number but increase with the increase of Grashoff number as well as mass Grashoff number. The temperature distribution decrease with the increase of both radiation parameter and Reynolds number. The Concentration field also decrease with the increase of both Schmidt number as well as Reynolds number. The shear stresses, the rate of heat transfer and mass transfer which are of physical interest are presented in the form of tables.

**Keywords.** Three-dimensional, radiation, incompressible, permeability, periodic-suction.

### **1. Introduction**

Free convective flow with heat and mass transfer has been a subject of interest of many reseachers due to its day-to day application in science and technology. Such phenomenon are observed in buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies such as earth, etc. Guria and Jana [1] studied the unsteady three dimensional flow and heat transfer along a porous vertical plate subjected to a periodic suction velocity distribution. Singh et al. [2] studied the flow of viscous incompressible fluid along an infinite porous plate subject to the sinusoidal suction velocity distribution fluctuating with time. Sing [3] extended this idea by applying transverse sinusoidal suction velocity in the presence of viscous dissipative heat. Guria and Jana [4] also have studied the effect of periodic suction on three dimensional vertical channel flow. Due to the periodic suction the flow becomes three dimensional In the above studies the radiation effect is ignored. It has important application in space vehicle re-entry problems. Many processes in engineering areas occur at high temperatures and it is important for the design of pertinent equipment. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft missiles, satellites and space vehicles are example of such engineering areas. At high temperature radiation effect can be quite significant. The heating of rooms and buildings by the use of radiators is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens etc surrounded by cooler air, are at least in part due to free convection. Hassan [5] and Raptis and Perdikis [6] studied the effect of

M. Guria

radiation on the flow of micropolar and viscoelastic fluid respectively. Seddeek [7] also studied the effect of radiation past a moving plate with variable viscosity. The effect of radiation on the flow past a vertical plate was discussed by Takhar et al. [8]. Guria et al. [9] investigated the effect of radiation on three dimensional flow in a vertical channel subjected to a periodic suction. Guria et al. [10] also studied the effect of radiation on steady three dimensional flow past a vertical porous plate in the presence of transverse magnetic field.

Sing and Thakar [11] discussed the effect of periodic suction on three dimensional mixed convection and mass transfer flow. Ahmed [12] also studied the effects of heat and mass transfer on the steady three dimensional flow of a viscous incompressible fluid along a moving vertical plate. Ahmed and Liu [13] studied the effects of heat and mass transfer on three dimensional flow past a vertical porous plate with uniform free stream velocity. Reddy and Reddy [14] studied radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate with viscous dissipation. However, the interaction of radiation with mass transfer in three dimension combined convection flow has received little attention.

The aim of this paper is to study the effect of radiation on three dimensional mixed convection flow and mass transfer past a vertical porous plate subject to the periodic suction velocity distribution with uniform free stream velocity.

## 2. Formulation of the problem

Consider the unsteady heat and mass transfer flow of viscous, incompressible fluid past a semi- infinite vertical porous plate. Here the  $x^*$ -axis is chosen along the vertical plate, that is, in the direction of the flow,  $y^*$ -axis is perpendicular to the plate and  $z^*$ -axis is normal to the  $x^*y^*$ -plane. All the fluid properties are considered constant except the influence of the density variation with temperature is considered only in the body force term. The plate is considered to be infinite length, all derivatives with respect to  $x^*$  vanish and so the physical variables are functions of  $y^*, z^*$ , and  $t^*$  only.

The plate is subjected to a periodic suction velocity distribution of the form

$$v^* = -V_0 \left[ 1 + \varepsilon \cos \left( \frac{\pi z^*}{l} - \omega^* t^* \right) \right], \quad (1)$$

where  $\varepsilon (\leq 1)$  is the amplitude of the suction velocity.  $V_0$  is the constant suction,  $\nu$  is the kinematic coefficient of viscosity and  $t^*$  is the time. Denoting velocity components  $u^*, v^*, w^*$  in the directions  $x^*, y^*$  and  $z^*$  axes respectively, under Bousinesq approximation, the flow is governed by the following equations

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \quad (2)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu \left( \frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) + g\beta(T^* - T_\infty) + g\beta(C^* - C_\infty), \quad (3)$$

Heat and mass transfer flow past a vertical porous plate in the presence of radiation

$$\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \left( \frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right), \quad (4)$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} + \nu \left( \frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}} \right), \quad (5)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}} \right) - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial y^*}, \quad (6)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} = D \left( \frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}} \right), \quad (7)$$

where  $\rho$  is the density of the fluid,  $p^*$  is the fluid pressure,  $g$  is the acceleration due to gravity,  $\beta$  is the coefficient of thermal expansion,  $k$  is the coefficient of heat conduction,  $C_p$  is the specific heat at constant pressure  $K^*$  is the permeability of the porous medium. The equation of conservation of radiative heat transfer per unit volume for all wavelength is

$$\nabla \cdot q_r^* = \int_0^\infty K_\lambda(T^*) (4e_{\lambda h}(T^*) - G_\lambda) d\lambda,$$

