Journal of Physical Sciences, Vol. 20, 2015, 111-132
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 24 December 2015

Comparative of Two Triangular Fuzzy Sets with α-cut

Salim Rezvani

Department of Mathematics, Imam Khomaini Mritime University of Nowshahr
Nowshahr, Iran, E-mail: *salim_rezvani@yahoo.com*

Received 05 September 2015; accepted 27 November2015

ABSTRACT
In this paper, We are going to show that α-cut method can be used for finding Comparative operation of triangular fuzzy sets.

Keywords: α-cut, triangular fuzzy sets

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [1] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [2]. Yong Sik Yun [3] presented a method for generalized triangular fuzzy sets. α-cut method is a standard method for performing different arithmetic operations like addition, multiplication, division, subtraction. Rezvani [7-17] introduced the concept of relation between Fuzzy Numbers. In [5] and [6] the authors argue that finding membership function for square root of *X* where *X* is a fuzzy number, is not possible by the standard alpha-cut method.

In [4] the authors proposed a method for construction of membership function with using α-cut. In this paper, we are going to show that α-cut method can be used for finding Comparative operation of triangular fuzzy sets.

2. Preliminaries

Definition 1. The set $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ is said to be the α-cut of a fuzzy set *A*.

The membership function of a fuzzy set *A* can be expressed is terms of the characteristic functions of its α-cut according to the formula

$$\mu_A(x) = \sup_{\alpha \in (0,1]} \min\left(\alpha, \mu_{A_\alpha}(x)\right) \tag{1}$$

where

$$\mu_{A_\alpha}(x) = \begin{cases} 1 & if \quad x \in A_\alpha \\ 0 & if \quad otherwise \end{cases} \tag{2}$$

It is easily checked that the following properties hold

$$(A \cup B)_\alpha = A_\alpha \cup B_\alpha, \ (A \cap B)_\alpha = A_\alpha \cap B_\alpha \tag{3}$$

Definition 2. A triangular fuzzy number is a fuzzy number A having membership function

$$\mu_A(x)=\begin{cases}0 & x<a_1, a_3\le x,\\ \dfrac{x-a_1}{a_2-a_1} & a_1\le x\le a_2,\\ \dfrac{a_3-x}{a_3-a_2} & a_2\le x\le a_3.\end{cases} \tag{4}$$

The above triangular fuzzy number is denoted by $A=(a_1,a_2,a_3)$.

Definition 3. We generalize the triangular fuzzy number. A generalized triangular fuzzy set is symmetric and may not have value 1.
A generalized triangular fuzzy set is a symmetric fuzzy set *A* having membership function

$$\mu_A(x)=\begin{cases}0 & x<a_1, a_3\le x,\\ \dfrac{2c(x-a_1)}{a_2-a_1} & a_1\le x<\dfrac{a_1+a_2}{2},\\ \dfrac{-2c(x-a_2)}{a_2-a_1} & \dfrac{a_1+a_2}{2}\le x<a_2.\end{cases} \tag{5}$$

where $a_1,a_2\in R$ and $0<c\le 1$.
The above generalized triangular fuzzy set is denoted by

$$A=((a_1,\mathrm{c},a_2)) \tag{6}$$

Let $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ and $B_\alpha=\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$ are the α-cuts of *A* and *B*, respectively.

Since $\alpha=\dfrac{-2c_1\left(a_1^{(\alpha)}-a_1\right)}{a_2-a_1}$ and $\alpha=\dfrac{-2c_1\left(a_2^{(\alpha)}-a_2\right)}{a_2-a_1}$, we have

$$A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]=\left[\frac{(a_2-a_1)\alpha}{2c_1}+a_1,\frac{(a_2-a_1)\alpha}{-2c_1}+a_2\right] \tag{7}$$

Similarly, we have

$$B_\alpha=\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{(b_2-b_1)\alpha}{2c_2}+b_1,\frac{(b_2-b_1)\alpha}{-2c_2}+b_2\right] \tag{8}$$

3. Addition of fuzzy Sets with using α-cut

Let $A=((a_1,\mathrm{c}_1,b_1))$ and $B=((a_2,\mathrm{c}_2,b_2))$ be two triangular fuzzy sets, Suppose the membership function of *A*, *B* is

$$\mu_A(x)=\begin{cases}0 & x<a_1, b_1\le x,\\ \dfrac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\dfrac{a_1+b_1}{2},\\ \dfrac{-2c_1(x-b_1)}{b_1-a_1} & \dfrac{a_1+b_1}{2}\le x<b_1.\end{cases} \quad (9)$$

And

$$\mu_B(x)=\begin{cases}0 & x<a_2, b_2\le x,\\ \dfrac{2c_2(x-a_2)}{b_1-a_1} & a_2\le x<\dfrac{a_2+b_2}{2},\\ \dfrac{-2c_2(x-b_2)}{b_2-a_2} & \dfrac{a_2+b_2}{2}\le x<b_2.\end{cases} \quad (10)$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right] \quad (11)$$

And

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right] \quad (12)$$

Therefore $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ and $B_\alpha=\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$ are the α-cuts of *A* and *B*, respectively. To calculate addition of fuzzy sets *A* and *B* we first add the α-cuts of *A* and *B* using interval arithmetic.

$$A_\alpha+B_\alpha=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]+\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

$$=\left[\frac{\left[c_2(b_1-a_1)+c_1(b_2-a_2)\right]\alpha}{2c_1c_2}+(a_1+a_2),\frac{\left[c_2(a_1-b_1)+c_1(a_2-b_2)\right]\alpha}{2c_1c_2}+(b_1+b_2)\right] \quad (13)$$

Now, we find the membership function $\mu_{A+B}(x)$

$$x=\frac{\left[c_2(b_1-a_1)+c_1(b_2-a_2)\right]\alpha}{2c_1c_2}+(a_1+a_2)$$

$$\Rightarrow \alpha=\frac{\left[x-(a_1+a_2)\right]2c_1c_2}{c_2(b_1-a_1)+c_1(b_2-a_2)} \qquad a_1+a_2\le x\le c_1+c_2 \quad (14)$$

