Journal of Physical Sciences, Vol. 20, 2015, 159-171
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 24 December 2015

Laser Diode to Elliptic-Core Step Index Single Mode Fiber Excitation via Parabolic Microlens on the Fiber Tip: Prediction of Coupling Efficiency by ABCD Matrix Formalism

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Received 21 May 2015; accepted 7 August 2015

ABSTRACT

We investigate theoretically the coupling optics involving laser diode and elliptic core step index monomode fiber via parabolic microlens on the fiber tip in absence of possible transverse and angular misalignments. By employing Gaussian field distributions for both the source and the fiber and also the derived ABCD matrix for parabolic microlens, we formulate analytical expressions for the concerned coupling efficiencies. Our formalism is very simple in comparison to the other existing methods involving cumbersome numerical integrations. Elliptic core single-mode fiber has already emerged as a potential candidate in polarisation conserving fiber optic sensor, coherent fiber optical communication etc.. The investigations have been performed for two different light emitting wavelengths of 1.3 μm and 1.5 μm for the said fibers. Therefore, such analysis which as per our knowledge is the first theoretical investigation, can also be treated as simple and novel prescription for choosing suitable parameters for the design of parabolic microlens on the tip of elliptic core step index fiber for optimum launch optics. In addition, this analysis will be useful in case of inherent non-circularity that may arise due to fabrication problem in circular core fiber and thereby generates interests among the experimentalists for verification and future investigations on parabolic microlens.

Keywords: Parabolic Microlens, Elliptic core step index fiber, Optical coupling.

1. Introduction

Recently, fabrication and design of microlenses on the fiber tip are of immense importance as far as source to single mode fiber (SMF) coupling is concerned [1-14]. These microlenses may have hyperbolic [2,3], hemispherical [2,3], upside down tapered [5,6] surfaces to modulate the spot size of laser diode (LD) light incident on them and transmit it into the SMF. In this respect, a hyperbolic microlens on the tip of a step index SMF has been shown to be most effective in comparison to other types of conventional microlenses [1-14]. The coupling optics involving hyperbolic microlens [7,8], hemispherical microlens [10], upside down taped microlens (UDTML) [12,13] on the tip of the circular core step index single mode fiber (CCSISMF) in absence of transverse and angular misalignments have been previously reported based on respective transformation ABCD matrix formalism. The application of the ABCD system matrix formalism [15-18] for prediction of the excitation efficiency for coupling of a LD to a SMF via hyperbolic microlens [7,8], hemispherical microlens [10], UDTML [12,13] on the tip of the CCSISMF has simplified calculations, nevertheless, yielding extremely accurate results. Furthermore, calculations by this formalism are simple and executable with very little computation as well. Such a study, therefore, should deserve the immediate attention of experimentalists.

But fabrication of hyperbolic microlens is limited by involved technique while the coupling efficiency of hemispherical microlens as well as UDTML is impaired due to limited aperture. Moreover, hemispherical microlens is not favourably recommended for practical implementation except for pedagogic appreciation.

Another recent candidate for such intrinsic microlensing scheme is parabolic microlens (PML) on the fiber tip. This type of microlens is observed to have higher coupling efficiency than similar lenses of hemispherical shape and other conventional coupling schemes. The coupling optics involving PML on the tip of CCSISMF has been recently reported based on very popular ABCD matrix formalism [19,20].

Moreover, elliptic core SMFs which can maintain the polarisation state of the propagating beam over long distances have a large number of applications in areas like coherent optical communication, fiber optics sensors etc. [21,22]. Sarkar et al. [23] have already presented a scalar variational analysis based on Gaussian approximation of the fundamental mode of elliptic core SMF. Using this formalism, Sarkar et al. [24] have also developed approximate analytical formulation to work out on lens excitation of the fundamental mode in elliptic core SMFs via ball lens by LDs. It is relevant to mention in this connection that analysis of the coupling of a LD to a hyperbolic microlens [9], hemispherical microlens [11], UDTML [14] on the tip of an ECSISMF has been reported recently. However, regarding non-circular core or, more specifically, elliptic core fibers, there is a fair amount of literature involving their propagation as well as birefringence characteristics, but no attempt has, so far, been made to study the efficiency of coupling of a LD to a ECSISMF via a PML on the fiber tip. In designing optimum launch optics involving such microlenses, theoretical computation of such coupling efficiencies require cumbersome and lengthy numerical integrations [3].

