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Optimal Velocity Forecast Model for a Car-Following Theory

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ABSTRACT

Previous car-following theory research does not consider the driver's forecast effect. In this Letter, we present a new car-following model with considering the optimal velocity forecast based on the full velocity difference model. The linear stability condition of the new model is obtained by using the linear stability theory. It can be found that the critical value of the sensitivity in the new model decreases and the stable region is apparently enlarged, compared with the FVDM. Finally the numerical results are in good agreement with the theoretical analysis.

Keywords: Car-following model; traffic flow; optimal velocity's forecast; stability analysis

1. Introduction

To today, many traffic models have been developed to explain various complex traffic phenomena since traffic problem has attracted peoples considerable attention [1,2]. And the optimal velocity model (OVM) proposed by Bando et al. [3], which was based on the idea that each vehicle has an optimal velocity, is one of favorable models. Subsequently, much work has been done based on the OVM [4-15]. Furthermore, the comparison with field data suggests that high acceleration and unrealistic deceleration appear in the OVM. To overcome the shortage of the OVM, Helbing and Tilch [4] presented a generalized force model (GFM). But Jiang et al. [5] pointed out that GFM can not describe the delay time and the kinematic wave speed at jam density properly. By taking both positive and negative velocity differences into account, Jiang et al. developed a full velocity difference model (FVDM) [5].

However, the above car-following models did not consider the driver's forecast effect. In particular, the factor about the diver's forecast effect may have important influence on traffic flow. In 2010, Tang et al [4] present a car-following model considering the driver's forecast. But, their model is too easy to describe the traffic phenomena. Their model ignores the leader car and following car's velocity.

In this Letter, we propose a new car-following model with the consideration of the difference of optimal velocity at the Future time and the optimal velocity at the present time on a single lane highway to study the effects of the optimal velocity difference, based on FVDM. Linear stability analysis and numerical simulation will be carried out to indicate that the new model is more reasonable than previous ones.

2. The new model

According to the above mentioned idea, a new optimal velocity forecast car-following model (for short, OVFM) is presented as follows:

$$\ddot{x}_n(t) = \alpha[V(\Delta x_n(t)) - v_n(t)] + k\Delta v_n + \gamma[V(\Delta x_n(t+\tau)) - V(\Delta x_n(t))] \tag{1}$$

where $x_n(t)$ is the position of car n at time t; $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ and $\Delta v_n(t) = v_{n-1}(t) - v_n(t)$ are the headway and the velocity difference between the preceding vehicle n+1 and the following vehicle n, respectively; α is the sensitivity of a driver; V(·) is the optimal velocity function (OVF); $\gamma[V(\Delta x_n(t+\tau)) - V(\Delta x_n(t))]$ is the optimal velocity difference term, γ is the response forecast coefficient of the optimal velocity difference between $V(\Delta x_n(t+\tau))$ and $V(\Delta x_n(t))$, τ is the forecast time. The new model conforms to the FVDM if γ=0. The optimal velocity function is adopted calibrated with observed data by Helbing [2]:

$$V(x) = V_1 + V_2 \tanh(C_1(x - l_c) - C_2) \tag{2}$$

where l_c =5 m is the length of the vehicles. The resulting optimal parameter values are κ=0.85 s−1, $V_1 = 6.75\text{m/s}$, $V_2 = 7.91\text{m/s}$, $C_1 = 0.13\text{m}^{-1}$ and $C_2 = 1.57$.

