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Laser Diode to Circular Core Graded Index Single Mode Fiber Excitation via Upside Down Tapered Microlens on the Fiber Tip and Identification of the Suitable Refractive Index Profile

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ABSTRACT

We investigate theoretically the coupling optics involving a laser diode and circular core graded index single mode fiber via upside down tapered microlens on the fiber tip in absence of possible transverse and angular misalignments to predict the nature of suitable refractive index profile for the first time in connection with optimum coupling. By employing Gaussian field distributions for both the source and the fiber and also ABCD matrix for upside down tapered microlens under paraxial approximation, we formulate analytical expressions for the concerned coupling efficiencies. The investigations are performed for two different light-emitting wavelengths of 1.3 μm and 1.5 μm for such fibers with different refractive index profile exponents. Further, it is observed that out of the studied refractive index profiles, triangular index profile having the dispersion-shifted merit comes out to be the most suitable profile to couple laser diode to such abovementioned fiber for two wavelengths of practical interest. The analysis should find use in ongoing investigations for optimum launch optics for the design of upside down tapered microlens either directly on the circular core graded index single mode fiber tip or such fiber attached to single mode fiber.

Keywords: Circular core graded index fiber, Upside down tapered microlens, Optical coupling.

1. Introduction

Optical packaging of semiconductor laser diode (LD) requires high coupling efficiencies and large misalignment tolerances from the point of view of its application based orientation for making the devices easier to handle, more reliable, and more rugged. For a highly efficient laser, usage of microlensed fibers are very common in practical semiconductor laser packages [1]. However, the advantage of alignment free and miniature optical design between the fiber and the microlens, demands the development of microlens on the fiber tip. The advantage of the design of more-compact optical components and modules also makes the design and fabrication of fiber microlens essentially attractive. Benefit of fiber sensing and optical recording also demands the use of fiber microlenses [2]. Therefore, the design and fabrication of microlenses on the fiber tip are of immense importance as far as source to single mode fiber (SMF) coupling is concerned [3-12]. Such microlenses include hyperbolic, hemispherical, upside down tapered surfaces to modulate the spot size of LD light incident on them and transmit it into the SMF.

In this respect, a hyperbolic microlens on the tip of a step index SMF has been shown to be most effective in comparison to other types of conventional microlenses [3-12]. According to geometrical optics, transformation of the incident spherical wave into a plane wave via hyperbolic microlens is possible. The coupling optics involving hyperbolic microlens on the tip of the circular core step index single mode fiber (CCSISMF) [6-9] as well as on the tip of elliptic core step index single mode fiber (ECSISMF) in absence [11] of transverse and angular misalignments has been previously reported. The reported theoretical models based on ABCD matrix [7,8] for hyperbolic microlens developed on the tip of CCSISMF [7,8] as well as on the tip of ECSISMF in absence [11] of transverse and angular misalignments have been shown to excellently interpret the available experimental findings [6] as well as theoretical predictions based on numerical integrations. Therefore the applicability of the model is tested and as such becomes acceptable to investigate and predict results for coupling via hyperbolic microlens developed on CCSISMF and ECSISMF as well as that on graded index SMF.

Moreover, the coupling optics involving hemispherical microlens on the tip of CCSISMF in absence [9] of possible misalignments as well as on the tip of the ECSISMF in absence of [10] transverse and angular misalignments have been already reported based on concerned theoretical ABCD matrix model [9].

But fabrication of hyperbolic microlens is limited by involved technique while the coupling efficiency of hemispherical microlens is impaired due to limited aperture. Recently, a new direction in lensed fiber configuration has already emerged to achieve low loss and long working distance (the air gap between LD and fiber) characteristics [13-15]. In this connection, a hyperbolic end shape on a GIF is proposed for coupling with SMF [16]**.** Moreover, hemispherical microlens is not favourably recommended for practical implementation except for pedagogic appreciation. Tapering of the fiber end is another state of art for the achievement of monolithic microlens integration which is also used in coupling optics. It may be relevant to mention in this connection that considerable amount of work has been reported regarding the geometrical optics of tapered gradient index rods [17], associated ABCD matrix in arbitrary tapered quadratic waveguides [18] and single mode coupling by tapered and non-tapered grin fiber lenses [19].