However, the simplicity and effectiveness of method of computation of coupling optics of PML involving ABCD matrix formalism motivates us to concentrate on the present work, which involves the study of the coupling optics of a PML on the non-circular core fiber in the framework of the ABCD formalism prescribed for a PML [19,20]. The present analysis, reported for the first time, contains significant new results and will be extremely important in coherent optical communications, fiber optic sensors etc.. In addition, if due to constructional problem, there is non-circularity in circular core fiber, such analysis will take care of the inherent non-circularity in the process of designing optimum launch optics.

In the first part of this paper, we theoretically investigate the coupling efficiencies between a semiconductor LD emitting light of wavelength λ = 1.3 μm [3] and three ECSISMFs [9] with elliptical configuration corresponding to different aspect ratios and spot sizes via PML of different focal parameters on the tip of these fibers in absence of possible transverse and angular mismatches. In the second part, we carry out the similar investigation for a LD emitting light of wavelength λ =1.5 μm [3]. A comparison between these two cases is performed. As stated earlier, this analysis is based on previously formulated simple ABCD matrix method for refraction by a parabolic interface [19,20]. In fact, prediction of coupling optics by ABCD matrix formalism has produced excellent results as far as coupling of LD via PML of specific focal parameter on the tip of CCSISMF [20] is concerned. Concerned calculations are easily executable with very little computations.

It may be relevant to mention in this connection that in the case of a semiconductor LD emitting a 1.3 μm wavelength of light, uncoated PMLs on the tip of a circular core fiber produce coupling efficiency of 92% [25]. On the other hand, an ideal LD having a symmetrical modal output and a PML with an anti-reflection coating, has a theoretical coupling efficiency of nearly 100% [20]. Therefore, the Fresnel backward reflection will not affect the coupling when a coated PML is on the tip of the ECSISMF**.**

In Reference [20], it had been shown, for the first time, that prediction of the coupling optics in the case of a PML on the CCSISMF tip has produced excellent results if we employ simple ABCD matrix formalism. In our paper, we use, simple ABCD matrix formalism for refraction by a parabolic interface in order to predict theoretically the coupling losses in the case of LD-to-ECSISMF excitation via a PML on the fiber tip. Further, we employ Gaussian field distributions for both the source and the fiber. The relevant calculations are executable with little computations. The results will be extremely important in the study of optimum coupling optics involving non-circular core fiber, or more specifically, ECSISMF.

2. Analysis

2.1. Formulation of microlens coupling scheme

The coupling scheme to be studied is shown in Figure 1. Here, L is the distance in between the PML end of the fiber and the laser source [20]. Elliptical intensity profiles of the optical beams emitted from LD are approximated by Gaussian spot sizes w_{1x} and w_{1y} along two mutually perpendicular directions, one perpendicular and the other parallel to the junction planes. Again, in our analysis, we use some usual approximations [1,7-10,14] like no transmission loss, Gaussian distributions for both the source and the fiber fields, perfect matching of the polarisation mode of the fiber field, and that on the microlens surface, sufficient angular width of the microlens for the interception of entire power radiated by the source for typical values of the microlens parameters employed. Again, the separation distance between the nearest point of the PML and the LD is also known as working distance.

