Journal of Physical Sciences, Vol. 20, 2015, 213-220
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 24 December 2015

EPR-Steering and Bell States for the Quadratically-Coupled Optomechanical System

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Received 12 September 2015; accepted 27 November 2015

ABSTRACT

The Hamiltonian of quadratically-coupled optomechanical system is solved by short time dynamics of the corresponding Heisenberg equation of motion for different field modes. Using these solutions, the possibility of EPR-Steering and Bell non-locality are investigated. Temporal variation of these properties also studied under different phase angle. Depth of Bell non-locality is more stronger than EPR-Steering under same parametric conditions.

***Keywords*:** nonclassical, EPR-Steering, Bell non-locality

1. Introduction

In 1935 Schrödinger introduced the concept of steering as a generalization of Einstein-Podolsky-Rosen(EPR) paradox [1]. In EPR paper [2] it was considered the general unfactorisable pure state of two system

$$|\Phi\rangle = \sum_{k=1}^{\infty} p_k |\alpha_k\rangle |\zeta_k\rangle = \sum_{k=1}^{\infty} c_k |\beta_k\rangle |\xi_k\rangle$$

For first system here two orthonormal basis are $|\alpha_k\rangle$ and $|\beta_k\rangle$. If the first set of basis corresponding to Alice and for Bob the second set. If Alice were to measure in the basis $|\alpha_k\rangle$ she would, then contiguously cause Bob's system to collapse into one the $|\zeta_k\rangle$ states. Similarly, if Alice measured in the $|\beta_k\rangle$ basis, then the Bob's system would collapse into one of the $|\xi_k\rangle$ system instantaneously. When $|\zeta_k\rangle$ is different to $|\xi_k\rangle$ was unsuitable to EPR as two system were remote, so they felt no real change could take place. In the context of EPR paper Schrödinger introduced the term steering to describe the affection of Bob state by Alice choice of measurement basis and entanglement to describe the states. For a mixed state there are three possibility of a state i.e. entanglement, steering and Bell non-locality. Werner[3] proposed a relationship between Bell non-locality and entanglement and stated that not all entangled state were Bell non-local. The exact relationship of above said three quantum correlation was introduced by Wiseman *et al.*[4, 5]. EPR-steering exists between the concepts of entanglement and Bell non-locality. Steerable states are superset of Bell non-local states and subset of entangled state. Wiseman *et al.* also gave the idea about multi-particle steering. To verify the existence of n-particle steering both for discrete and continuous variable an inductive model was proposed by Reid and He [6]. Z.Ou *et al.*[7] had given the first experimental evidence of steering using continuous variable optical beams. Quantum steering in Gaussian regime was demonstrated by Bowen *et al.*[8] and continuous variable regime by Samblowski *et al.*[9] and Steinlechner *et al.* [10]. Bell non-locality was discussed for continuous variable entangled system by Garngier *et al.* [11],Praxmayer *et al.* [12]. These quantum correlations have large utility in quantum cryptography, quantum teleportation, quantum metrology. Motivated by these fact we are interested to investigate for the existence of nonclassical correlation in quadratically coupled optomechanical system(OMS). As OMS is a very promising platform for demonstrating various nonclassical correlation which are very much useful for quantum information theory. Steering is observed for several optomechanical model such as hybrid microwave optical steering for two separate cavities separated by a mechanical oscillator [13], experimental demonstration of EPR steering for OMS [14,15]. For quadratically coupled OMS the existence of squeezing and entanglement is already discussed by some of us [16, 17]. As EPR-steering and Bell non-locality are more stronger quantum correlation than entanglement so, here we discussed about the possibility of the existence of EPR-steering and Bell non-locality for quadratically coupled OMS.

Our article is organised as follows, at first we describe the model Hamiltonian and its perturbative solutions for Heisenberg equation of motions corresponds to cavity field mode and phonon mode. Then using the Cavalcanti *et al.* [18] criterions we have to investigate the existence of steerable state and Bell states for quadratic coupled OMS and discuss the results. At last,we gave the main conclusions.

2. The model Hamiltonian

Quadratic-coupled OMS is such a system configuration in which a dielectric membrane is placed at the middle between two macroscopic, rigid and high-finesse cavity. The Hamiltonian for such a system is given by(with $\hbar = 1$)[19, 20]

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + g a^\dagger a (b^\dagger + b)^2 \tag{1}$$

The cavity field mode characterized by the annihilation(creation) operators a ($a^\dagger$)and the field operators b ($b^\dagger$) corresponds to the mechanical motion of the membrane, where these operators satisfy bosonic commutation relation $[a,a^\dagger] = [b,b^\dagger] = 1$. The cavity resonant frequency and frequency for mechanical mode is given by ω_c and ω_m respectively. The coupling strength between cavity field and the membrane is g.