where  $e_{\lambda h}$  is the Plank's function and the incident radiation  $G_\lambda$  is defined as

$$G_\lambda = \frac{1}{\pi} \int_{\Omega=4\pi} e_\lambda(\Omega) d\Omega,$$

$\nabla \cdot q_r^*$  is the radiative flux divergence and  $\Omega$  is the solid angle. Now, for an optically thin fluid exchanging radiation with an isothermal flat plate at temperature  $T_0$  and according to the above definition for the radiative flux divergence and Kirchhoffs law, the incident radiation is given by  $G_\lambda = 4e_{\lambda h}(T_0)$  then,

$$\nabla \cdot q_r^* = 4 \int_0^\infty K_\lambda(T^*) (e_{\lambda h}(T^*) - e_{\lambda h}(T_0)) d\lambda,$$

Expanding  $K_\lambda(T^*)$  and  $e_{\lambda h}(T_0)$  in a Taylor series around  $T_0$ , for small  $(T^* - T_0)$ , we can rewrite the radiative flux divergence as

$$\nabla \cdot q_r^* = 4(T^* - T_0) \int_0^\infty K_{\lambda_0} \left( \frac{\partial e_{\lambda h}}{\partial T} \right)_0 d\lambda,$$

where  $K_{\lambda_0} = K_{\lambda(T_0)}$ .

Hence an optical thin limit for a non-gray gas near equilibrium, the following relation holds

$$\nabla \cdot q_r^* = 4(T^* - T_0) I,$$

and hence

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_0) I,$$

M. Guria

where  $I = \int_0^\infty K_{\lambda_0} \left( \frac{\partial e_{\lambda h}}{\partial T} \right)_0 d\lambda$ .

The boundary conditions of the problem are

$$u^* = 0, v^* = -V_0 \left[ 1 + \varepsilon \cos \left( \frac{\pi z^*}{l} - \omega^* t^* \right) \right], w^* = 0, T^* = T_w, C^* = C_w \text{ at } y^* = 0,$$

$$u^* = U_0, v^* = -V_0, w^* = 0, p^* = p_0, T^* = T_0, C^* = C_0 \text{ as } y^* \rightarrow \infty. \quad (8)$$

Introduce the non-dimensional variables

$$y = \frac{y^*}{l}, z = \frac{z^*}{l}, t = \omega^* t^*, p = \frac{p^* l^2}{\rho V^2},$$

$$u = \frac{u^*}{V_0}, v = \frac{v^*}{V_0}, w = \frac{w^*}{V_0}, \theta = \frac{(T^* - T_0)}{T_w - T_0}, C = \frac{(C^* - C_0)}{C_w - C_0}. \quad (9)$$

Using (2.9), equations (2.2)-(2.7) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (10)$$

$$\omega \frac{\partial u}{\partial t} + Re \left( v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + Re(Gr\theta + G_m C), \quad (11)$$

$$\omega \frac{\partial v}{\partial t} + Re \left( v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{1}{Re} \frac{\partial p}{\partial y} + \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \quad (12)$$

$$\omega \frac{\partial w}{\partial t} + Re \left( v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{1}{Re} \frac{\partial p}{\partial z} + \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad (13)$$

$$\omega \frac{\partial \theta}{\partial t} + Re \left( v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) - FRe\theta, \quad (14)$$

$$\omega \frac{\partial C}{\partial t} + Re \left( v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} \right) = \frac{1}{S} \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right), \quad (15)$$

where  $Gr = \frac{g\beta(T_w - T_0)\nu}{V_0^2}$ , the Grashoff number,  $G_m = \frac{g\beta(C_w - C_0)l}{V_0^2}$ , mass Grashoff

number,  $\omega = \frac{cV}{u_\infty^2}$ , the frequency parameter and  $Pr = \frac{\nu}{\alpha}$ , the Prandtl number,

$F = \frac{4Il}{\rho C_p V_0}$ , the radiation parameter,  $S = \frac{\nu}{D}$ , the Schmidt number,  $Re = \frac{V_0 l}{\nu}$ , the

Reynolds number. The boundary conditions (2.8) become

$$u = 0, w = 0, v = -[1 + \varepsilon \cos(\pi z - t)], \theta = 1, C = 1 \text{ at } y = 0,$$

$$u = U, w = 0, v = -1, \theta = 0, C = 0 \text{ as } y \rightarrow \infty, \quad (16)$$

Heat and mass transfer flow past a vertical porous plate in the presence of radiation where  $U = U_0/V_0$ , is the non-dimensional free stream velocity.