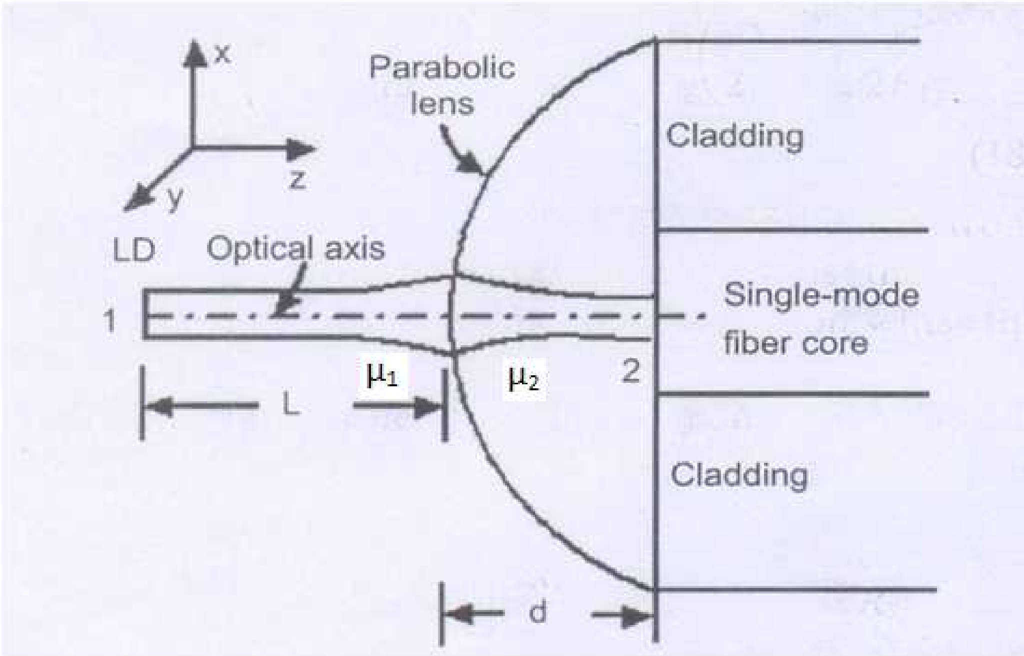

Figure 1: Geometry of laser diode to elliptic core single mode fiber coupling via parabolic microlens on the fiber tip ; μ_1 and μ_2 stand for refractive indices of incident and microlens media respectively.

The field Ψ_u representing the output of the LD at a distance *u* from the PML surface is given by [23,24,26]

$$\Psi_u = \exp\left[-\left(\frac{x^2}{w_{1x}^2}+\frac{y^2}{w_{1y}^2}\right)\right]\exp\left[-\frac{ik_1}{2}.\frac{x^2+y^2}{R_1}\right] \tag{1}$$

Here, w_{1x} and w_{1y} represent the spot sizes of the light beams emitted from the LD along two perpendicular directions X and Y, k_1 is the wave number in the incident medium and R_1 is the radius of curvature of the wavefronts from the laser source. Our analysis is restricted to single frequency laser emitting only one spatial mode with a Gaussian intensity profile.

It has been, already, reported [23,24] that the Gaussian approximations for fundamental mode in an ECSISMF present sufficiently accurate results with respect to coupling losses [9,14]. The corresponding fundamental modal field in the elliptic-core fiber is represented as [9,11,14,24]

$$\Psi_f = \exp\left[-\left(\frac{x^2}{w_{fx}^2}+\frac{y^2}{w_{fy}^2}\right)\right] \tag{2}$$

where w_{fx} and w_{fy} are the respective elliptic-core fiber spot sizes in the X and Y directions and can be analysed with a variational method [23]. Actually, variational technique is one of the conventional methods for prediction of modal solution for step and graded index fibers for the fundamental mode (LP_{01}).It is relevant to mention in this connection that some detailed discussions on variational analysis of propagation characteristics for graded index fibers have been elucidated in References [27,28]..

The PML transformed laser field Ψ_v on the fiber plane 2 as indicated in Figure 1 can be expressed by [23,24]

$$\Psi_v = \exp\left[-\left(\frac{x^2}{w_{2x}^2}+\frac{y^2}{w_{2y}^2}\right)\right]\exp\left[-\frac{ik_2}{2}\left(\frac{x^2}{R_{2x}}+\frac{y^2}{R_{2y}}\right)\right] \tag{3}$$

where k_2 is the wave number in the microlens medium and w_{2x}, w_{2y} are, respectively, microlens transformed spot sizes and R_{2x}, R_{2y} being the respective transformed radii of curvature of the refracted wavefronts in the X and Y directions. It deserves mentioning in this connection that the method of finding w_{2x}, w_{2y}, R_{2x} and R_{2y} in terms of w_{1x}, w_{1y} and R_1 with the relevant ABCD matrix for PML on the fiber tip [20] is once again reported in Appendix for ready reference.