Recently upside down tapered microlens (UDTM) on the fiber tip, designed by tapering the fiber end, has also emerged [20-22] side by side as a new lensing scheme in coupling optics. Therefore, the study of coupling optics involving UDTM on the tip of SMF is of huge importance. This novel lensing scheme, UDTM [21-23] can be drawn from the fiber end where the fiber tip is tapered into a large hemispherical shape. UDTM may be utilized to accumulate a huge amount of light from the source [21]. A detailed work on formation and power distribution properties of UDTM has been already reported [21] while the corresponding structure of the UDTM fiber end and the refractive index distribution has already been highlighted [22]. The transformation ABCD matrix of the UDTM developed on CCSISMF end has already been prescribed [23]. It is to be noted that in case of semiconductor LD emitting light of wavelength $1.3\mu m$, the coupling efficiency between the LD with a CCSISMF via a uncoated UDTM on the tip of the step index SMF comes out to be 97% [24]. Moreover, the study of the coupling losses in absence of possible transverse and angular mismatches in case of LD to ECSISMF coupling via UDTM on the fiber tip has already been reported [12]. It deserves mentioning in this connection that the Fresnel backward reflection can be neglected to investigate the coupling efficiencies in case of coated UDTM on the tip of the CCSISMF and ECSISMF [5,12] .

Again, we know that graded index fibers (GIF) are tremendously important due to its high bandwidth. Side by side, a GIF shows low sensitivity to micro- and macrobending. Despite numerous studies on different types of microlens on the tip of step index SMF, very recently study on hyperbolic microlens [25] on the tip of circular core graded index single mode fiber (CCGISMF) have already been reported. However, no such information is available regarding a study of coupling optics involving LD and CCGISMF via UDTM on the fiber tip. However, it is pertinent that one should know the exact nature of a refractive index profile to support maximum coupling between LD and UDTM not only developed monolithically on the CCGISMF but also on the GIF to be attached with a SMF. Such investigation, reported for the first time to the best of our knowledge, is important from the standpoint for assessing the sensitivity of the said coupler. Such a study, therefore, should deserve the immediate attention of experimentalists.

In the first part of this paper, we theoretically investigate the coupling efficiencies between a semiconductor LD emitting light of wavelength $\lambda = 1.3\ \mu m$ [6] and a series of CCGISMFs with different profile exponents in refractive index profile via UDTM on the tip of these fibers in absence of possible transverse and angular mismatches. In the second part, we carry out the similar investigation for a LD emitting light of wavelength $\lambda = 1.5\ \mu m$ [6]. A comparison between these two cases is performed. As stated earlier, this analysis is based on previously formulated [23,24] simple ABCD matrix method for refraction by a upside down tapered interface. In fact, prediction of coupling optics by ABCD matrix formalism has produced excellent results as far as coupling of LD via UDTM on the tip of CCSISMF [23, 24] as well as on the tip of ECSISMF [12] is concerned. Concerned calculations are easily executable with very little computations.

Further, we employ Gaussian field distributions for both the source and the fiber. The results should be extremely important for the designers who can, accordingly, mould and shape the desired UDTM at the fiber end to achieve optimum coupling optics involving CCGISMF.

2. Analysis

2.1. Preview of UDTM structure and spot sizes of CCGISMF

In this analysis, we consider the structure of the UDTM fiber end drawn from a SMF of core radius '*a*' as shown in Figure 1. Here, the radius of curvature R_0 and height '*h*' of the spherical end are related to the radius of aperture '*d*' by [22,23]

$$R_0 = \frac{h^2 + d^2}{2h} \tag{1}$$

where the radially symmetric axis OZ is actually the fiber axis and z and r represent the respective axial and radial coordinates, in the tapered region. The tapered surface equation is given by [22,23]

$$r = d\ \left(1 - \frac{z}{L}\right) \tag{2}$$

where L is the length of the cone including the tapered region.

For graded index profiles, the refractive index distribution is written as [26]

$$n(r) = n_{core}\left[1 - 2\left(\frac{r}{a}\right)^g \Delta\right]^{0.5} \quad \text{for } r < a$$

$$= n_{clad} = n_{core}\left[1 - 2\Delta\right]^{0.5} \quad \text{for } r >= a \tag{3a}$$

where g is the exponent of power law, and Δ, the grading parameter is defined as

$$\Delta = \frac{n_{core}^2 - n_{clad}^2}{2n_{core}^2} \tag{3b}$$

where n_{core} and n_{clad} being refractive indices of core axis and cladding, respectively,

It may be noted that g = 1, 2 and ∞ correspond to triangular, parabolic and step profile distributions respectively.