### 3. Solution of the problem

To solve the equations (10)-(15), we assume the solution of the following form

$$\begin{aligned} u(y, z, t) &= u_0(y) + \varepsilon u_1(y, z, t) + \varepsilon^2 u_2(y, z, t) + \dots, \\ v(y, z, t) &= v_0(y) + \varepsilon v_1(y, z, t) + \varepsilon^2 v_2(y, z, t) + \dots, \\ w(y, z, t) &= w_0(y) + \varepsilon w_1(y, z, t) + \varepsilon^2 w_2(y, z, t) + \dots, \\ p(y, z, t) &= p_0(y) + \varepsilon p_1(y, z, t) + \varepsilon^2 p_2(y, z, t) + \dots, \\ \theta(y, z, t) &= \theta_0(y) + \varepsilon \theta_1(y, z, t) + \varepsilon^2 \theta_2(y, z, t) + \dots, \\ C(y, z, t) &= C_0(y) + \varepsilon C_1(y, z, t) + \varepsilon^2 C_2(y, z, t) + \dots, \end{aligned} \quad (17)$$

Substituting (17) in equations (10) to (15), comparing the term free from  $\varepsilon$  and the coefficients of  $\varepsilon$  from both sides and neglecting those of  $\varepsilon^2$ . The term free from  $\varepsilon$  are

$$v_0' = 0, \quad (18)$$

$$u_0'' + Re(Gr\theta_0 + GmC_0) + Rev_0 u_0' = 0, \quad (19)$$

$$\theta_0'' - RePrv_0 \theta_0' - FRePr\theta_0 = 0, \quad (20)$$

$$C_0'' - ReSv_0 \theta_0' = 0, \quad (21)$$

where the primes denote differentiation with respect to  $y$ .

The boundary conditions become

$$\begin{aligned} u_0 = 0, v_0 = -1, w_0 = 0, \theta_0 = 1, C_0 = 1 \quad \text{at } y = 0, \\ u_0 = 1, v_0 = -1, w_0 = 0, \theta_0 = 0, C_0 = 0 \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (22)$$

The solutions of (18)- (21) under the boundary conditions (22) are

$$\begin{aligned} v_0(y) &= -1, \quad \theta_0(y) = e^{-\lambda_1 y}, \quad C_0(y) = e^{-SRey} \\ u_0(y) &= U(1 - e^{-Rey}) + A_1(e^{-\lambda_1 y} - e^{-Rey}) + A_2(e^{-SRey} - e^{-Rey}), \end{aligned}$$

$$\text{where, } \lambda_1 = \frac{ReP_r + \sqrt{Re^2 Pr^2 + 4FRePr}}{2}, \quad (23)$$

$$A_1 = \frac{-ReGr}{\lambda_1^2 - Re\lambda_1}, \quad A_2 = \frac{-Gm}{SRe(S-1)}.$$

Equating the coefficient of  $\varepsilon$  from both sides, we get

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad (24)$$

$$\omega \frac{\partial u_1}{\partial t} + Re \left( v_1 \frac{du_0}{dy} - \frac{\partial u_1}{\partial y} \right) = \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) + Re(Gr\theta_1 + GmC_1), \quad (25)$$

$$\omega \frac{\partial v_1}{\partial t} - Re \frac{\partial v_1}{\partial y} = -\frac{1}{Re} \frac{\partial p_1}{\partial y} + \left( \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right), \quad (26)$$

M. Guria

$$\omega \frac{\partial w_1}{\partial t} - Re \frac{\partial w_1}{\partial y} = -\frac{1}{Re} \frac{\partial p_1}{\partial z} + \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right), \quad (27)$$

$$\omega \frac{\partial \theta_1}{\partial t} + Re \left( v_1 \frac{d\theta_0}{dy} - \frac{\partial \theta_1}{\partial y} \right) = \frac{1}{Pr} \left( \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right) - FRe \theta_1, \quad (28)$$

$$\omega \frac{\partial C_1}{\partial t} + Re \left( v_1 \frac{dC_0}{dy} - \frac{\partial C_1}{\partial y} \right) = \frac{1}{S} \left( \frac{\partial^2 C_1}{\partial y^2} + \frac{\partial^2 C_1}{\partial z^2} \right). \quad (29)$$

The boundary conditions become

$$u_1 = 0, v_1 = -\cos(\pi z - t), w_1 = 0, \theta_1 = 0, C_1 = 0 \quad \text{at } y = 0, \\ u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0, C_1 = 0 \quad \text{as } y \rightarrow \infty. \quad (30)$$

These are the linear partial differential equations describing the three-dimensional flow. We assume the velocity components, pressure and temperature distribution in the following form

$$u_1(y, z, t) = u_{11}(y) e^{i(\pi z - t)}, \quad v_1(y, z, t) = v_{11}(y) e^{i(\pi z - t)}, \\ w_1(y, z, t) = \frac{-1}{i\pi} v'_{11}(y) e^{i(\pi z - t)}, \quad p_1(y, z, t) = p_{11}(y) e^{i(\pi z - t)}, \quad (31)$$

$$\theta_1(y, z, t) = \theta_{11}(y) e^{i(\pi z - t)}, \quad C_1(y, z, t) = C_{11}(y) e^{i(\pi z - t)},$$