The source to fiber coupling efficiency via PML on the fiber tip is expressed in terms of well known overlap integral by [7,8,24]

$$\eta = \frac{\left|\iint \Psi_v \Psi_f^* dxdy\right|^2}{\iint \left|\Psi_v\right|^2 dxdy \iint \left|\Psi_f\right|^2 dxdy} \tag{4}$$

Employing Eqs. (2) and (3) in Eq. (4), the coupling efficiency is found as [9,11,14,24]

$$\eta_0 = \frac{4w_{2x}w_{2y}w_{fx}w_{fy}}{\left[\left(w_{fx}^2+w_{2x}^2\right)^2+\frac{k_2^2 w_{2x}^4 w_{fx}^4}{4R_{2x}^2}\right]^{1/2}\left[\left(w_{2y}^2+w_{fy}^2\right)^2+\frac{k_2^2 w_{2y}^4 w_{fy}^4}{4R_{2y}^2}\right]^{1/2}} \tag{5}$$

This equation has been employed in the next section to estimate the coupling efficiency from the knowledge of w_{fx},w_{fy}, w_{2x},w_{2y}, R_{2x} and R_{2y}.

3. Results and discussions

3.1. Optogeometrical parameters under consideration

Our formalism employs the ABCD matrix under the paraxial approximation in order to predict the optics involved in the coupling of a LD to ECSISMF via a PML. In order to estimate coupling efficiencies in absence of any possible transverse and angular misalignments for a PML of specific focal parameter on the tip of ECSISMF, we firstly use a LD emitting light of wavelength $\lambda = 1.3\mu m$ with

$w_{1x} = 1.081\mu m, w_{1y} = 1.161\mu m$ [3] and then LD emitting light of wavelength $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$ [3]. The LD parameters used in this investigation are mentioned in Table 1. For the LDs emitting light of abovementioned first wavelength, we study the coupling efficiencies for three ECSISMFs having different aspect ratios. In this context, we use [9,24] three ECSISMFs, numbered as #1, #2 and #3, having same *V* number, 1.4, but different core diameters $(\mu m \times \mu m)$ namely, 9.8×9.7, 8.0 ×4.8, 6.9 ×2.9, with typically low, intermediate and high aspect ratios respectively and thus different w_{fx} ,w_{fy}. In this study, fiber #1 having w_{fx} = 8.32987 μm and w_{fy} = 8.35205 μm , fiber #2 having w_{fx} = 3.73584 μm and w_{fy} = 4.504 μm and fiber #3 having w_{fx} = 2.20734 μm and w_{fy} = 3.17883 μm are used [9]. Again the maximum depth of the microlens *d* is taken as 6 μm [7]. The refractive index $\mu(= \mu_2/\mu_1)$ of the material of the microlens with respect to surrounding medium is once again taken as 1.55 [3,7]. We employ only planar wave model [3,7,8] for the incident ray for the purpose of evaluating the coupling efficiency, since it has been shown that prediction of coupling optics on the basis of spherical model differs slightly from that on the basis of planar wave model. Our aim is to study the crucial role of aspect ratio of the elliptic core fiber in connectivity of PML and ECSISMFs of various aspect ratios where aspect ratio is the ratio of the length of semi-major axis to the length of semi-minor axis in elliptic core.

LD	Wavelength λ in μm	Spot size w_{1x} in μm	Spot size w_{1y} in μm	λ_1 in μm	K_2 in μm^{-1}
#1	1.3	1.081	1.161	0.4138	7.4915
#2	1.5	0.843	0.857	0.4775	6.4926

Table 1: Laser diode parameters

For the values of focal parameter (p) 8.0 [20], 10.0, 12.0 [20], 14.0 and 16.0 μm [20], respectively, we calculate the coupling efficiency changing with the working distance, and the results are shown in Table 2. Then we use a LD emitting light of wavelength $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$ [3]. We compute again relevant working distances with resulting maximum coupling efficiencies for the above same set used in the first part of this investigation and present in Table 3.

Maximum depth of the microlens (d) = 6 μm; Refractive index of the material of the microlens with respect to surrounding medium (μ) = 1.55

Fiber #	p = 8.0 μm	p = 10.0 μm	p = 12.0 μm	p =14.0 μm	P =16.0 μm

	$L(\mu m)$	η_0	$L(\mu m)$	η_0	$L(\mu m)$	η_0	$L(\mu m)$	η_0	$L(\mu m)$	η_0
1	14.5	0.8266	18.0	0.9517	21.5	0.9970	25.0	0.9863	28.4	0.9423
2	13.3	0.9318	15.7	0.8212	17.5	0.7126	18.9	0.6224	19.8	0.5514
3	9.0	0.7249	8.6	0.6348	7.6	0.5793	6.4	0.5456	5.3	0.5248