Again, the normalised frequency V is given by $V = k_0 a\left(n_{core}^2 - n_{clad}^2\right)^{\frac{1}{2}}$ with k_0 being the free space wave number.

Further, the Gaussian beam width parameter w_f for CCGISMF as a function of normalised frequency V and the exponent g of the power law profile is approximated as [26-28]

$$\frac{w_f}{a} = \left[\frac{A^/}{V^{2/(g+2)}} + \frac{B^/}{V^{3/2}} + \frac{C^/}{V^6}\right] \tag{4}$$

where parameter optimisation of the functions leads to expressions for $A^/$, $B^/$ and $C^/$ given as [26-28]

$$A^/ = \left\{\frac{2}{5}\left[1 + 4\left(\frac{2}{g}\right)^{5/6}\right]\right\}^{1/2} \tag{5a}$$

$$B^/ = e^{0.298/g} - 1 + 1.478(1 - e^{-0.077g}) \tag{5b}$$

$$C^{/} = 3.76 + \exp\left(4.19 / g^{0.418}\right) \tag{5c}$$

with their validity for $1.5 < V < \infty$.

It must be noted that the fiber spot size w_f is approximated for step index fiber as [29]

$$w_f = a\left[0.65 + \frac{1.619}{V^{1.5}} + \frac{2.879}{V^{6}}\right] \tag{6}$$

2.2. Formulation of microlens coupling scheme

The coupling scheme to be studied has been presented in Figure 1. Here, '*u*' is the distance of separation between the UDTM end of the fiber and the LD. Elliptical intensity profiles of the optical beams emitted from LD are approximated by Gaussian spot sizes w_{1x} and w_{1y} along two mutually perpendicular directions, one perpendicular and the other parallel to the junction planes. Again, in our analysis we use some usual approximations [3,7-12] like no transmission loss, Gaussian distributions for both the source field and the fiber field, perfect matching of the polarisation mode of the fiber field and that on the microlens surface, sufficient angular width of the microlens for the interception of entire power radiated by the source for typical values of the microlens parameters employed. However, in relation to the approximation of the Gaussian source field, it can be pertinently pointed out that the field in the direction perpendicular to the epitaxial layer departs from the Gaussian since the dimension of the junction of the most sources in this direction is much less than that in the other direction, the later being close to the wavelength of propagation [24]. However, for the purpose of estimation of coupling optics using such source and various kind of microlenses [7-12,30], we simply use Gaussian beam with the elliptical waist spot sizes since the laser sources we usually employ have dimensions of the parallel and perpendicular junctions fairly comparable [24].

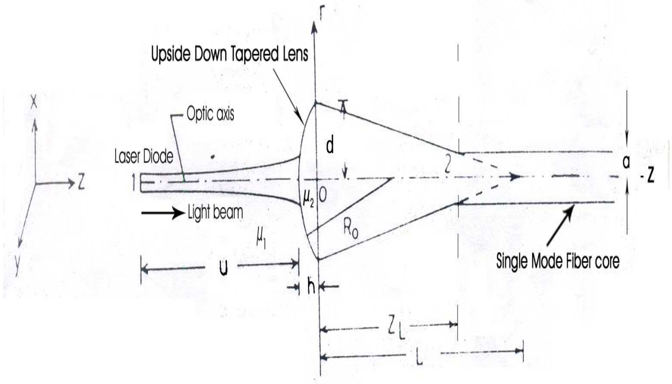

Figure 1: Geometry of laser diode to circular core single mode fiber coupling via upside down tapered microlens on the fiber tip ; μ_1 and μ_2 stand for refractive indices of incident and microlens media respectively.

The field Ψ_u representing the output of the LD at a distance *u* from the UDTM surface is taken as [12,31]

$$\Psi_u = \exp\left[-\left(\frac{x^2}{w_{1x}^2}+\frac{y^2}{w_{1y}^2}\right)\right]\exp\left[-\frac{ik_1}{2}.\frac{x^2+y^2}{R_1}\right] \tag{7}$$

Here, w_{1x} and w_{1y} represent the spot sizes along two perpendicular directions X and Y, k_1 is the wave number in the incident medium and R_1 is the radius of curvature of the wavefronts from the laser source. Our analysis is applicable to single frequency laser emitting only one spatial mode with a Gaussian intensity profile.