Substituting (3.31) in (25)-(29), we get the following set of differential equations

$$v''_{11} + Rev'_{11} - (\pi^2 - i\omega) v_{11} = \frac{1}{Re} p'_{11}, \quad (32)$$

$$v'''_{11} + Rev''_{11} - (\pi^2 - i\omega) v'_{11} = \frac{\pi^2}{Re} p_{11}, \quad (33)$$

$$\theta''_{11} + RePr\theta'_{11} - (FRePr + \pi^2 - iPr\omega) \theta_{11} = RePrv_{11}\theta'_0, \quad (34)$$

$$u''_{11} + Reu'_{11} - (\pi^2 - i\omega) u_{11} = Rev_{11}u'_0 - Re(Gr\theta_{11} + GmC_1), \quad (35)$$

$$C''_{11} + SReC'_{11} - (\pi^2 - iS\omega) C_{11} = SRev_{11}C'_0 \quad (36)$$

The boundary conditions become

$$u_{11} = 0, \quad v_{11} = -1, \quad v'_{11} = 0, \quad \theta_{11} = 0, \quad C_{11} = 0 \quad \text{at } y = 0, \\ u_{11} = 0, \quad v_{11} = 0, \quad v'_{11} = 0, \quad \theta_{11} = 0, \quad C_{11} = 0 \quad \text{as } y \rightarrow \infty. \quad (37)$$

Solving (32)- (36), under the boundary conditions (37), and on using (33), we get

$$v_1(y, z, t) = \frac{1}{(\lambda - \pi)} \left[ \pi e^{-\lambda y} - \lambda e^{-\pi y} \right] e^{i(\pi z - t)},$$

$$w_1(y, z, t) = \frac{\lambda}{i(\lambda - \pi)} \left[ e^{-\lambda y} - e^{-\pi y} \right] e^{i(\pi z - t)},$$

$$\theta_1(y, z, t) = \left[ A_3 \left\{ e^{-(\lambda + \lambda_1)y} - e^{-\mu y} \right\} + A_4 \left\{ e^{-(\pi + \lambda_1)y} - e^{-\mu y} \right\} \right] e^{i(\pi z - t)},$$

$$C(y, z, t) = \left[ A_5 \left\{ e^{-(\lambda + SRe)y} - e^{-\lambda_2 y} \right\} + A_6 \left\{ e^{-(\pi + SRe)y} - e^{-\lambda_2 y} \right\} \right] e^{i(\pi z - t)},$$

Heat and mass transfer flow past a vertical porous plate in the presence of radiation

$$\begin{aligned}
 u_1(y, z, t) = & \left[ B_1 \left\{ e^{-(\lambda+\lambda_1)y} - e^{-\lambda y} \right\} + B_2 \left\{ e^{-\mu y} - e^{-\lambda y} \right\} \right. \\
 & + B_3 \left\{ e^{-(\pi+\lambda_1)y} - e^{-\lambda y} \right\} + B_4 \left\{ e^{-(\lambda+SRe)y} - e^{-\lambda y} \right\} \\
 & \left. B_5 \left\{ e^{-\lambda_2 y} - e^{-\lambda y} \right\} + B_6 \left\{ e^{-(\pi+SRe)y} - e^{-\lambda y} \right\} + B_7 \left\{ e^{-(\pi+Re)y} - e^{-\lambda y} \right\} + B_8 \left\{ e^{-(\lambda+Re)y} - e^{-\lambda y} \right\} \right] e^{i(\pi z - t)},
 \end{aligned}
 \tag{38}$$

where

$$\begin{aligned}
 \lambda &= \frac{Re + \sqrt{Re^2 + 4(\pi^2 - i\omega)}}{2}, \\
 \mu &= \frac{RePr_r + \sqrt{Re^2 Pr^2 + 4(FRePr + \pi^2 - iPr\omega)}}{2}, \\
 \lambda_2 &= \frac{SRe + \sqrt{S^2 Re^2 + 4(\pi^2 - iS\omega)}}{2}, \\
 K &= \frac{RePr\lambda_1}{(\lambda - \pi)} \quad K_1 = \frac{-S^2 Re^2}{(\lambda - \pi)} \\
 A_3 &= \frac{-K\pi}{(\lambda + \lambda_1)^2 - RePr(\lambda + \lambda_1) - (FRePr + \pi^2 - iPr\omega)}, \\
 A_4 &= \frac{K\lambda}{(\pi + \lambda_1)^2 - RePr(\pi + \lambda_1) - (FRePr + \pi^2 - iPr\omega)}, \\
 A_5 &= \frac{K_1\pi}{(\lambda^2 - \pi^2 + iS\omega)}, \quad A_6 = \frac{-K_1\lambda}{S(\pi Re + i\omega)}, \\
 B_1 &= \frac{-(ReGrA_3 + K_2\pi A_1\lambda)}{\lambda_1(\lambda_1 + 2\lambda - Re)}, \\
 B_2 &= \frac{ReGr(A_3 + A_4)}{(\mu^2 - \mu Re - (\pi^2 - i\omega))}, \\
 B_3 &= \frac{-(ReGrA_4 - K_2\lambda A_1\lambda_1)}{(\pi + \lambda_1)^2 - Re(\pi + \lambda_1) - (\pi^2 - i\omega)}, \\
 B_4 &= \frac{-(ReGmA_5 + K_2\pi A_2 SRe)}{(2\lambda SRe + S^2 Re^2 - SRe^2)}, \\
 B_5 &= \frac{ReGm(A_5 + A_6)}{(\lambda_2^2 - \lambda_2 Re - (\pi^2 - i\omega))}, \\
 B_6 &= \frac{-(ReGmA_6 - K_2\lambda A_2 SRe)}{(\pi + SRe)^2 - Re(\pi + SRe) - (\pi^2 - i\omega)}, \\
 B_7 &= \frac{-K_2\lambda Re(U + A_1 + A_2)}{(\pi Re + i\omega)},
 \end{aligned}
 \tag{39}$$