3. Linear stability analysis

Supposing the vehicles running with the uniform headway b and the optimal velocity $V(b)$, solution of the uniformly steady state for Eq. (1) can be written as follows:

$$x_n^0(t) = bn + V(b)t, \quad \text{with}\, b = L/N, \tag{3}$$

where N is the total number of vehicles, and L is the road length. Let $y_n(t)$ be a small deviation from the uniform solution $x_n^0(t)$, $x_n(t) = x_n^0(t) + y_n(t)$,then ,we have

$$y_n(t) = x_n(t) - x_n^0(t),\ \dot{y}_n(t) = \dot{x}_n(t) - V(b)\ \ \ddot{y}_n(t) = \ddot{x}_n(t), \tag{4}$$

$$\Delta x_n(t) = x_{n-1}(t) - x_n(t) = b + y_{n-1}(t) - y_n(t) = b + \Delta y_n(t), \tag{5}$$

$$\Delta v_n(t) = \dot{x}_{n-1}(t) - \dot{x}_n(t) = \dot{y}_{n-1}(t) - \dot{y}_n(t) = \Delta \dot{y}_n(t). \tag{6}$$

Substituting Eq.(5) into $\Delta x_n(t+\tau)$, we hold

$$\begin{aligned} \Delta x_n(t+\tau) &= \Delta x_n(t) + \Delta \dot{x}_n(t)\tau \\ &= \Delta x_n(t) + \Delta v_n(t)\tau \\ &= b + \Delta y_n(t) + \Delta \dot{y}_n(t)\tau. \end{aligned} \tag{7}$$

Substituting Eq.(4-7) into (1) and linearizing the resulting equation

$$\ddot{y}_n(t) = \alpha[V'(b)\Delta y_n(t) - \dot{y}_n(t)] + k\Delta \dot{y}_n + \gamma\tau V'(b)\Delta \dot{y}_n(t). \tag{8}$$

Assume that $y_n(t) = e^{n\alpha_k i + zt}$, we can get

$$\dot{y}_n(t) = ze^{n\alpha_k i + zt},\ \ddot{y}_n(t) = z^2 e^{n\alpha_k i + zt}, \tag{9}$$

$$\Delta y_n(t) = e^{n\alpha_k i + zt}(e^{\alpha_k i} - 1),\quad \Delta \dot{y}_n(t) = ze^{n\alpha_k i + zt}(e^{\alpha_k i} - 1). \tag{10}$$

Substituting Eq.(9-10) into (8), the Eq.(8) can be change as follow

$$z^2 + [a - k(e^{\alpha_k i} - 1) - \gamma\tau V'(b)(e^{\alpha_k i} - 1)]z = aV'(b)(e^{\alpha_k i} - 1). \tag{11}$$

Solving Eq.(11) with respect to z, we find that the leading term of z is the order of ik. Since $z \to 0$ when $ik \to \infty$, z can be expressed by a long wave as $z = z_1(ik) + z_2(ik)^2 + \cdots$.

Substituting it into Eq.(11) and neglecting the terms with order greater than 2, the two roots of z are obtained

$$z_1 = V'(b)\,,\quad z_2 = \frac{\alpha + 2k + 2\gamma\tau V'(b)}{2\alpha} V'(b) - \frac{(V'(b))^2}{\alpha}\,. \tag{12}$$

If $z_2 < 0$, the uniform steady-state flow becomes unstable, while the uniform flow is stable when $z_2 > 0$. Then, we get the following neutral stability condition

$$V'(b) = (\frac{a}{2} + k + \gamma\tau V'(b))\,. \tag{13}$$

For small disturbances with long wavelengths, the uniform traffic flow is stable if

$$\alpha \geq 2(V'(b) - k - \gamma\tau V'(b))\,. \tag{14}$$

As γ=0, $\tau = 0$, the result of stable conditions is the same as that of FVDM [5]:

$$\alpha \geq 2(V'(b) - k)\,. \tag{15}$$

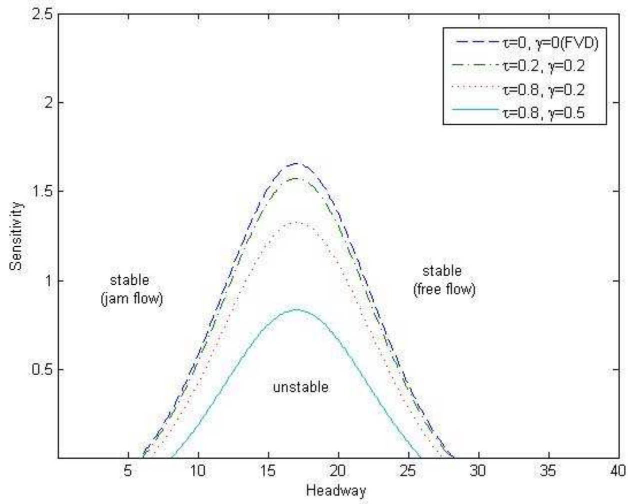

Figure 1: The neutral stability under different τ, γ

Eq.(13) shows that the neutral stability curve will decrease with the forecast effect coefficient γ and the forecast time τ, so the stability of traffic flow will be improved with the increase of the forecast effect coefficient γ and the forecast time τ. Fig.1 shows the neutral stability curves in the space (b,α) under the different parameters (τ,γ). According to the Map, OVFM will put down the neutral stability curve, so the stability region in the space (b,α) will be enlarged.

4. Numerical simulation

In this section, we use numerical simulation to test whether Eq.(1) can describe the effects of optimal velocity forecast. The following simulation is carried out under a periodic boundary condition. The total car number $N = 100$ and circuit length $L = 1500$ m. The related parameters are taken as $\alpha = 1s^{-1}$. The initial disturbance is same as that in Ref. [3]:

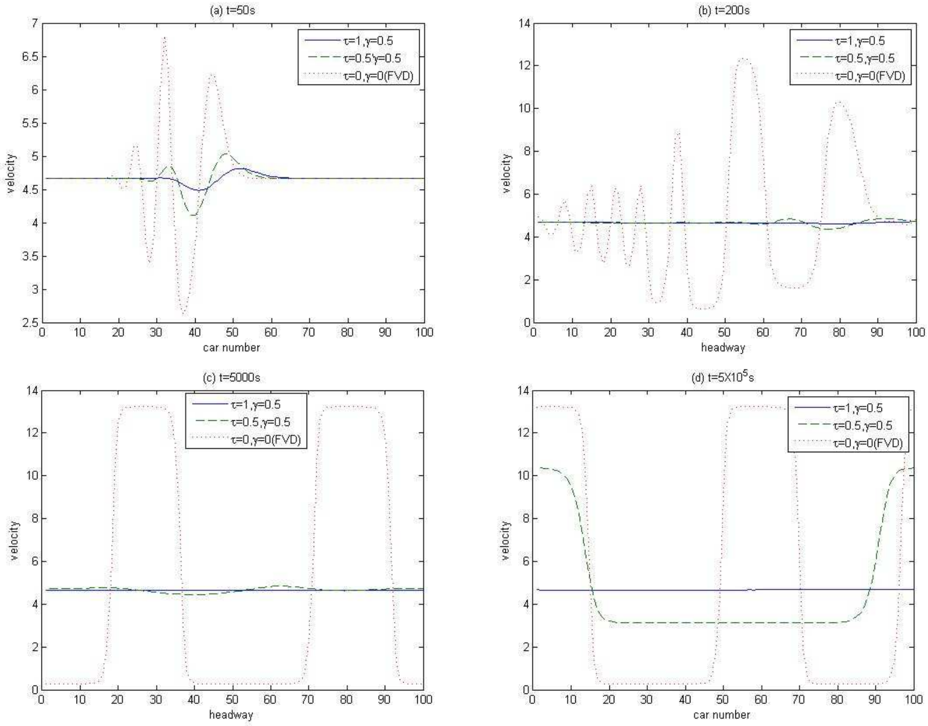

Figure 2: Snapshot of the velocities of all vehicles at different values τ, λ

$$x_1(0) = 10\text{m},\ x_n(0) = (n-1)\frac{L}{N},\ \text{for } n \neq 1,\ \ v_n(0) = V(\frac{L}{N}).$$