2.1. Equations of motion

To investigate the existence of EPR-steering and Bell states in quadratically coupled OMS we use Heisenberg equations of motion $\frac{dA}{dt} = -i[A,H]$ for cavity field mode a and mechanical mode b corresponds to Hamiltonian of equation (1) and we express these in the following equations(2).

$$\frac{da(t)}{dt} = -i\left[\omega_c a(t) + g a(t)\left\{b^{\dagger 2}(t) + b^2(t) + 2b^\dagger(t)b(t) + 1\right\}\right]$$

$$\frac{db(t)}{dt} = -i\left[\omega_m b(t) + 2ga^{\dagger}(t)a(t)\left\{b^{\dagger}(t) + b(t)\right\}\right] \tag{2}$$

2.2. Solutions

Expanding any operator $A(t)$ in Taylor's expansion as $A(t) = A(0) + t\dot{A}(t)|_{t=0} + ...$ where $\dot{A}(t) = i[H, A(0)]$. As the interaction time is taken to be short, so we keep terms up to first-order in gt. In this short-time approximated solution we do not obtain any term which arises from $\ddot{A}(t)$, also beyond that as $gt < 1$ for this perturbation calculations holds good. From the evaluation of the commutator we get the time development of operator $a(t)$ as well as $b(t)$ interms of $a(0)$ and $b(0)$. The solutions of the equations(2) are

$$a(t) = f(t)[a(0) + F(t)a(0)b^{\dagger 2}(0) - F^{*}(t)a(0)b^{2}(0) - 2igta(0)b^{\dagger}(0)b(0) - igta(0)]$$
$$b(t) = h(t)[b(0) - 2igta^{\dagger}(0)a(0)b(0) + 2F(t)a^{\dagger}(0)a(0)b^{\dagger}(0)] \tag{3}$$

where the functions are $f(t) = e^{-i\omega_c t}$, $h(t) = e^{-i\omega_m t}$ and $F(t) = \frac{g}{2\omega_m}(1 - e^{2i\omega_m t})$. For checking the solutions we use Equal time commutation relation (ETCR) as $[a(t), a^{\dagger}(t)] = [b(t), b^{\dagger}(t)] = 1$ where the operator $a(0)$ commutes with $b^{\dagger}(0)$ and $b(0)$ .Similarly, $b(0)$ commutes with $a^{\dagger}(0)$ and $a(0)$.

3. EPR-Steering and bell states

EPR-steering is a quantum correlation which can be interpreted as the remote manipulation of a quantum state due to operations on a non-seperable or entangled system. In order to finding the possibility of EPR-steering and Bell state for quadratically coupled OMS, here we use inequality by Cavalcanti *et al.*[18], which is a continuous-variable nonlocal inequality. These inequalities are based on non-negativity of variances for local hidden variable state and local uncertainty relation of quadrature (position and momentum) operator. Cavalcanti *et al.* criterion for two modes EPR-steering written as

$$|\langle a_i a_j^{\dagger}\rangle|^2 \leq \langle a_i^{\dagger} a_i (a_j^{\dagger} a_j + \frac{1}{2})\rangle \tag{4}$$

The violation of the inequality denotes the possibility of EPR-steering. According to overworked Alice and Bob, if Alice measures mode a_i and Bob measures mode a_j then the violation of this inequality signify that Bob would be able to steer Alice and vice versa. From this inequality we may define a correlation function $\xi_{a_i a_j}$ which signifies the presence of EPR-steering which when it has a value of greater than zero i.e. $\xi_{a_i a_j} > 0$.

$$\xi_{a_i a_j} = |\langle a_i a_j^{\dagger}\rangle|^2 - \langle a_i^{\dagger} a_i (a_j^{\dagger} a_j + \frac{1}{2})\rangle \tag{5}$$

Using solutions and above said conditions we get

$$\xi_{ab}(t) = 2\{-F^{*}(t)\,|\alpha|^{2}\,\beta^{2} + c.c.\} - \frac{1}{2}|f(t)|^{2}|\alpha|^{2} \qquad (6)$$