M. Guria

$$B_8 = \frac{K_2 \pi Re(U + A_1 + A_2)}{\lambda + Re)^2 - Re(\lambda + Re) - (\pi^2 - i\omega)}.$$

#### 4. Results and discussion

I have computed the numerical value of the velocity, temperature, shear stresses, and rate of heat and mass transfer for different values of the non dimensional parameters and plotted in the diagram. The value of dimensionless parameter  $Gr$  is taken as positive. The positive value corresponds to an extremely cooled plate by the free convection currents. The value of Prandtl number is taken equal to 0.71 and this value corresponds to the air. The values of Grashof numbers ( $Gr$ ) are taken to be large from the physical point of view. The large Grashof number values correspond to free convection problem. The Schmidt number ( $S$ ) are taken for helium ( $S=0.3$ ), water vapor ( $S=0.60$ ), oxygen ( $S=0.66$ ) and ammonia ( $S=0.78$ ). The effect of radiation parameter, Prandtl number, permeability parameter and suction parameter on main flow velocity is shown in Figs.2-5. The effect of radiation parameter  $F$  on the primary velocity is shown in Fig.2. It is seen that the primary velocity decreases with the increase of the radiation parameter. Figs.3. and 4 show the effect of thermal Grashoff number and mass Grashoff number on the primary velocity profile. It is noticed that the velocity profile increase with the increase of the both thermal and mass Grashoff number. Fig.5 shows the effects of Schmidt number on the primary velocity. It is noticed that the fluid velocity decreases with the increase of Schmidt number. Figs.6 and 7 show the effect of  $F$  and  $Re$  on the temperature profile. It is clear that temperature decrease with the increase of both  $F$  and  $Re$ . Therefore using radiation we can control the flow characteristic and temperature distribution. In Figs. 8 and 9 I have presented the concentration field for several values of Schmidt number and Reynolds number. It is found that the concentration field decrease with increase in both Schmidt number and Reynolds number. The non dimensional shear stress component due to primary flow can be expressed as

$$\begin{aligned} \tau_x &= \left( \frac{\partial u}{\partial y} \right)_{y=0} = u'_0(0) + \varepsilon u'_1(0) = u'_0(0) + \varepsilon u'_{11}(0) e^{i(\pi z - t)}, \\ &= Re(U + A_1 + A_2) - A_2 S Re - A_1 \lambda_1 \\ &+ \varepsilon [t_r \cos(\pi z - t) - t_i \sin(\pi z - t)]. \end{aligned} \quad (40)$$

The shear stress due to primary flow in terms of  $\tau_x$  are given in Table.1 for several values of frequency parameter and radiation parameter. From the Table.1 I observe that  $\tau_x$  decrease with the increase of both frequency parameter or radiation parameter. In Table.2 I have presented  $\tau_x$  for several values of Schmidt number and mass Grashoff number. It is found that  $\tau_x$  decreases with the increase of Schmidt number but increases with the increase of mass Grashoff number.



Heat and mass transfer flow past a vertical porous plate in the presence of radiation

	$\tau_x$			
$F \backslash \omega$	2	3	4	5
5	31.32	26.03	25.71	25.43
6	31.07	25.91	25.58	25.29
7	30.81	25.79	25.46	25.16
8	30.56	25.67	25.34	25.04

**Table 1:** Shear stress component due to primary flow for  $Gr = 5.0, Gm = 5, Re = 5.0, U = 1, S = 0.3, z = 0, t = 0.5$ .

	$\tau_x$			
$S \backslash Gm$	2	3	4	5
0.3	21.55	24.80	28.06	31.32
0.60	18.37	20.03	21.69	23.35
0.66	18.24	19.84	21.44	23.04
0.78	17.89	19.31	20.73	22.16

**Table 2:** Shear stress component due to primary flow for  $Gr = 5.0, \omega = 10, F = 2, Re = 5.0, U = 1, z = 0, t = 0.5$ .