It is already known that Gaussian approximations for fundamental mode in CCSISMF [7-9,24] represent sufficiently accurate results in the context of coupling losses. The corresponding fundamental modal field in such fiber is taken as [29]

$$\Psi_f = \exp\left[-\frac{x^2+y^2}{w_f^2}\right] \tag{8}$$

where w_f being the fiber spot size corresponding to Eqs. (4) and (5a-5c) for CCGISMF and Eq. (6) for CCSISMF.

The UDTM transformed laser field Ψ_v on the fiber plane 2 as indicated in Figure 1 can be expressed as [31]

$$\Psi_v = \exp\left[-\left(\frac{x^2}{w_{2x}^2}+\frac{y^2}{w_{2y}^2}\right)\right]\exp\left[-\frac{ik_2}{2}\left(\frac{x^2}{R_{2x}}+\frac{y^2}{R_{2y}}\right)\right] \tag{9}$$

where k_2 is the wave number in the microlens medium and w_{2x}, w_{2y} are respectively microlens transformed spot sizes and R_{2x}, R_{2y} being the respective transformed radii of curvature of the refracted wavefronts in the X and Y directions. The method of finding w_{2x}, w_{2y}, R_{2x} and R_{2y} in terms of w_{1x}, w_{1y} and R_1 with the relevant ABCD matrix for UDTM [24] on the fiber tip is once again presented in the Appendix for ready reference.

The source to fiber coupling efficiency via UDTM on the fiber tip is expressed in terms of well known overlap integral as mentioned below [7-9,30]

$$\eta = \frac{\left|\iint \Psi_v \Psi_f^* dxdy\right|^2}{\iint |\Psi_v|^2 dxdy \iint |\Psi_f|^2 dxdy} \tag{10}$$

Therefore η_0 for circular core fiber is given by

$$\eta_0 = \frac{4w_{2x}w_{2y}w_f^2}{\left[\left(w_f^2+w_{2x}^2\right)^2+\frac{k_2^2w_{2x}^4w_f^4}{4R_{2x}^2}\right]^{1/2}\left[\left(w_{2y}^2+w_f^2\right)^2+\frac{k_2^2w_{2y}^4w_f^4}{4R_{2y}^2}\right]^{1/2}} \tag{11}$$

However, Eq.(11) can be obtained by employing Eqs. (8) and (9) in Eq. (10) [7-9,25].

The above formulations are used in the next section to calculate coupling efficiencies in absence of possible transverse and angular misalignments in case of coupling in between LD and CCGISMF for a specific *V* value corresponding to each profile exponent *g* in refractive index profile.

3. Results and discussions

3.1. Optogeometrical parameters under consideration

In order to estimate coupling efficiencies in absence of any possible transverse and angular misalignments for a UDTM on the tip of CCGISMF, we use first a LD emitting light of wavelength $\lambda = 1.3\mu m$ with $w_{1x} = 1.081\mu m, w_{1y} = 1.161\mu m$ [6]. The LD parameters used in this investigation are mentioned in Table 1. For the LD emitting light of above wavelength, we study the coupling efficiencies for a series of CCGISMFs having different profile exponents *g* [32] in refractive index profile as mentioned in Table 2. We choose *V* value corresponding to each *g* value (g = 1, 2, 4, 8, 10, 20, ∞) as 1.924 [6]. Different fiber spot sizes w_f for this specific *V* value corresponding to each *g* value (g = 1, 2, 4, 8, 10, 20, ∞) [32] are then calculated [26] and shown in Table 2 where we also present the relevant source position with corresponding maximum coupling efficiencies for $\lambda = 1.3\mu m$.

LD	Wavelength λ in μm	Spot size w_{1x} in μm	Spot size w_{1y} in μm	λ_1 in μm	k_2 in μm^{-1}
#1	1.3	1.081	1.161	0.4138	7.4915
#2	1.5	0.843	0.857	0.4775	6.4926

Table 1: Laser diode parameters

Again as taken in all previous cases, the maximum depth of the microlens *h* is taken as $6\mu m$ [7]. The refractive index $\mu(=\mu_2/\mu_1)$ of the material of the microlens with respect to surrounding medium is once again taken as 1.55 [6,7]. The core and cladding refractive indices are chosen as 1.46 and 1.45 respectively. The core diameter is taken as $2a = 4\ \mu m$. An UDTM to be drawn from those typical fibers is chosen with $2d = 6\ \mu m$ with UDTM length z_L as 23.3 μm corresponding to *L* being 70 μm. The radius of curvature R_0 of the spherical end of the UDTM is taken as 90 μm [24]. Further, as explained in earlier cases, since estimation of coupling efficiency on the basis of planar wave model differs slightly from that on the basis of spherical wave model [7,8], we consider planar wave model for the input beam from the laser facet for the sake of simplicity.