Table 2: Results for optimum coupling efficiency for elliptic core step index fibers excited with LD#1

Maximum depth of the microlens (d) = 6 μm; Refractive index of the material of the microlens with respect to surrounding medium (μ) = 1.55

Fiber #	$p = 8.0\ \mu m$		$p = 10.0\ \mu m$		$p = 12.0\ \mu m$		$p = 14.0\ \mu m$		$p = 16.0\ \mu m$	
	L (μm)	η_0	L (μm)	η_0	L (μm)	η_0	L (μm)	η_0	L (μm)	η_0
1	14.4	0.9992	18.0	0.9617	21.4	0.8700	24.8	0.7657	28.1	0.6671
2	12.9	0.6711	15.0	0.5277	16.5	0.4288	17.4	0.3608	17.9	0.3134
3	8.0	0.4550	7.0	0.3889	5.6	0.3549	4.4	0.3366	3.3	0.3264

Table 3: Results for optimum coupling efficiency for elliptic core step index fibers excited with LD#2

3.2. Results for coupling scheme without misalignment consideration

From Table 2 and 3, it is clear that the focal parameter of the PML is a key parameter and affects directly the coupling efficiency, which implies that the coupling efficiency can be improved through optimizing the focal parameter. Now from Table 2 for a fit focal parameter, it is observed that the coupling efficiency can reach close to 100% (exactly 99.70%) when p =12.0 μm and L = 21.5 μm for fiber #1 with w_{fx} = 8.32987 μm and w_{fy} = 8.35205 μm when excited with a LD emitting the abovementioned first wavelength.

It is again observed from Table 3 for a fit focal parameter the coupling efficiency can again reach close to 100% (exactly 99.92%) when p = 8.0 μm and L = 14.4 μm for fiber #1 with w_{fx} = 8.32987 μm and w_{fy} = 8.35205 μm when excited with a LD emitting the abovementioned second wavelength.

It is seen from Table 2 and 3 that for LD emitting light of wavelengths $\lambda = 1.3\mu m$ and $\lambda = 1.5\mu m$, this promising aspect is also excellently supported for fiber #1 having $w_{fx} = 8.32987\,\mu m$ and $w_{fy} = 8.35205\,\mu m$ where we have obtained maximum coupling efficiency in both cases. However, the comparison between these results of coupling efficiencies for ECSISMF #1 with $w_{fx} = 8.32987\,\mu m$ and $w_{fy} = 8.35205\,\mu m$ excited with two wavelengths predicts that this specific fiber is most suitable in the context of the aforesaid coupling optics involving ECSISMFs and this excitement is uniquely excellent for a LD #2 emitting specially light of wavelength $\lambda = 1.5\mu m$. Also it is revealed in our study that so far as the demand of achieving the merit of longest working distance to have maximum coupling efficiency using PML [20] on the tip of ECSISMF is concerned, this merit is acquired by the respective fiber #1.

It is easy to see that the core of fiber #1 departs slightly from perfect circularity. The maximum coupling efficiency for a circular-core SMF having w_f of the order of 8.34096 μm [9], which happens to be the mean of w_{fx} and w_{fy} of fiber #1, is nearly 100% (exact value 99.92 % at working distance 14.4 μm for focal parameter 8.0 μm at 1.5 μm and exact value 99.71% at working distance 21.5 μm for focal parameter 12.0 μm at 1.3 μm) and eventually we find that our estimations match excellently with the results found for fiber #1 and hence the validity of our formalism is verified.

From the above analysis, it is evident that for the two LDs emitting light of wavelength $\lambda = 1.3\mu m$ with $w_{1x} = 1.081\mu m, w_{1y} = 1.161\mu m$ and $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$, fiber #1 shows best coupling in both cases. However, the coupling efficiency is maximum for this fiber when excited with LD #2 emitting $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$. Therefore fiber #1 is also most suitable for excitation by a LD #2 emitting light of wavelength $\lambda = 1.5\mu m$ with $w_{1x} = 0.843\mu m, w_{1y} = 0.857\mu m$ from the point of view of excitation efficiencies. Moreover, it may be relevant to mention in this connection that the wavelength at and around 1.5 μm is the region where the erbium doped fiber amplifier and Raman gain fiber amplifier operate efficiently and elegantly.