And

$$x = \frac{\left[c_2\left(a_1 - b_1\right) + c_1\left(a_2 - b_2\right)\right]\alpha}{2c_1c_2} + \left(b_1 + b_2\right)$$

$$\Rightarrow \alpha = \frac{\left[x - \left(b_1 + b_2\right)\right]2c_1c_2}{c_2\left(a_1 - b_1\right) + c_1\left(a_2 - b_2\right)} \qquad c_1 + c_2 \le x \le b_1 + b_2 \qquad (15)$$

Therefore membership function $\mu_{A+B}(x)$ is

$$\mu_{A+B}(x) = \begin{cases} \dfrac{\left[x - \left(a_1 + a_2\right)\right]2c_1c_2}{c_2\left(b_1 - a_1\right) + c_1\left(b_2 - a_2\right)} & a_1 + a_2 \le x \le c_1 + c_2 \\ \dfrac{\left[x - \left(b_1 + b_2\right)\right]2c_1c_2}{c_2\left(a_1 - b_1\right) + c_1\left(a_2 - b_2\right)} & c_1 + c_2 \le x \le b_1 + b_2 \end{cases} \qquad (16)$$

Example 1. Let $A = \left(\left(2, \frac{12}{5}, 5\right)\right)$ and $B = \left(\left(1, \frac{6}{5}, 3\right)\right)$ be two triangular fuzzy sets,

Suppose the membership function of *A*, *B* is

$$\mu_A(x) = \begin{cases} 0 & x < 2,\ 5 \le x, \\ \frac{8}{5}(x-2) & 2 \le x < \frac{7}{2}, \\ -\frac{8}{5}(x-5) & \frac{7}{2} \le x < 5. \end{cases} \qquad \mu_B(x) = \begin{cases} 0 & x < 1,\ 3 \le x, \\ \frac{6}{5}(x-1) & 2 \le x < 2, \\ -\frac{6}{5}(x-3) & 2 \le x < 3. \end{cases}$$

And

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right],$$

$$B_\alpha = \left[a_2^{(\alpha)}, b_2^{(\alpha)}\right] = \left[\frac{5}{6}\alpha + 1,\ -\frac{5}{6}\alpha + 3\right].$$

Therefore

$$A_\alpha + B_\alpha = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right] + \left[\frac{5}{6}\alpha + 1,\ -\frac{5}{6}\alpha + 3\right]$$

$$= \left[\frac{35}{24}\alpha + 3,\ -\frac{35}{24}\alpha + 8\right]$$

Now, with use *(14),(15)* we get

$$x = \frac{35}{24}\alpha + 3 \Rightarrow \alpha = \frac{24}{35}(x - 3) \qquad 3 \le x \le \frac{18}{5}$$

$$x = -\frac{35}{24}\alpha + 8 \Rightarrow \alpha = \frac{24}{35}(8 - x) \qquad \frac{18}{5} \le x \le 8$$

which gives

$$\mu_{A+B}(x)=\begin{cases}\frac{24}{35}(x-3) & 3\le x\le\frac{18}{5},\\ \frac{24}{35}(8-x) & \frac{18}{5}\le x\le 8.\end{cases}$$

4. Subtraction of fuzzy Sets with using α-cut

Let $A=((a_1,c_1,b_1))$ and $B=((a_2,c_2,b_2))$ be two triangular fuzzy sets, Suppose the membership function of *A*, *B* is

$$\mu_A(x)=\begin{cases}0 & x<a_1, b_1\le x,\\ \frac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\frac{a_1+b_1}{2},\\ \frac{-2c_1(x-b_1)}{b_1-a_1} & \frac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

And

$$\mu_B(x)=\begin{cases}0 & x<a_2, b_2\le x,\\ \frac{2c_2(x-a_2)}{b_1-a_1} & a_2\le x<\frac{a_2+b_2}{2},\\ \frac{-2c_2(x-b_2)}{b_2-a_2} & \frac{a_2+b_2}{2}\le x<b_2.\end{cases}$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]$$

And

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

Therefore $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ and $B_\alpha=\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$ are the α-cuts of *A* and *B*, respectively. To calculate subtraction of fuzzy sets *A* and *B* we first add the α-cuts of *A* and *B* using interval arithmetic.

$$A_\alpha - B_\alpha = \left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1, \frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right] - \left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2, \frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

$$= \left[\frac{\left[c_2(b_1-a_1)+c_1(b_2-a_2)\right]\alpha}{2c_1c_2}+(a_1-b_2), \frac{\left[c_2(a_1-b_1)+c_1(a_2-b_2)\right]\alpha}{2c_1c_2}+(b_1-a_2)\right] \quad (17)$$

Now, we find the membership function $\mu_{A-B}(x)$

$$x = \frac{\left[c_2(b_1-a_1)+c_1(b_2-a_2)\right]\alpha}{2c_1c_2}+(a_1-b_2)$$

$$\Rightarrow \alpha = \frac{\left[x-(a_1-b_2)\right]2c_1c_2}{c_2(b_1-a_1)+c_1(b_2-a_2)} \qquad a_1-b_2 \le x \le c_1-c_2 \qquad (18)$$

And

$$x = \frac{\left[c_2(a_1-b_1)+c_1(a_2-b_2)\right]\alpha}{2c_1c_2}+(b_1-a_2)$$

$$\Rightarrow \alpha = \frac{\left[x-(b_1-a_2)\right]2c_1c_2}{c_2(a_1-b_1)+c_1(a_2-b_2)} \qquad c_1-c_2 \le x \le b_1-a_2 \qquad (19)$$

Therefore membership function $\mu_{A-B}(x)$ is

$$\mu_{A-B}(x) = \begin{cases} \dfrac{\left[x-(a_1-b_2)\right]2c_1c_2}{c_2(b_1-a_1)+c_1(b_2-a_2)} & a_1-b_2 \le x \le c_1-c_2 \\[2ex] \dfrac{\left[x-(b_1-a_2)\right]2c_1c_2}{c_2(a_1-b_1)+c_1(a_2-b_2)} & c_1-c_2 \le x \le b_1-a_2 \end{cases} \qquad (20)$$