Then we use a LD emitting light of wavelength $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$ [6]. We compute again relevant source position with resulting maximum coupling efficiencies for the above same set used in the first part of this investigation and present in Table 2.

$d = 6.0\ \mu m$, $\mu = 1.55$, $a = 2.0\ \mu m$, $D = 3.0\ \mu m$, $R_0 = 90\ \mu m$, $L = 70\ \mu m$

g	A', B', C'	V	w_f (μm)	λ =1.3 μm		λ =1.5 μm	
				u (μm)	η_0	u (μm)	η_0
1.0	A'=1.803 B'=0.457 C'=69.783	1.924	9.901	5.6	0.7466	5.5	0.9819
2.0	A'=1.414 B'=0.372 C'=26.773	1.924	6.156	5.3	0.9920	5.3	0.8950
4.0	A'=1.139 B'=0.469 C'=14.216	1.924	5.008	5.0	0.9848	4.8	0.7612
8.0	A'=0.951 B'=0.718 C'=9.554	1.924	4.714	4.9	0.9684	4.6	0.7186
10.0	A'=0.905 B'=0.824 C'=8.715	1.924	4.715	4.9	0.9685	4.6	0.7187
20.0	A'=0.797 B'=1.176 C'=7.703	1.924	4.858	5.0	0.9773	4.7	0.7398
∞	A'=0.632 B'=1.478 C'=4.76	1.924	4.672	4.8	0.9655	4.6	0.7123

Table 2: Results for optimum coupling efficiency for graded index fibers excited with LD #1 and LD #2

3.2. Results for coupling scheme without misalignment consideration

It is seen from Table 2 that for LD #2 emitting light of wavelength $\lambda = 1.5\mu m$, the maximum coupling efficiency of 98.19% is excellently achieved for CCGISMF having triangular index profile (g = 1) for source position 5.5 μm. However, for the LD #1 emitting light of wavelength $\lambda = 1.3\mu m$, this promising aspect of maximum coupling efficiency of 99.20% is also similarly supported for fiber with parabolic index profile having profile exponent g = 2 for source position of 5.3 μm. Hence, the comparison between these results for CCGISMF excited with two wavelengths predicts that fibers with profile exponent g = 2 appear to be more suitable in the context of the aforesaid coupling optics involving GIFs and this excitement is uniquely excellent for a LD #1 emitting specially light of wavelength $\lambda = 1.3\mu m$ in absence of any misalignment. But it may be worthwhile to mention that the triangular profile *g* = 1 possesses dispersion-shifted merit in the lowest loss in wavelength region of λ = 1.5 μm and for more or less same focal length and maximum coupling efficiency stands as a better candidate for technological importance.

For a typical estimation of knowledge of excitation via UDTM, we present the variation of coupling efficiencies versus the source position for fibers with specific *V* value corresponding to profile exponent *g* = 1 excited with LD #2 emitting light of wavelength $\lambda = 1.5\mu m$ and profile exponent *g* = 2 excited with LD #1 emitting light of wavelength $\lambda = 1.3\mu m$ as shown in Figure 2. In this Figure, solid line (____________ denoting EFF0) corresponds to fibers with profile exponent *g* = 2 excited with LD #1 emitting wavelength $\lambda = 1.3\mu m$ and dashed line (----------- denoting EFF0DS) to with profile exponent *g* = 1 excited with LD #2 emitting wavelength $\lambda = 1.5\mu m$.