It is seen that our LDs emitting light for wavelength of 1.3 μm and 1.5 μm have comparable spot sizes with small non-circularity in X and Y direction. Such non-circularity appears to be compatible with the non-circularity of fiber #1 chosen by us out of three fibers and hence produce maximum coupling efficiency in excitation of light. We also see that coupling efficiency with same LD decreases when aspect ratio of the elliptic core fiber increases with the increase of corresponding non-circularity. Hence, one should be alert to recommend and use of LDs with relevant non-circular spot sizes compatible with the non-circularity of elliptic core fibers as supplied.

Thus, the method and results should find wide applications in prediction of optimal launch optics involving PML, which has emerged as a potential candidate.

For a typical estimation of knowledge of excitation via PML with focal parameter 8.0 μm excited with LD #2 emitting light of wavelength $\lambda = 1.5\mu m$ and PML with focal parameter 12.0 μm excited with LD #1 emitting light of wavelength $\lambda = 1.3\mu m$, we present the variation of coupling efficiencies versus the working distance for fiber #1 having w_{fx} = 8.32987 μm and w_{fy} = 8.35205 μm as shown in Figure 2. In this Figure, solid line (____________ denoting EFF0) corresponds to $\lambda = 1.3\mu m$ and focal parameter p = 12.0 μm, dashed line (------------ denoting EFF0DS) to $\lambda = 1.5\mu m$ and focal parameter p = 8.0 μm. We see that although the values of the maximum coupling efficiencies at each typical working distance corresponding to curves are more or less the same, the tolerance is relatively better observed when the corresponding fiber is excited with LD #2 emitting light of wavelength $\lambda = 1.5\mu m$, in addition to achievement of longest working distance.

4. Conclusion

Employing an ABCD matrix for refraction by a PML on the tip of a ECSISMF, we present a simple but accurate theoretical formalism for evaluation of the coupling efficiency of a LD to a fiber via a PML with different focal parameters in absence of possible transverse and angular mismatches for three typical ECSISMFs. Analytical expressions for the analysis are prescribed. The application of an ABCD matrix has simplified the analysis so that the relevant calculations are executable easily. The analysis will be extremely important and useful for the design of optimum launch optics in order to obtain maximum light coupling efficiency in polarisation-maintaining fiber optic sensors and also in coherent fiber optic communication systems. The analysis can also take care of deviation from circularity in case of circular-core SMFs. The PML on the tip of ECSISMF having spot sizes 8.32987 μm and 8.35205 μm in the X and Y directions respectively and lowest aspect ratio has been found to be most suitable in this context. However, the aspect ratio of the elliptic core fiber as well as the focal parameter of the PML are simultaneously playing crucial roles for optimum coupling optics. Moreover, LD emitting light of wavelength λ= 1.5 μm is observed to be superior to that emitting light of wavelength λ= 1.3 μm from the standpoint of relevant coupling optics. However, the excellent agreement of our results for ECSISMF in comparison with the available theoretical results for CCSISMF certifies the correctness of our simple formalism which is executable with little computations. This methodology will benefit the system engineers and packagers working in the field of optimum launch optics involving PML on the fiber tip.