Example 2. Let $A = \left(\left(2, \frac{12}{5}, 5\right)\right)$ and $B = \left(\left(1, \frac{6}{5}, 3\right)\right)$ be two triangular fuzzy sets,

Suppose the membership function of *A*, *B* is

$$\mu_A(x) = \begin{cases} 0 & x < 2,\ 5 \le x, \\ \frac{8}{5}(x-2) & 2 \le x < \frac{7}{2}, \\ -\frac{8}{5}(x-5) & \frac{7}{2} \le x < 5. \end{cases} \qquad \mu_B(x) = \begin{cases} 0 & x < 1,\ 3 \le x, \\ \frac{6}{5}(x-1) & 2 \le x < 2, \\ -\frac{6}{5}(x-3) & 2 \le x < 3. \end{cases}$$

And

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right],$$

$$B_\alpha = \left[a_2^{(\alpha)}, b_2^{(\alpha)}\right] = \left[\frac{5}{6}\alpha + 1,\ -\frac{5}{6}\alpha + 3\right].$$

Therefore

$$A_\alpha - B_\alpha = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right] - \left[\frac{5}{6}\alpha + 1,\ -\frac{5}{6}\alpha + 3\right]$$
$$= \left[\frac{35}{24}\alpha - 1,\ -\frac{35}{24}\alpha + 4\right]$$

Now, with use *(18),(19)* we get

$$x = \frac{35}{24}\alpha - 1 \Rightarrow \alpha = \frac{24}{35}(x+1) \qquad -1 \le x \le \frac{6}{5}$$

$$x = -\frac{35}{24}\alpha + 4 \Rightarrow \alpha = \frac{24}{35}(4-x) \qquad \frac{6}{5} \le x \le 4$$

This gives

$$\mu_{A-B}(x) = \begin{cases} \frac{24}{35}(x+1) & -1 \le x \le \frac{6}{5}, \\ \frac{24}{35}(4-x) & \frac{6}{5} \le x \le 4. \end{cases}$$

5. Multiplication of fuzzy sets with using α-cut

Let $A = ((a_1, c_1, b_1))$ and $B = ((a_2, c_2, b_2))$ be two triangular fuzzy sets, Suppose the membership function of *A, B* is

$$\mu_A(x) = \begin{cases} 0 & x < a_1, b_1 \le x, \\ \frac{2c_1(x-a_1)}{b_1 - a_1} & a_1 \le x < \frac{a_1+b_1}{2}, \\ \frac{-2c_1(x-b_1)}{b_1 - a_1} & \frac{a_1+b_1}{2} \le x < b_1. \end{cases}$$

And

$$\mu_B(x) = \begin{cases} 0 & x < a_2, b_2 \le x, \\ \frac{2c_2(x-a_2)}{b_1 - a_1} & a_2 \le x < \frac{a_2+b_2}{2}, \\ \frac{-2c_2(x-b_2)}{b_2 - a_2} & \frac{a_2+b_2}{2} \le x < b_2. \end{cases}$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]$$

And

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

Therefore $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ and $B_\alpha=\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$ are the α-cuts of *A* and *B*, respectively. To calculate multiplication of fuzzy sets *A* and *B* we first add the α-cuts of *A* and *B* using interval arithmetic.

$$A_\alpha.B_\alpha=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right].\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

$$=\left[\alpha^2\left(\frac{(b_1-a_1)(b_2-a_2)}{4c_1c_2}\right)+\alpha\left(\frac{(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1}{2c_1c_2}\right)+a_1a_2,\right. \qquad (21)$$

$$\left.\alpha^2\left(\frac{(b_1-a_1)(b_2-a_2)}{4c_1c_2}\right)+\alpha\left(\frac{(a_1-b_1)b_2c_2+(a_2-b_2)b_1c_1}{2c_1c_2}\right)+b_1b_2\right]$$

Now, we find the membership function $\mu_{A.B}(x)$

$$x=\alpha^2\left(\frac{(b_1-a_1)(b_2-a_2)}{4c_1c_2}\right)+\alpha\left(\frac{(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1}{2c_1c_2}\right)+a_1a_2$$

$$\Rightarrow\alpha=\left[-(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1\right. \qquad (22)$$

$$\left.+2c_1c_2\sqrt{\left(\frac{(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1}{2c_1c_2}\right)^2-\frac{(b_1-a_1)(b_2-a_2)(a_1a_2-x)}{c_1c_2}}\right]/(b_1-a_1)(b_2-a_2)$$

$$a_1a_2\le x\le c_1c_2$$

And

$$x=\alpha^2\left(\frac{(b_1-a_1)(b_2-a_2)}{4c_1c_2}\right)+\alpha\left(\frac{(a_1-b_1)b_2c_2+(a_2-b_2)b_1c_1}{2c_1c_2}\right)+b_1b_2$$

$$\Rightarrow\alpha=\left[-(a_1-b_1)b_2c_2+(b_2-a_2)b_1c_1\right. \qquad (23)$$

$$\left.+2c_1c_2\sqrt{\left(\frac{(a_1-b_1)b_2c_2+(a_2-b_2)b_1c_1}{2c_1c_2}\right)^2-\frac{(b_1-a_1)(b_2-a_2)(b_1b_2-x)}{c_1c_2}}\right]/(b_1-a_1)(b_2-a_2)$$

$$c_1c_2\le x\le b_1b_2$$

Therefore membership function $\mu_{A.B}(x)$ is

$$\mu_{A.B}(x)$$

$$=\begin{cases} \Bigg[-(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1 \\ +2c_1c_2\sqrt{\left(\dfrac{(b_1-a_1)a_2c_2+(b_2-a_2)a_1c_1}{2c_1c_2}\right)^2-\dfrac{(b_1-a_1)(b_2-a_2)(a_1a_2-x)}{c_1c_2}}\Bigg]/(b_1-a_1)(b_2-a_2) \\ \qquad a_1a_2\le x\le c_1c_2, \\ \Bigg[-(a_1-b_1)b_2c_2+(b_2-a_2)b_1c_1 \\ +2c_1c_2\sqrt{\left(\dfrac{(a_1-b_1)b_2c_2+(a_2-b_2)b_1c_1}{2c_1c_2}\right)^2-\dfrac{(b_1-a_1)(b_2-a_2)(b_1b_2-x)}{c_1c_2}}\Bigg]/(b_1-a_1)(b_2-a_2) \\ \qquad c_1c_2\le x\le b_1b_2 \end{cases}$$