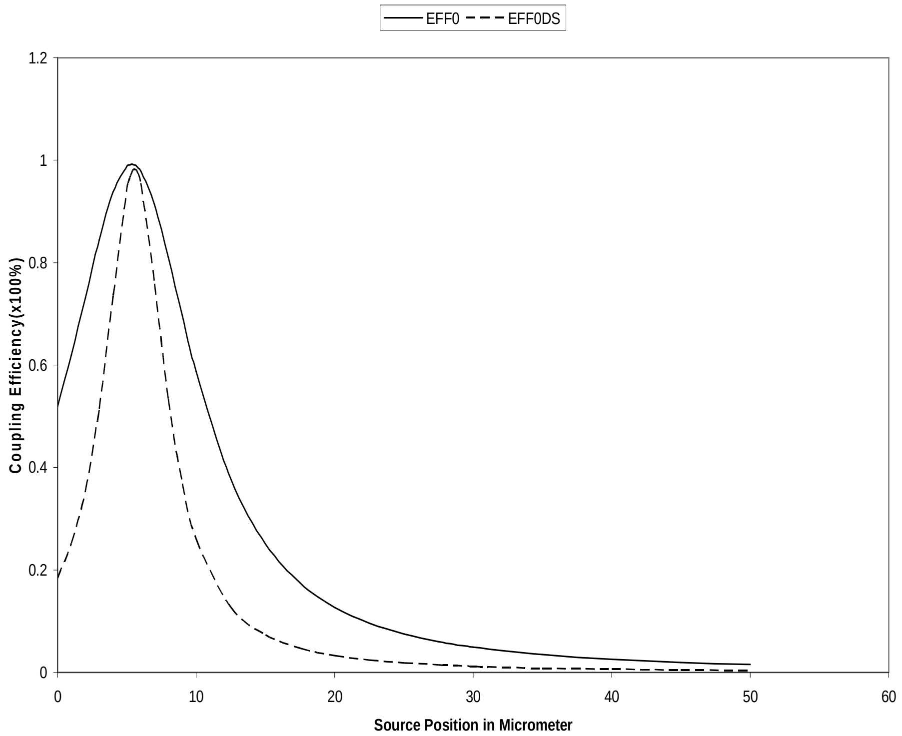

Figure 2: Variation of coupling efficiencies versus the source position for fibers with specific *V* value corresponding to profile exponent *g* = 1 excited with LD #2 and profile exponent *g* = 2 excited with LD#1. Solid line (____________ denoting EFF0) corresponds to fibers with profile exponent *g* = 2 excited with LD #1 and dashed line (----------- denoting EFF0DS) to with profile exponent *g* = 1 excited with LD #2.

It is relevant to mention in this connection that V =1.924 value really corresponds to low V region for triangular index fiber which has the first higher order mode cut off frequency of V_{c1} = 4.38 as well as for parabolic index fiber for which first higher order mode cut off frequency of V_{c1} = 3.5. This low V region is very well known for evanescent wave coupling. Therefore if one likes to exploit our results for use in optical directional couplers they can easily choose such region of V for practical purpose.

Although step index fiber is usually used for communication, GIF having dispersion-shifted criteria stands as a potential candidate for its zero dispersion wavelength being shifted to the region of 1.55 μm to exploit the well known low loss window. Further, triangular index profile has a strong dispersion-shifted merit. Side by side, it may be relevant to mention in this connection that the wavelength at and around 1.5 μm is the region where the erbium doped fiber amplifier and Raman gain fiber amplifier efficiently and elegantly operate. Coupled with these favourable factors for the 1.5 μm wavelength region, one can recommend use of CCGISMF having triangular index profile (g = 1) safely as a microlensing scheme in optimum coupling optics. It is important to note that the source position is nearly constant in both cases. Our present analysis and results point out the merit of triangular index profile in coupling of LD and CCGISMF via UDTM and present the relevant optimum coupling optics. However, the assembly of a LD in the close proximity of the UDTM within 5.3 μm is challenging to achieve in practice. But with the advent of new progress in nanotechnology, we are optimistic that the future technologist will involve any breakthrough to realize our result and test it experimentally.

4. Conclusion

With an aim to explore the nature of suitable refractive index profiles, the coupling optics in case of source to CCGISMF excitation via UDTM on the fiber tip in absence of possible transverse and angular mismatches is formulated and investigated for the first time to the best of our knowledge. For fibers with a typical V value corresponding to each profile exponent g value, appropriate source positions are chosen so as to give maximum coupling for the respective fiber. The best maximum coupling is achieved for triangular index fibers when excited with LD emitting light of wavelength $\lambda = 1.5\mu m$.The application of ABCD matrix has simplified the analysis and the concerned calculations need little computations. Such study should be extremely useful for the designers and packagers of the microlenses on the fiber tip.