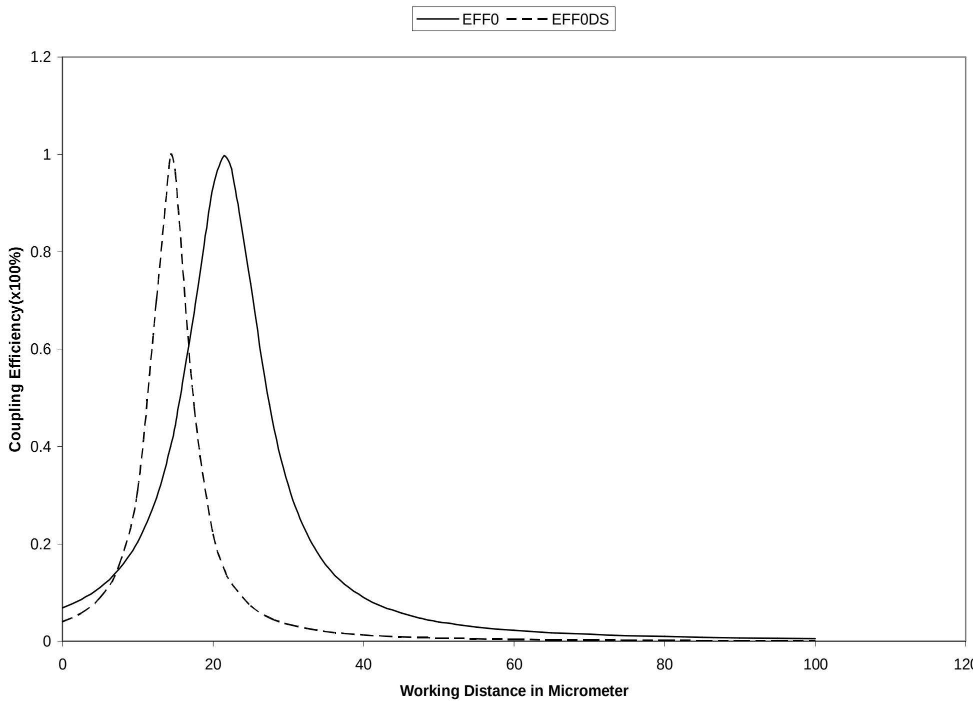

Figure 2. Variation of coupling efficiency versus working distance in Micrometer for specific fiber # 1, excited with LD emitting wavelengths λ = 1.3 μm and λ = 1.5 μm. Solid line (____________ denoting EFF0) corresponds to λ = 1.3 μm and focal parameter p = 12.0 μm, dashed line (------------ denoting EFF0DS) to λ = 1.5 μm and focal parameter p = 8.0 μm.

Appendix:

The relation between input and output parameters (q_1, q_2) of the light beam is given by

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \tag{A1}$$

where

$$\frac{1}{q_{1,2}} = \frac{1}{R_{1,2}} - \frac{i\lambda_0}{\pi w_{1,2}^2 \mu_{1,2}} \tag{A2}$$

with symbols having their usual meanings as already described.

The ray matrix M for the PML on the fiber tip is given by [20]

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1-\mu}{\mu p} & \frac{1}{\mu} \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \tag{A3}$$

where

$$A = 1 + \frac{d(1-\mu)}{\mu p} \tag{A4a}$$

$$B = L + \frac{(1-\mu)Ld}{\mu p} + \frac{d}{\mu} \tag{A4b}$$

$$C = \frac{1-\mu}{\mu p} \tag{A4c}$$

$$D = \frac{1}{\mu} + \frac{(1-\mu)L}{\mu p} \tag{A4d}$$

where p is the focal parameter of the parabola, and L is the working distance which is also the distance of the LD from the PML.

Again, the refractive index of the material of the microlens with respect to the incident medium is represented by $\mu(= \mu_2/\mu_1)$. The transformed beam spot sizes and radii of curvature in the X and Y directions are found by using Eqs. (A4a-A4d) in Eqs. (A1) and (A2) and can be expressed as

$$w_{2x,2y}^2 = \frac{A_1^2 w_{1x,1y}^2 + \frac{\left(\lambda_1^2 B^2\right)}{w_{1x,1y}^2}}{\mu(A_1 D - BC_1)} \tag{A5}$$

$$\frac{1}{R_{2x,2y}} = \frac{A_1 C_1 w_{1x,1y}^2 + \frac{\left(\lambda_1^2 BD\right)}{w_{1x,1y}^2}}{A_1^2 w_{1x,1y}^2 + \frac{\left(\lambda_1^2 B^2\right)}{w_{1x,1y}^2}} \tag{A6}$$

where

$$\lambda_1 = \frac{\lambda}{\pi}\ , \lambda = \frac{\lambda_0}{\mu_1}\ ,\ A_1 = A + \frac{B}{R_1} \quad \text{and } C_1 = C + \frac{D}{R_1}. \tag{A7}$$

In plane wavefront model, the radius of curvature R_1 of the wavefront from the laser facet $\rightarrow\infty$. This leads to $A_1=A$ and $C_1=C$.

Acknowledgements: The author is grateful to University Grants Commission (UGC) for providing financial assistance in a UGC-Minor Research Project (No. PSW-076/14-15 (ERO)). He is also grateful to Prof. S.N. Sarkar, Professor of the Department of Electronic Science, University of Calcutta for his helpful discussions and constant encouragement.

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