Example 3. Let $A=\left(\left(2,\frac{12}{5},5\right)\right)$ and $B=\left(\left(1,\frac{6}{5},3\right)\right)$ be two triangular fuzzy sets, Suppose the membership function of A, B is

$$\mu_A(x)=\begin{cases} 0 & x<2,\ 5\le x, \\ \frac{8}{5}(x-2) & 2\le x<\frac{7}{2}, \\ -\frac{8}{5}(x-5) & \frac{7}{2}\le x<5. \end{cases} \qquad \mu_B(x)=\begin{cases} 0 & x<1,\ 3\le x, \\ \frac{6}{5}(x-1) & 2\le x<2, \\ -\frac{6}{5}(x-3) & 2\le x<3. \end{cases}$$

And

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right],$$

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{5}{6}\alpha+1,\ -\frac{5}{6}\alpha+3\right].$$

Therefore

$$A_\alpha.B_\alpha=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right].\left[\frac{5}{6}\alpha+1,\ -\frac{5}{6}\alpha+3\right]$$

$$=\left[\frac{25}{48}\alpha^2+\frac{55}{24}\alpha+2,\ \frac{25}{48}\alpha^2-\frac{145}{24}\alpha+15\right]$$

Now, with use *(22)*,*(23)* we get

$$x=\frac{25}{48}\alpha^2+\frac{55}{24}\alpha+2\Rightarrow\alpha=\frac{-55+\sqrt{625+1200x}}{25}\qquad 2\le x\le\frac{72}{25}$$

$$x=\frac{25}{48}\alpha^2-\frac{145}{24}\alpha+15\Rightarrow\alpha=\frac{145+\sqrt{3025+1200x}}{25}\qquad \frac{72}{25}\le x\le 15$$

thich gives

$$\mu_{A.B}(x)=\begin{cases}\dfrac{-55+\sqrt{625+1200x}}{25} & 2\le x\le\dfrac{72}{25},\\ \dfrac{145+\sqrt{3025+1200x}}{25} & \dfrac{72}{25}\le x\le 15.\end{cases}$$

6. Division of fuzzy sets with using α-cut

Let $A=((a_1,\mathrm{c}_1,b_1))$ and $B=((a_2,\mathrm{c}_2,b_2))$ be two triangular fuzzy sets, Suppose the membership function of *A*, *B* is

$$\mu_A(x)=\begin{cases}0 & x<a_1,b_1\le x,\\ \dfrac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\dfrac{a_1+b_1}{2},\\ \dfrac{-2c_1(x-b_1)}{b_1-a_1} & \dfrac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

And

$$\mu_B(x)=\begin{cases}0 & x<a_2,b_2\le x,\\ \dfrac{2c_2(x-a_2)}{b_1-a_1} & a_2\le x<\dfrac{a_2+b_2}{2},\\ \dfrac{-2c_2(x-b_2)}{b_2-a_2} & \dfrac{a_2+b_2}{2}\le x<b_2.\end{cases}$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]$$

And

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2,\frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

Therefore $A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ and $B_\alpha = \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$ are the α-cuts of A and B, respectively. To calculate Division of fuzzy sets A and B we first add the α-cuts of A and B using interval arithmetic.

$$A_\alpha / B_\alpha = \left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1, \frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right] / \left[\frac{(b_2-a_2)\alpha}{2c_2}+a_2, \frac{(b_2-a_2)\alpha}{-2c_2}+b_2\right]$$

$$= \left[\frac{\frac{(b_1-a_1)\alpha}{2c_1}+a_1}{\frac{(b_2-a_2)\alpha}{-2c_2}+b_2}, \frac{\frac{(b_1-a_1)\alpha}{-2c_1}+b_1}{\frac{(b_2-a_2)\alpha}{2c_2}+a_2}\right] \qquad (25)$$

Now, we find the membership function $\mu_{A/B}(x)$

$$x = \frac{\frac{(b_1-a_1)\alpha}{2c_1}+a_1}{\frac{(b_2-a_2)\alpha}{-2c_2}+b_2}$$

$$\Rightarrow \alpha = \frac{2c_1c_2(xb_2-a_1)}{c_1x(b_2-a_2)+c_2(b_1-a_1)} \qquad \frac{a_1}{b_2} \le x \le \frac{c_1}{c_2} \qquad (26)$$

And

$$x = \frac{\frac{(b_1-a_1)\alpha}{-2c_1}+b_1}{\frac{(b_2-a_2)\alpha}{2c_2}+a_2}$$

$$\Rightarrow \alpha = \frac{2c_1c_2(b_1-xa_2)}{c_1x(b_2-a_2)+c_2(b_1-a_1)} \qquad \frac{c_1}{c_2} \le x \le \frac{b_1}{a_2} \qquad (27)$$

Therefore membership function $\mu_{A/B}(x)$ is

$$\mu_{A-B}(x) = \begin{cases} \frac{2c_1c_2(xb_2-a_1)}{c_1x(b_2-a_2)+c_2(b_1-a_1)} & \frac{a_1}{b_2} \le x \le \frac{c_1}{c_2} \\ \frac{2c_1c_2(b_1-xa_2)}{c_1x(b_2-a_2)+c_2(b_1-a_1)} & \frac{c_1}{c_2} \le x \le \frac{b_1}{a_2} \end{cases} \qquad (28)$$