Appendix:

Considering the distance u of the LD from the UDTM end, q parameters of the Gaussian beams at the input laser facet and the output microlens fiber interface can be related by the ABCD matrix as follows:

The input and output parameters (q_1,q_2) of the light beam is related by

$$q_2 = \frac{Aq_1 + Au + B}{Cq_1 + Cu + D} \tag{A1}$$

where

$$\frac{1}{q_{1,2}} = \frac{1}{R_{1,2}} - \frac{i\lambda_0}{\pi w_{1,2}^2 \mu_{1,2}} \tag{A2}$$

The ray matrix *M* for the UDTL on the fiber tip is given by [23,24]

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{A3}$$

where

$$A = r_2(z) - \frac{(1-\mu) r_1(z)}{\mu R_0} \tag{A4a}$$

$$B = \frac{r_1(z)}{\mu} \tag{A4b}$$

$$C = -\frac{(1-\mu)}{\mu R_0}\frac{dr_1(z)}{dz} + \frac{dr_2(z)}{dz} \tag{A4c}$$

$$D = \frac{1}{\mu}\frac{dr_1(z)}{dz} \tag{A4d}$$

The refractive index of the material of the microlens with respect to the incident medium is represented by $\mu (= \mu_2 / \mu_1)$.

The *z* dependence of the above matrix elements can be explicitly expressed by substituting [23,24]

$$r_1(z) = -\frac{L}{\alpha}\left(1 - \frac{z}{L}\right)^{1/2} \sin k(z) \tag{A5a}$$

$$\frac{dr_1(z)}{dz} = \frac{1}{\left(1 - \frac{z}{L}\right)^{1/2}} \left\{\cos k(z) + \frac{1}{2\alpha}\sin k(z)\right\} \tag{A5b}$$

$$r_2(z) = \left(1 - \frac{z}{L}\right)^{1/2} \left\{\cos k(z) - \frac{1}{2\alpha}\sin k(z)\right\} \tag{A5c}$$

$$\frac{dr_2(z)}{dz} = \frac{A_0^2 L}{\left(1 - \frac{z}{L}\right)^{1/2} \alpha} \sin k(z) \tag{A5d}$$

where

$$k(z) = \alpha \ln\left(1 - \frac{z}{L}\right) \tag{A6a}$$

and

$$\alpha = \left(A_0^2 L^2 - 1/4\right)^{1/2} \tag{A6b}$$

L being the tapered length or length of the cone including tapered region and A_0 is a constant given by

$$A_0 = \frac{1}{d}\left(2\ln\frac{n_{core}}{n_{clad}}\right)^{1/2} \tag{A7}$$

For a UDTM having aperture 2d

$$z_L = \frac{L(d \ - a)}{d} \tag{A8}$$

In order to obtain $w_{2x,2y}$, the matrix is evaluated for $z = z_L$.

The transformed beam spot sizes and radii of curvature in the X and Y directions are found by using Eqs. (A4a-A4d) in Eqs. (A1) and (A2) and are given by

$$w_{2x,2y}^2 = \frac{A_1^2 w_{1x,1y}^2 + \dfrac{\left(\lambda_1^2 B_1^{\ 2}\right)}{\pi^2 w_{1x,1y}^2}}{\mu(A_1 D_1 - B_1 C_1)} \tag{A9}$$

$$\frac{1}{R_{2x,2y}} = \frac{A_1 C_1 w_{1x,1y}^2 + \dfrac{\left(\lambda_1^2 B_1 D_1\right)}{\pi^2 w_{1x,1y}^2}}{A_1^2 w_{1x,1y}^2 + \dfrac{\left(\lambda_1^2 B_1^{\ 2}\right)}{\pi^2 w_{1x,1y}^2}} \tag{A10}$$

where

$$\lambda_1 = \frac{\lambda_0}{\mu_1} \tag{A11}$$

$$A_1 = A + \frac{B_1}{R_1} \tag{A12a}$$

$$B_1 = Au + B \tag{A12b}$$

$$C_1 = C + \frac{D_1}{R_1}. \tag{A12c}$$

$$D_1 = Cu + D. \tag{A12d}$$

In plane wavefront model, the radius of curvature R_1 of the wavefront from the laser facet →∞. This leads to $A_1=A$ and $C_1=C$.

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