We substitute the above parameters in the inequality, where the traffic flow is unstable both in FVDM and the OVFM($\tau = 0.5$). But the traffic flow in the OVFM ($\tau = 1$) is actually stable, according to our simulation, although OVFM ($\tau = 0.5$) does not converge, but its volatility is always very small in $t = 10^4$s ago. Figure 2 and table 1 show the information of the velocities of all vehicles at $t = 50$s, $t = 200$s, $t = 5000$s and $t = 5\times10^5$s for OVFM, respectively. When $\tau = 0$, $\gamma = 0$, the OVFM is change to FVDM.

We can discover from figure2 (a) that the initial disturbance caused the cars' velocity fluctuate nearby 4.6647m/s in OVFM ($\tau = 0.5$) and FVDM, but the velocity perturbation scope in the FVDM is bigger than in the OVFM, the unstable traffic flow in the FVDM evolves into stop-and-go traffic flow. The velocity's fluctuation around 4.6647m/s is not obvious in the OVFM ($\tau = 1, \tau = 0.5$), the traffic flow is still homogenous flow (figure 2(b) and table 1(b)), but the fluctuation of velocity in the OVFM ($\tau = 0.5$)is larger than in the OVFM ($\tau = 1$). When $t = 5000$s, the disturbance of velocity presents obvious changes (figure 2(c)), but the fluctuation of velocity in the OVFM($\tau = 0.5$) is still small, the traffic flow in the OVFM ($\tau = 1$) is homogeneous. Until $t = 5\times10^5$s the vehicles stop at jam region in the FVD model; car's velocity of OVFM ($\tau = 0.5$) at jam region is 3.1223m/s (see table1 (d)), this state is low speed travel while not stationary, but the vehicle are still homogenous flow in the OVFM.

This is consistent with the theoretical analysis. Thus, it can be found that the jam condition in the FVDM is more serious than in the OVFM. It is suggested that the optimal velocity forecast item can weaken the traffic jam. Moreover, the OVFM is similar to the FVDM, the phase change can be found from free flow to jam flow.

Table.1(a)

$t = 50\text{s}$	$\tau = 1, \lambda = 0.5$	$\tau = 0.5, \lambda = 0.5$	$\tau = 0, \lambda = 0$ (FVDM)
Max	4.8116	5.0320	6.8062
Mean	4.6649	4.6656	4.6821
Min	4.4821	4.1128	2.6314
(Max-Mean)/Mean	0.0314	0.0785	0.4537
(Mean-Min)/Mean	0.0392	0.1185	0.4380

Max , Mean , Min represents the maximum, average and minimum speed of one hundred vehicles. (Max-Mean)/Mean and (Mean-Min)/Mean denote the upward and downward volatility.

Table.1(b)

$t = 200\text{s}$	$\tau = 1, \lambda = 0.5$	$\tau = 0.5, \lambda = 0.5$	$\tau = 0, \lambda = 0$ (FVDM)
Max	4.7083	4.8500	12.3715
Mean	4.6647	4.6652	4.9226
Min	4.6135	4.3591	0.6387
(Max-Mean)/Mean	0.0093	0.0396	1.5132
(Mean-Min)/Mean	0.0110	0.0656	0.8703

Table.1(c)

$t = 5000\text{s}$	$\tau = 1, \lambda = 0.5$	$\tau = 0.5, \lambda = 0.5$	$\tau = 0, \lambda = 0$ (FVDM)
Max	4.6655	4.8400	13.2246
Mean	4.6647	4.6652	5.2330
Min	4.6639	4.4491	0.2754
(Max-Mean)/Mean	0.0002	0.0375	1.5271
(Mean-Min)/Mean	0.0002	0.0463	0.9474

Table.1(d)