Now we calculate the rate of heat transfer. The rate of heat transfer at the plate  $y = 0$  is given by

$$\begin{aligned}
 Nu &= \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = -\theta'_0(0) - \varepsilon \theta'_1(0), \\
 &= -\theta'_0(0) - \varepsilon \theta'_{11}(0) e^{i(\pi z - t)}, \\
 &= \lambda_1 - \varepsilon \left[ \theta_r \cos(\pi z - t) - \theta_i \sin(\pi z - t) \right]. \tag{41}
 \end{aligned}$$

The non-dimensional mass flux at the plate  $y = 0$  in terms of Sherwood number  $Sh$  is

$$\begin{aligned}
 Sh &= \left( \frac{\partial C}{\partial y} \right)_{y=0} = -C'_0(0) - \varepsilon C'_1(0), \\
 &= -C'_0(0) - \varepsilon C'_{11}(0) e^{i(\pi z - t)}, \\
 &= SRe - \varepsilon \left[ C_r \cos(\pi z - t) + C_i \sin(\pi z - t) \right]. \tag{42}
 \end{aligned}$$

In Table 3 I have presented the rate of heat transfer in terms of Nusselt number  $Nu$  for  $F = 2$  and rate of mass transfer in terms of Sherwood number. It is observed that  $Nu$  increase with the increase of both Prandtl number as well as Reynolds number.  $Sh$  also increase frequency parameter and radiation parameter the increase in both Schmidt number as well as Reynolds number.

$Pr$ $Re$	$Nu$				$S$ $Re$	$Sh$			
	0.025	0.71	7.0	11.4		0.30	0.60	0.66	0.78
2	0.34	2.47	14.48	22.10	2	0.59	1.18	1.30	1.53
3	0.42	3.27	19.42	28.20	3	0.89	1.78	1.96	2.31
4	0.50	4.03	23.28	31.23	4	1.20	2.39	2.63	3.10
5	0.56	4.76	26.19	31.63	5	1.50	3.00	3.30	3.90

**Table 3:** Nusselt number and Sherwood number for  $\omega = 10$ ,  $z = 0$ ,  $t = 0.5$ .

### 5. Conclusion

In this paper I have studied the three dimensional mixed convection and mass transfer flow past a vertical porous plate in the presence of radiation with uniform free stream velocity. The solutions have been obtained for the velocity, temperature and concentration fields, shear stresses, rate of heat transfer and mass transfer using perturbation technique. It is found that the primary velocity decrease with the increase of both radiation parameter and Schmidt number but increase with the increase of Grashoff number as well as mass Grashoff number. The temperature distribution decrease with the increase of both radiation parameter and Reynolds number. The Concentration field also decrease with the increase of both Schmidt number as well as Reynolds number.

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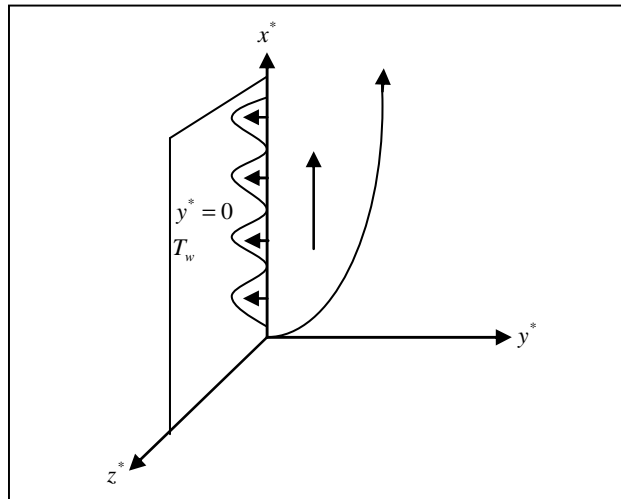
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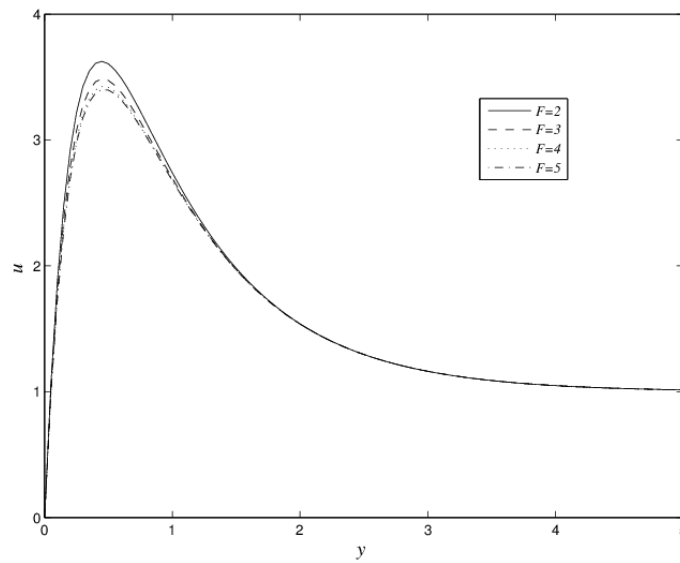
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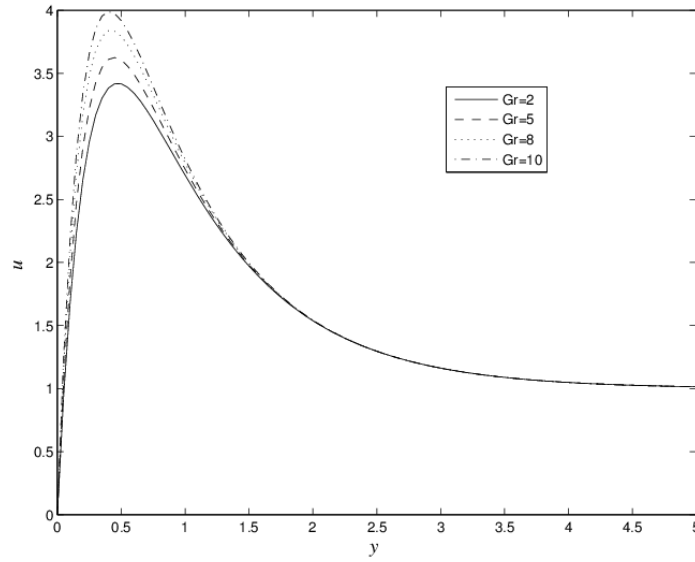