Example 4. Let $A = \left(\left(2, \frac{12}{5}, 5\right)\right)$ and $B = \left(\left(1, \frac{6}{5}, 3\right)\right)$ be two triangular fuzzy sets, Suppose the membership function of A, B is

$$\mu_A(x)=\begin{cases}0 & x<2,\ 5\le x,\\ \frac{8}{5}(x-2) & 2\le x<\frac{7}{2},\\ -\frac{8}{5}(x-5) & \frac{7}{2}\le x<5.\end{cases}\qquad \mu_B(x)=\begin{cases}0 & x<1,\ 3\le x,\\ \frac{6}{5}(x-1) & 2\le x<2,\\ -\frac{6}{5}(x-3) & 2\le x<3.\end{cases}$$

And

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right],$$

$$B_\alpha=\left[a_2^{(\alpha)},b_2^{(\alpha)}\right]=\left[\frac{5}{6}\alpha+1,\ -\frac{5}{6}\alpha+3\right].$$

Therefore

$$A_\alpha/B_\alpha=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right]/\left[\frac{5}{6}\alpha+1,\ -\frac{5}{6}\alpha+3\right]$$

$$=\left[\frac{15\alpha+48}{72-20\alpha},\ \frac{120-15\alpha}{20\alpha+24}\right]$$

Now, with use *(26)*,*(27)* we get

$$x=\frac{15\alpha+48}{72-20\alpha}\Rightarrow\alpha=\frac{72x-48}{15+20x}\qquad \frac{2}{3}\le x\le 2$$

$$x=\frac{120-15\alpha}{20\alpha+24}\Rightarrow\alpha=\frac{120-24x}{20\alpha+15}\qquad 2\le x\le 5$$

which gives

$$\mu_{A/B}(x)=\begin{cases}\frac{72x-48}{15+20x} & \frac{2}{3}\le x\le 2,\\ \frac{120-24x}{20\alpha+15} & 2\le x\le 5.\end{cases}$$

7. Inverse of fuzzy sets with using α-cut

Let $A=((a_1,\mathrm{c}_1,b_1))$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<a_1,b_1\le x,\\ \frac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\frac{a_1+b_1}{2},\\ \frac{-2c_1(x-b_1)}{b_1-a_1} & \frac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

Then

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{(b_1 - a_1)\alpha}{2c_1} + a_1, \frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right]$$

Therefore $A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ is the α-cuts of *A*. To calculate inverse of fuzzy sets *A* we first add the α-cuts of *A* using interval arithmetic.

$$\frac{1}{A_\alpha} = \frac{1}{\left[\frac{(b_1 - a_1)\alpha}{2c_1} + a_1, \frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right]} = \left[\frac{1}{\frac{(b_1 - a_1)\alpha}{-2c_1} + b_1}, \frac{1}{\frac{(b_1 - a_1)\alpha}{2c_1} + a_1}\right] \quad (29)$$

Now, we find the membership function $\mu_{1/A}(x)$

$$x = \frac{1}{\frac{(b_1 - a_1)\alpha}{-2c_1} + b_1} \Rightarrow \alpha = \frac{2c_1(xb_1 - a_1)}{x(b_1 - a_1)} \qquad \frac{1}{b_1} \le x \le \frac{1}{c_1} \quad (30)$$

And

$$x = \frac{1}{\frac{(b_1 - a_1)\alpha}{2c_1} + a_1} \Rightarrow \alpha = \frac{2c_1(1 - xa_1)}{x(b_1 - a_1)} \qquad \frac{1}{c_1} \le x \le \frac{1}{a_1} \quad (31)$$

Therefore membership function $\mu_{1/A}(x)$ is

$$\mu_{1/A}(x) = \begin{cases} \frac{2c_1(xb_1 - a_1)}{x(b_1 - a_1)} & \frac{1}{b_1} \le x \le \frac{1}{c_1} \\ \frac{2c_1(1 - xa_1)}{x(b_1 - a_1)} & \frac{1}{c_1} \le x \le \frac{1}{a_1} \end{cases} \quad (32)$$

Example 5. Let $A = \left(\left(2, \frac{12}{5}, 5\right)\right)$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x) = \begin{cases} 0 & x < 2,\ 5 \le x, \\ \frac{8}{5}(x - 2) & 2 \le x < \frac{7}{2}, \\ -\frac{8}{5}(x - 5) & \frac{7}{2} \le x < 5. \end{cases}$$

And

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right].$$

Therefore

$$1/A_\alpha = \left[\frac{1}{-\frac{5}{8}\alpha + 5},\ \frac{1}{\frac{5}{8}\alpha + 2}\right]$$

Now, with use *(30),(31)* we get

$$x = \frac{1}{-\frac{5}{8}\alpha + 5} \Rightarrow \alpha = \frac{40x - 8}{5x} \qquad \frac{1}{5} \le x \le \frac{5}{12}$$

$$x = \frac{1}{\frac{5}{8}\alpha + 5} \Rightarrow \alpha = \frac{8 - 16x}{5x} \qquad \frac{5}{12} \le x \le \frac{1}{2}$$

which gives

$$\mu_{1/A}(x) = \begin{cases} \dfrac{40x - 8}{5x} & \dfrac{1}{5} \le x \le \dfrac{5}{12}, \\ \dfrac{8 - 16x}{5x} & \dfrac{5}{12} \le x \le \dfrac{1}{2}. \end{cases}$$

8. Exponential of fuzzy sets with using α-cut

Let $A = ((a_1, \mathrm{c}_1, b_1))$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x) = \begin{cases} 0 & x < a_1, b_1 \le x, \\ \dfrac{2c_1(x - a_1)}{b_1 - a_1} & a_1 \le x < \dfrac{a_1 + b_1}{2}, \\ \dfrac{-2c_1(x - b_1)}{b_1 - a_1} & \dfrac{a_1 + b_1}{2} \le x < b_1. \end{cases}$$

Then

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{(b_1 - a_1)\alpha}{2c_1} + a_1, \frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right]$$

Therefore $A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ is the α-cuts of *A*. To calculate exponential of fuzzy sets *A* we first add the α-cuts of *A* using interval arithmetic.