$t = 5\times10^5\text{s}$	$\tau = 1, \lambda = 0.5$	$\tau = 0.5, \lambda = 0.5$	$\tau = 0, \lambda = 0$ (FVDM)
Max	4.6696	10.3650	13.2246
Mean	4.6647	4.7735	5.2329
Min	4.6588	3.1223	0.2754
(Max-Mean)/Mean	0.0010	1.1714	1.5272
(Mean-Min)/Mean	0.0013	0.3459	0.9474

Table 1: The maximum, average and minimum etc. of the 100 car distribution at different times,(Table.1(a), Table.1(b), Table.1(c), and Table.1(d), indicates $t = 50\text{s}$, $t = 200\text{s}$, $t = 5000\text{s}$ and $t = 5\times10^5\text{s}$ respectively.)

We can also see from Figure 2, in the FVDM, the perturbation propagation speed is quick. When $t = 200\text{s}$, the jam can be found, the velocity of vehicle fluctuates between

0.6387~12.3715m/s (see table1(b)). When $t = 5000\mathrm{s}$, traffic flow in OVFM($\tau = 0.5$) model is still homogenous flow, the velocity of vehicle fluctuates between 4.4491~4.8400m/s (see table1(c)). Although the FVDM and OVFM($\tau = 0.5$) with given parameters are unstable, but compared with the FVDM, the free flow in the OVFM evolved into jam flow requires long time, which suggests that the headway of the initial small perturbation propagates slowly.

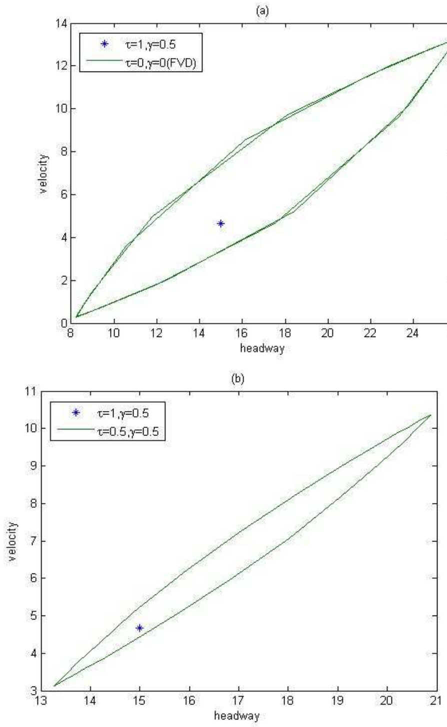

Figure 3: Loops for the OVFM at different values of τ and γ

Moreover, in the phase space (s-v space), the "hysteresis loop" of car motion can be found after enough time as shown in Figure 3 (here, we take $t = 5\times10^5\mathrm{s}$), which suggests that the phase transition from free flow to congestion can also be found in the OVFM. When $\lambda = \tau = 0$, the OVFM model is the same as FVD model. Along with the enlargement of value λ or τ, the "hysteresis loop" is in reduction. When $\tau = 1, \gamma = 0.5$, the condition (14) is satisfied and the system is stable, the "hystresis loop" will not be generated, and in the phase space, there will be only a point on the curve instead.