Figure (1) and (2) of gave the graphical representation of ξ_{ab} with rescaled time t. The right hand side of equation (6) is not so simple, from graphical representation of it one can easily understand the existence of quantum steering for the system. From equation (6) it is clear that the value of ξ_{ab} is independent of photon phase but depends on phase of phonon mode, here we assume $\beta = |\beta| e^{i\phi}$. In figure (1) we plot this result for phase angle $\phi = 0$, $\phi = \frac{\pi}{4}$ and for phase angle $\phi = \frac{\pi}{2}$, $\phi = \frac{3\pi}{4}$ is shown in figure (2). The positivity of ξ_{ab} is observed for phase angle $\frac{\pi}{4}$, $\frac{\pi}{2}$, and $\frac{3\pi}{4}$. So, there are existence for steerable state for quadratically-coupled OMS for that phase angle of phonon.

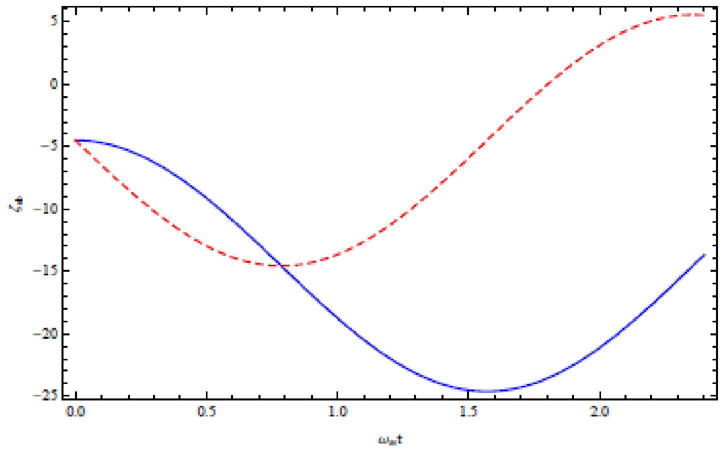

Figure 1: (color online)Plot of ξ_{ab} with rescaled time $\omega_m t$ for $|\alpha| = 3$, $|\beta| = 2$, $\omega_c = 2\pi \times 2.4GHz$, $g = 2\pi \times 14KHz$, $g/\omega_m = 0.14$, for phase angle of β with $\phi = 0$ (solid curve) and $\phi = \frac{\pi}{4}$ (dashed curve).

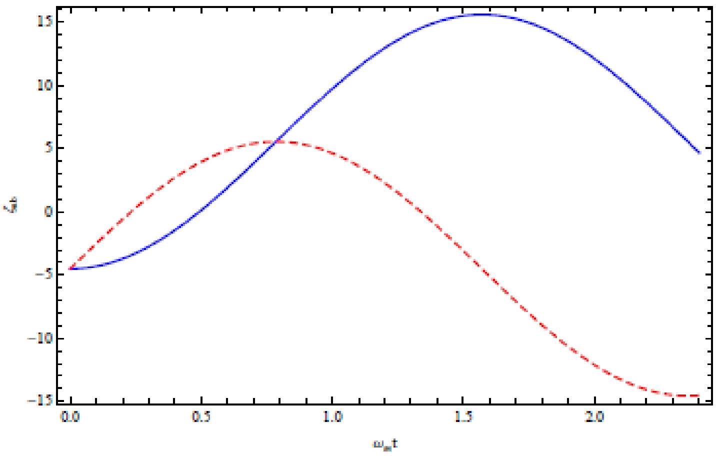

Figure 2: (color online)Plot of ξ_{ab} with rescaled time $\omega_m t$ for $|\alpha|=3$, $|\beta|=2$, $\omega_c = 2\pi\times 2.4GHz$, $g = 2\pi\times 14KHz$, $g/\omega_m = 0.14$, for phase angle of β with $\phi = \frac{\pi}{2}$ (solid curve) and $\phi = \frac{3\pi}{4}$ (dashed curve).

Again, Bell state inequality is expressed as

$$|\langle a_i a_j^\dagger \rangle|^2 \le \langle (a_i^\dagger a_i + \frac{1}{2})(a_j^\dagger a_j + \frac{1}{2})\rangle \tag{7}$$

From this inequality we may define Bell correlation function $\zeta_{a_i a_j}$, for which a positive value.