**Figure 1:** Physical model and Co-ordinates system

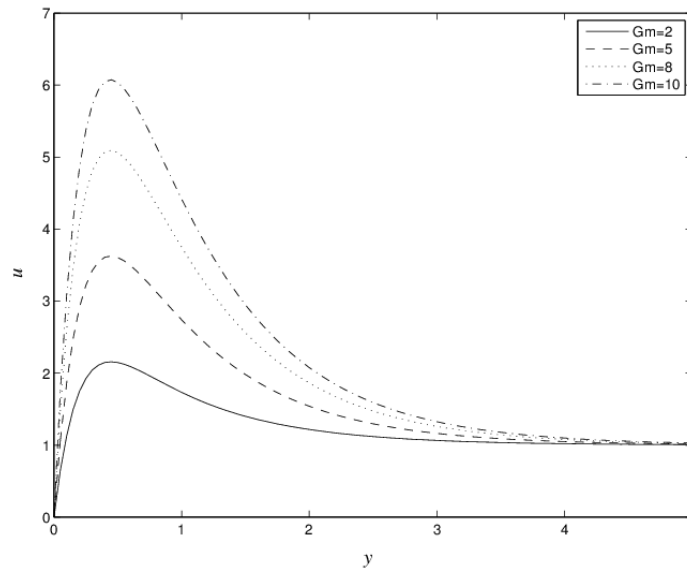


**Figure 2:** Variation of primary velocity for  $Pr = 0.71, Gr = 5, Gm = 5, \omega = 10, Re = 4, S = 0.3, U = 1$

Heat and mass transfer flow past a vertical porous plate in the presence of radiation