$$\exp\left(A_\alpha\right)=\exp\left(\left[\frac{\left(b_1-a_1\right)\alpha}{2c_1}+a_1,\frac{\left(b_1-a_1\right)\alpha}{-2c_1}+b_1\right]\right)$$
$$=\left[\exp\left(\frac{\left(b_1-a_1\right)\alpha}{2c_1}+a_1\right),\exp\left(\frac{\left(b_1-a_1\right)\alpha}{-2c_1}+b_1\right)\right] \quad (33)$$

Now, we find the membership function $\mu_{\exp(A)}(x)$

$$x=\exp\left(\frac{\left(b_1-a_1\right)\alpha}{2c_1}+a_1\right)\Rightarrow\alpha=\frac{2c_1\left(\ln x-a_1\right)}{b_1-a_1},\quad \exp\left(a_1\right)\le x\le\exp\left(c_1\right) \quad (34)$$

And

$$x=\exp\left(\frac{\left(b_1-a_1\right)\alpha}{-2c_1}+b_1\right)\Rightarrow\alpha=\frac{2c_1\left(\ln x-b_1\right)}{a_1-b_1},\quad \exp\left(c_1\right)\le x\le\exp\left(b_1\right) \quad (35)$$

Therefore membership function $\mu_{\exp(A)}(x)$ is

$$\mu_{\exp(A)}(x)=\begin{cases}\dfrac{2c_1\left(\ln x-a_1\right)}{b_1-a_1}, & \exp\left(a_1\right)\le x\le\exp\left(c_1\right)\\ \dfrac{2c_1\left(\ln x-b_1\right)}{a_1-b_1}, & \exp\left(c_1\right)\le x\le\exp\left(b_1\right)\end{cases} \quad (36)$$

Example 6. Let $A=\left(\left(2,\frac{12}{5},5\right)\right)$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<2,\ 5\le x,\\ \frac{8}{5}(x-2) & 2\le x<\frac{7}{2},\\ -\frac{8}{5}(x-5) & \frac{7}{2}\le x<5.\end{cases}$$

And

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right].$$

Therefore

$$exp\left(A_\alpha\right)=\left[exp\left(\frac{5}{8}\alpha+2\right),\ exp\left(-\frac{5}{8}\alpha+5\right)\right]$$

Now, with use *(34),(35)* we get

$$x=\exp\left(\frac{5}{8}\alpha+2\right)\Rightarrow\alpha=\frac{8}{5}(\ln x-2)\qquad \exp(2)\le x\le\exp\left(\frac{12}{5}\right)$$

$$x=\exp\left(-\frac{5}{8}\alpha+5\right)\Rightarrow\alpha=\frac{8}{5}(5-\ln x)\qquad \exp\left(\frac{12}{5}\right)\le x\le\exp(5)$$

which gives

$$\mu_{\exp(A)}(x)=\begin{cases}\frac{8}{5}(\ln x-2) & \exp(2)\le x\le\exp\left(\frac{12}{5}\right),\\ \frac{8}{5}(5-\ln x) & \exp\left(\frac{12}{5}\right)\le x\le\exp(5).\end{cases}$$

9. Logarithm of fuzzy sets with using α-cut

Let $A=((a_1,\mathrm{c}_1,b_1))$ be two triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<a_1, b_1\le x,\\ \frac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\frac{a_1+b_1}{2},\\ \frac{-2c_1(x-b_1)}{b_1-a_1} & \frac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]$$

Therefore $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ is the α-cuts of *A*. To calculate Logarithm of fuzzy sets *A* we first add the α-cuts of *A* using interval arithmetic.

$$\ln(A_\alpha)=\ln\left(\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]\right)$$

$$=\left[\ln\left(\frac{(b_1-a_1)\alpha}{2c_1}+a_1\right),\ln\left(\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right)\right]\qquad(37)$$

Now, we find the membership function $\mu_{\ln(A)}(x)$

$$x=\ln\left(\frac{(b_1-a_1)\alpha}{2c_1}+a_1\right)\Rightarrow\alpha=\frac{2c_1(\exp x-a_1)}{b_1-a_1},\quad \ln(a_1)\le x\le\ln(c_1)\qquad(38)$$

And

$$x=\ln\left(\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right)\Rightarrow\alpha=\frac{2c_1(\mathrm{b}_1-\exp x)}{b_1-a_1},\quad \ln(c_1)\le x\le\ln(b_1) \tag{39}$$

Therefore membership function $\mu_{\ln(A)}(x)$ is

$$\mu_{\ln(A)}(x)=\begin{cases}\dfrac{2c_1(\exp x-a_1)}{b_1-a_1}, & \ln(a_1)\le x\le\ln(c_1)\\ \dfrac{2c_1(\mathrm{b}_1-\exp x)}{b_1-a_1}, & \ln(c_1)\le x\le\ln(b_1)\end{cases} \tag{40}$$

Example 7. Let $A=\left(\left(2,\frac{12}{5},5\right)\right)$ be a triangular fuzzy sets, Suppose the membership function of A is

$$\mu_A(x)=\begin{cases}0 & x<2,\ 5\le x,\\ \frac{8}{5}(x-2) & 2\le x<\frac{7}{2},\\ -\frac{8}{5}(x-5) & \frac{7}{2}\le x<5.\end{cases}$$

And

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right].$$

Therefore

$$\ln(A_\alpha)=\left[\ln\left(\frac{5}{8}\alpha+2\right),\ \ln\left(-\frac{5}{8}\alpha+5\right)\right]$$

Now, with use *(38)*,*(39)* we get

$$x=\ln\left(\frac{5}{8}\alpha+2\right)\Rightarrow\alpha=\frac{8}{5}(\exp(x)-2) \qquad \ln(2)\le x\le\ln\left(\frac{12}{5}\right)$$

$$x=\ln\left(-\frac{5}{8}\alpha+5\right)\Rightarrow\alpha=\frac{8}{5}(5-\exp(x)) \qquad \ln\left(\frac{12}{5}\right)\le x\le\ln(5)$$

which gives

$$\mu_{\ln(A)}(x)=\begin{cases}\frac{8}{5}(\exp(x)-2) & \ln(2)\le x\le\ln\left(\frac{12}{5}\right),\\ \frac{8}{5}(5-\exp(x)) & \ln\left(\frac{12}{5}\right)\le x\le\ln(5).\end{cases}$$