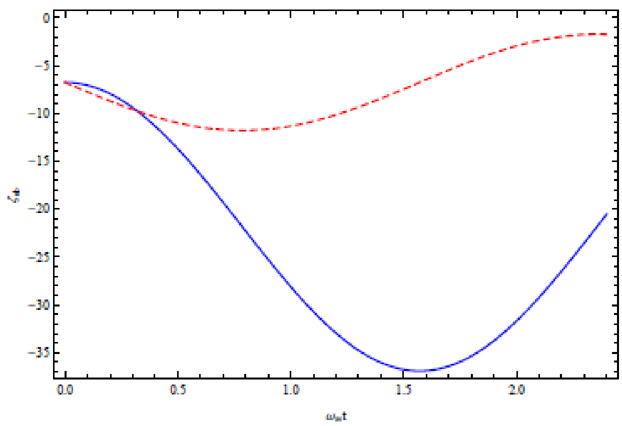

Figure 3: (color online)Plot of ζ_{ab} with rescaled time $\omega_m t$ for $|\alpha|=3$, $|\beta|=2$, $\omega_c = 2\pi\times 2.4GHz$, $g = 2\pi\times 14KHz$, $g/\omega_m = 0.14$, for phase angle of β with $\phi = 0$ (solid curve) and $\phi = \frac{\pi}{4}$ (dashed curve).

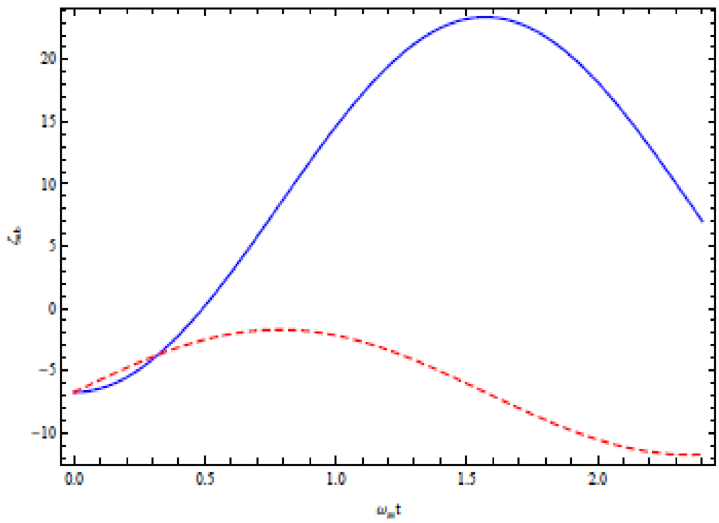

Figure 4: (color online)Plot of ζ_{ab} with rescaled time $\omega_m t$ for $|\alpha|=3$, $|\beta|=2$, $\omega_c = 2\pi \times 2.4GHz$, $g = 2\pi \times 14KHz$, $g/\omega_m = 0.14$, for phase angle of β with $\phi = \frac{\pi}{2}$ (solid curve) and $\phi = \frac{3\pi}{4}$ (dashed curve).

state the presence of Bell states.

$$\zeta_{a_i a_j} = |\langle a_i a_j^\dagger \rangle|^2 - \langle (a_i^\dagger a_i + \frac{1}{2})(a_j^\dagger a_j + \frac{1}{2}) \rangle \quad (8)$$

Using Bell correlation function and solutions for mode a and b we find out

$$\zeta_{ab}(t) = -\frac{1}{2}\{|f(t)|^2|\alpha|^2 + |h(t)|^2|\beta|^2 + \frac{1}{2}\} + [\{-2F^*(t) - F(t)\}|\alpha|^2 \beta^2 + c.c.] \quad (9)$$

The graphical representation of equation (9) is shown in figure (3) and (4) for different phase angle of phonon mode. From equation (9) it is also cleared that the value of ζ_{ab} is dependent on phase angle of phonon mode only. The positive value of ζ_{ab} is occurred for phase angle $\frac{\pi}{2}$ so, the violation of Bell state inequality is possible only for phase angle $\frac{\pi}{2}$.

4. Conclusions

We explored the existence of EPR steering and Bell non-locality for quadratically coupled OMS. Both quantum correlation are independent of phase angle of cavity field mode but dependent on phonon phase. Steering is observed for phase angle $\frac{\pi}{4}$, $\frac{\pi}{2}$, and $\frac{3\pi}{4}$ but Bell non-locality is possible only for phase angle $\frac{\pi}{2}$ which confirm that all steerable state does not show Bell non-locality. The depth of Bell non-locality is more prominent than steerable state for phase angle $\frac{\pi}{2}$ under same parametric condition. Our result should be useful for cryptographic key distribution for quantum information theory.

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