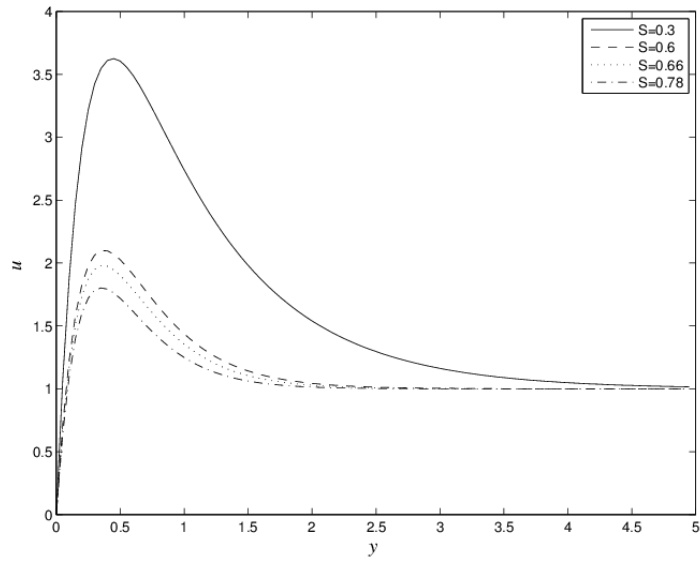


**Figure 3:** Variation of primary velocity for  $Pr = 0.71, F = 2, Gm = 5, \omega = 10, Re = 4, S = 0.3, U = 1$

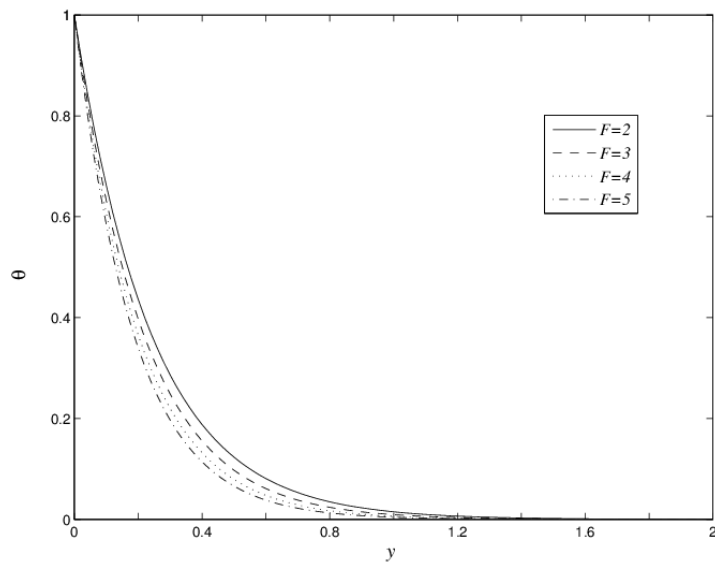


**Figure 4:** Variation of primary velocity for  $Pr = 0.71, Gr = 5, F = 2, \omega = 10, Re = 4, S = 0.3, U = 1$

M. Guria

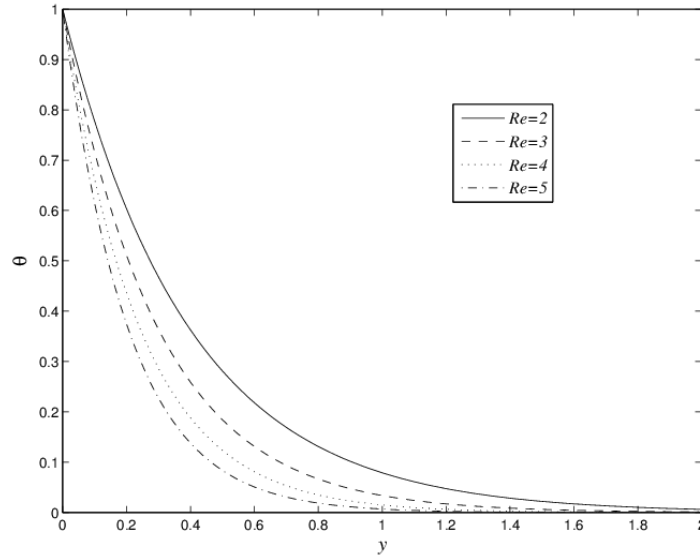


**Figure 5:** Variation of primary velocity for  $Pr = 0.71, Gr = 5, Gm = 5, \omega = 10, Re = 4, F = 2, U = 1$

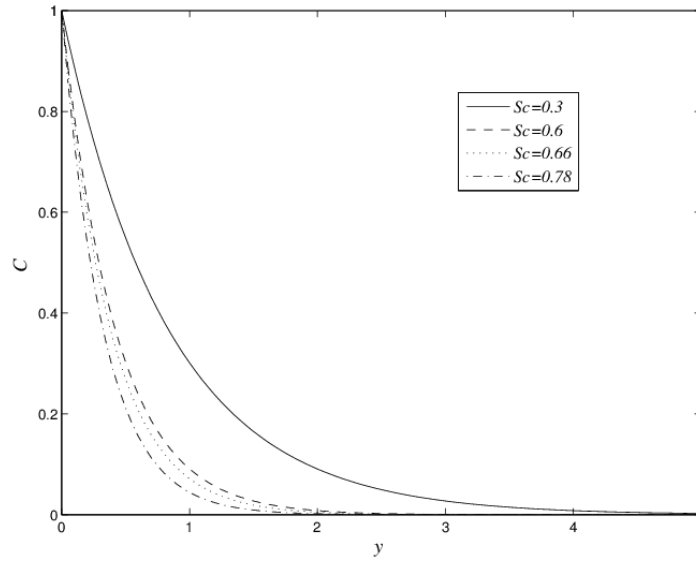


**Figure 6:** Variations of temperature profile for  $Pr = 0.71, \omega = 10, Re = 4$ .

Heat and mass transfer flow past a vertical porous plate in the presence of radiation

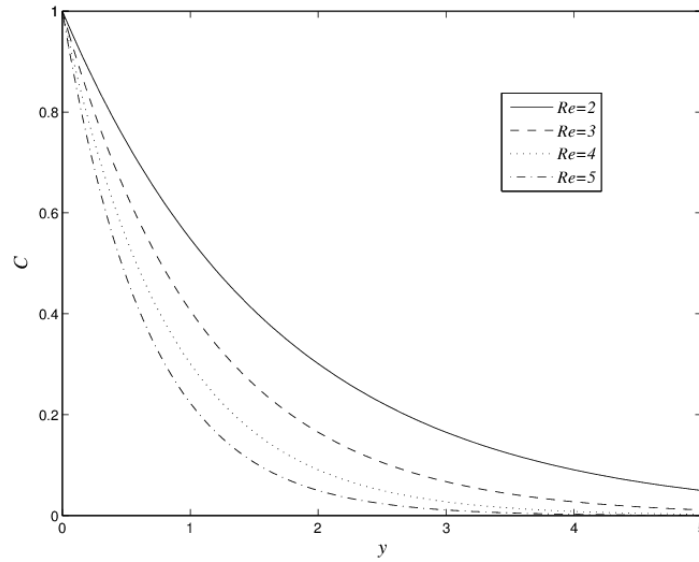


**Figure 7:** Variations of temperature profile for  $Pr = 0.71, \omega = 10, F = 2$ .



**Figure 8:** Variations of concentration field for  $\omega = 10, Re = 4$ .

M. Guria



**Figure 9:** Variations of concentration field for  $\omega = 10, S = 0.3$ .