10. Square root of fuzzy sets with using α-cut

Let $A=\left(\left(a_1,\mathrm{c}_1,b_1\right)\right)$ be two triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<a_1,b_1\le x,\\ \dfrac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\dfrac{a_1+b_1}{2},\\ \dfrac{-2c_1(x-b_1)}{b_1-a_1} & \dfrac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

Then

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]$$

Therefore $A_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right]$ is the α-cuts of *A*. To calculate square root of fuzzy sets *A* we first add the α-cuts of *A* using interval arithmetic.

$$\sqrt{(A_\alpha)}=\sqrt{\left[\frac{(b_1-a_1)\alpha}{2c_1}+a_1,\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right]}$$

$$=\left[\sqrt{\left(\frac{(b_1-a_1)\alpha}{2c_1}+a_1\right)},\sqrt{\left(\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right)}\right] \qquad (41)$$

Now, we find the membership function $\mu_{\sqrt{(A)}}(x)$

$$x=\sqrt{\left(\frac{(b_1-a_1)\alpha}{2c_1}+a_1\right)}\Rightarrow\alpha=\frac{2c_1\left(x^2-a_1\right)}{b_1-a_1},\quad \sqrt{(a_1)}\le x\le\sqrt{(c_1)} \qquad (42)$$

And

$$x=\sqrt{\left(\frac{(b_1-a_1)\alpha}{-2c_1}+b_1\right)}\Rightarrow\alpha=\frac{2c_1\left(\mathrm{b}_1-x^2\right)}{b_1-a_1},\quad \sqrt{(c_1)}\le x\le\sqrt{(b_1)} \qquad (43)$$

Therefore membership function $\mu_{\sqrt{(A)}}(x)$ is

$$\mu_{\ln(A)}(x)=\begin{cases}\dfrac{2c_1\left(x^2-a_1\right)}{b_1-a_1}, & \sqrt{(a_1)}\le x\le\sqrt{(c_1)}\\ \dfrac{2c_1\left(\mathrm{b}_1-x^2\right)}{b_1-a_1}, & \sqrt{(c_1)}\le x\le\sqrt{(b_1)}\end{cases} \qquad (44)$$

Example 8. Let $A=\left(\left(2,\frac{12}{5},5\right)\right)$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<2,\ 5\le x,\\ \frac{8}{5}(x-2) & 2\le x<\frac{7}{2},\\ -\frac{8}{5}(x-5) & \frac{7}{2}\le x<5.\end{cases}$$

And

$$A_\alpha=\left[a_1^{(\alpha)},b_1^{(\alpha)}\right]=\left[\frac{5}{8}\alpha+2,\ -\frac{5}{8}\alpha+5\right].$$

Therefore

$$\sqrt{(A_\alpha)}=\left[\sqrt{\left(\frac{5}{8}\alpha+2\right)},\ \sqrt{\left(-\frac{5}{8}\alpha+5\right)}\right]$$

Now, with use *(42)*,*(43)* we get

$$x=\sqrt{\left(\frac{5}{8}\alpha+2\right)}\Rightarrow\alpha=\frac{8}{5}\left(x^2-2\right)\qquad \sqrt{2}\le x\le\sqrt{\left(\frac{12}{5}\right)}$$

$$x=\sqrt{\left(-\frac{5}{8}\alpha+5\right)}\Rightarrow\alpha=\frac{8}{5}\left(5-x^2\right)\qquad \sqrt{\left(\frac{12}{5}\right)}\le x\le\sqrt{5}$$

which gives

$$\mu_{\sqrt{(A)}}(x)=\begin{cases}\frac{8}{5}\left(x^2-2\right) & \sqrt{2}\le x\le\sqrt{\left(\frac{12}{5}\right)},\\ \frac{8}{5}\left(5-x^2\right) & \sqrt{\left(\frac{12}{5}\right)}\le x\le\sqrt{5}.\end{cases}$$

11. n^{th} root of fuzzy sets with using α-cut

Let $A=\left(\left(a_1,c_1,b_1\right)\right)$ be two triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x)=\begin{cases}0 & x<a_1,b_1\le x,\\ \frac{2c_1(x-a_1)}{b_1-a_1} & a_1\le x<\frac{a_1+b_1}{2},\\ \frac{-2c_1(x-b_1)}{b_1-a_1} & \frac{a_1+b_1}{2}\le x<b_1.\end{cases}$$

Then

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{(b_1 - a_1)\alpha}{2c_1} + a_1, \frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right]$$

Therefore $A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ is the α-cuts of *A*. To calculate n^{th} root of fuzzy sets *A* we first add the α-cuts of *A* using interval arithmetic.

$$(A_\alpha)^{\frac{1}{n}} = \left[\frac{(b_1 - a_1)\alpha}{2c_1} + a_1, \frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right]^{\frac{1}{n}}$$

$$= \left[\left(\frac{(b_1 - a_1)\alpha}{2c_1} + a_1\right)^{\frac{1}{n}}, \left(\frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right)^{\frac{1}{n}}\right] \qquad (45)$$

Now, we find the membership function $\mu_{(A)^{\frac{1}{n}}}(x)$

$$x = \left(\frac{(b_1 - a_1)\alpha}{2c_1} + a_1\right)^{\frac{1}{n}} \Rightarrow \alpha = \frac{2c_1(x^n - a_1)}{b_1 - a_1}, \quad \sqrt[n]{(a_1)} \le x \le \sqrt[n]{(c_1)} \qquad (46)$$

And

$$x = \left(\frac{(b_1 - a_1)\alpha}{-2c_1} + b_1\right)^{\frac{1}{n}} \Rightarrow \alpha = \frac{2c_1(\mathrm{b}_1 - x^n)}{b_1 - a_1}, \quad \sqrt[n]{(c_1)} \le x \le \sqrt[n]{(b_1)} \qquad (47)$$

Therefore membership function $\mu_{(A)^{\frac{1}{n}}}(x)$ is

$$\mu_{(A)^{\frac{1}{n}}}(x) = \begin{cases} \dfrac{2c_1(x^n - a_1)}{b_1 - a_1}, & \sqrt[n]{(a_1)} \le x \le \sqrt[n]{(c_1)} \\ \dfrac{2c_1(\mathrm{b}_1 - x^n)}{b_1 - a_1}, & \sqrt[n]{(c_1)} \le x \le \sqrt[n]{(b_1)} \end{cases} \qquad (48)$$

Example 9. Let $A = \left(\left(2, \frac{12}{5}, 5\right)\right)$ be a triangular fuzzy sets, Suppose the membership function of *A* is

$$\mu_A(x) = \begin{cases} 0 & x < 2,\ 5 \le x, \\ \frac{8}{5}(x - 2) & 2 \le x < \frac{7}{2}, \\ -\frac{8}{5}(x - 5) & \frac{7}{2} \le x < 5. \end{cases}$$

And

$$A_\alpha = \left[a_1^{(\alpha)}, b_1^{(\alpha)}\right] = \left[\frac{5}{8}\alpha + 2,\ -\frac{5}{8}\alpha + 5\right].$$

Therefore

$$(A_\alpha)^{\frac{1}{n}} = \left[\left(\frac{5}{8}\alpha + 2\right)^{\frac{1}{n}},\ \left(-\frac{5}{8}\alpha + 5\right)^{\frac{1}{n}}\right]$$

Now, with use *(46),(47)* we get

$$x = \left(\frac{5}{8}\alpha + 2\right)^{\frac{1}{n}} \Rightarrow \alpha = \frac{8}{5}\left(x^n - 2\right) \qquad 2^{\frac{1}{n}} \le x \le \left(\frac{12}{5}\right)^{\frac{1}{n}}$$

$$x = \left(-\frac{5}{8}\alpha + 5\right)^{\frac{1}{n}} \Rightarrow \alpha = \frac{8}{5}\left(5 - x^n\right) \qquad \left(\frac{12}{5}\right)^{\frac{1}{n}} \le x \le 5^{\frac{1}{n}}$$

which gives

$$\mu_{(A)^{\frac{1}{n}}}(x) = \begin{cases} \frac{8}{5}\left(x^n - 2\right) & 2^{\frac{1}{n}} \le x \le \left(\frac{12}{5}\right)^{\frac{1}{n}}, \\ \frac{8}{5}\left(5 - x^n\right) & \left(\frac{12}{5}\right)^{\frac{1}{n}} \le x \le 5^{\frac{1}{n}}. \end{cases}$$

REFERENCES

1. L.A. Zadeh, Fuzzy set, *Information and Control*, 8(3) (1965) 338-353.
2. D. Dubois and H. Prade, Operations on fuzzy numbers. *International Journal of Systems Science*, 9(6) (1978) 613-626.
3. Y.S. Yun, S.U. Ryu and J.W. Park, The generalized triangular fuzzy sets, *Journal of the chungcheong mathematical society,* 22(2) (2009).
4. P. Dutta, H. Boruah and T. Ali, Fuzzy Arithmetic with and without using αcut method: A Comparative Study, *International Journal of Latest Trends in Computing,* 2(1) 2011.
5. R. Chutia, S. Mahanta and H.K. Baruah. An alternative method of finding the membership of a fuzzy number, *International journal of latest trends in computing,* 1(2) (2010) 69-72.
6. S. Mahanta, R. Chutia and H.K. Baruah, Fuzzy Arithmetic without using the method of α-cut International journal of latest trends in computing, 1(2) (2010) 73-80.
7. S. Rezvani, Multiplication Operation on Trapezoidal Fuzzy Numbers, Journal *of Physical Sciences,* 15 (2011) 17-26.
8. S. Rezvani, New Method for Ranking in Perimeters of two Generalized Trapezoidal Fuzzy Numbers, *International Journal of Applied Operational Research*, 2(3) (2012) 83-90.

9. S. Rezvani, A New Approach Ranking of Exponential Trapezoidal Fuzzy Numbers, *Journal of Physical Sciences*, 16 (2012) 45-57.
10. S. Rezvani, A New Method for Ranking in Areas of two Generalized Trapezoidal Fuzzy Numbers, International Journal of Fuzzy Logic Systems (IJFLS), 3(1) (2013) 17-24.
11. S. Rezvani, Ranking Generalized Trapezoidal Fuzzy Numbers with Euclidean Distance by the Incentre of Centroids, *Mathematica Aeterna*, 3(2) (2013) 103-114.
12. S. Rezvani, Ranking Method of Trapezoidal Intuitionistic Fuzzy Numbers, *Annals of Fuzzy Mathematics and Informatics*, 5(3) (2013) 515-523.
13. S. Rezvani, Ranking Exponential Trapezoidal Fuzzy Numbers by Median Value, *Journal of Fuzzy Set Valued Analysis*, 2013 (2013) 1-9.
14. S. Rezvani, Ranking exponential trapezoidal fuzzy numbers with cardinality, *Communications of the Korean Mathematical Society*, 29(1) (2014) 187–193.
15. S. Rezvani, A New Method for Ranking Fuzzy Numbers with Using TRD Distance Based on mean and standard deviation, *International Journal of Mechatronics, Electrical and Computer Technology (IJMEC)*, 4(12) (2014) 840-856.
16. S. Rezvani, Rank, Mode, Divergence and Spread on Generalized Triangular Fuzzy Numbers, *Mathematica Aeterna*, 5(3) (2015) 403 – 415.
17. S. Rezvani, Ranking generalized exponential trapezoidal fuzzy numbers based on variance, *Applied Mathematics and Computation*, 262 (2015) 191-198.