Model	α	k	γ	τ	δt	c_j
FVDM	0.41	0	0	0	1.4	19.03
OVDM	0.41	0.5	0.5	1	1.2	22.20

Table 2: δt and c_j in OVFM and FVD

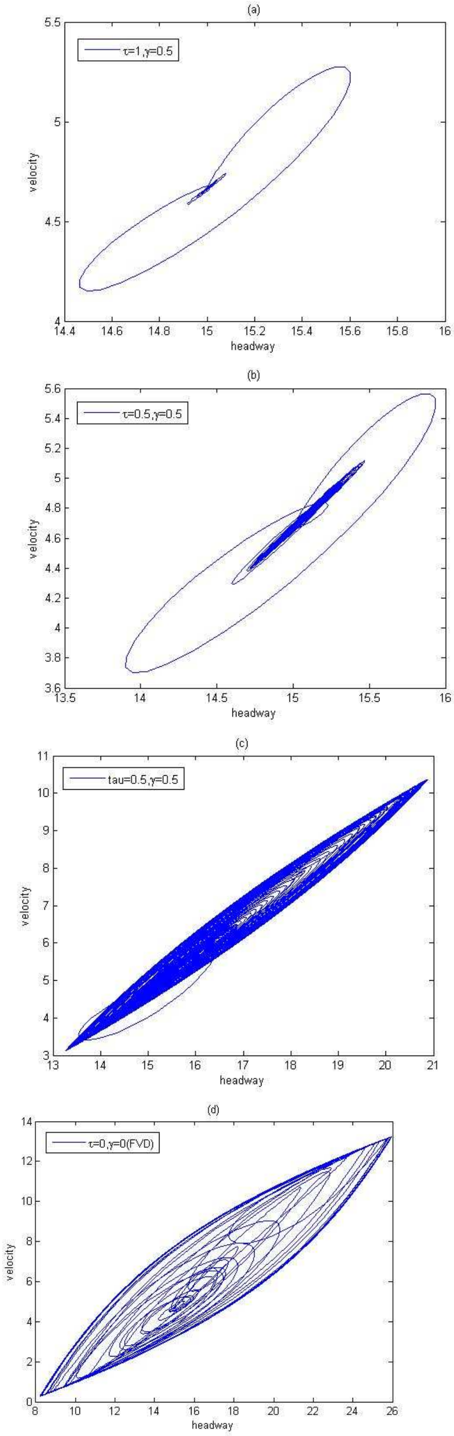

Figure 4: The time evolution curve of “hysteresis loop” ((a),(c): $t \in [0,\ 5\times10^5]$; (b): $t \in [0,\quad 3000]$; (d): $t \in [0,\quad 200]$)

This phenomenon also can be found by the time evolution curve of "hysteresis loop" form the initial conditions of the traffic(Fig. 4). With the time growth, OVFM tends to steady state(figure 4(a)). Hysteresis curve converges to a small area, the lag effect was weaken. The time evolution curve of "hysteresis loop" can be found from time 0s to time 5×10^5 s as shown in Figure 4 (b),(c,),(d) which suggests that the phase transition from free flow to congestion can be found in the OVFM($\tau = 0.5$) and FVDM. But compared with the FVDM, the free flow in the OVFM ($\tau = 0.5$) evolved into jam flow requires very long time.

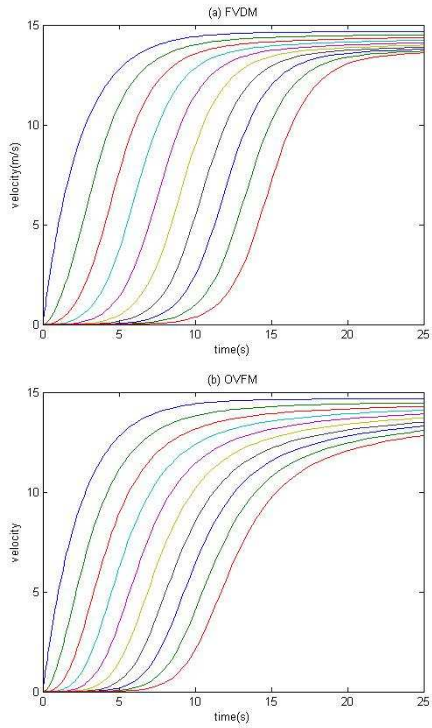

Figure 5: Motions of cars 1-10starting from a traffic signal

Next, we considering ten cars initially at rest with a headway of 7.4m, the leading car is unobstructed. At $t = 0$, the ten cars start up according to OVFM and FVDM respectively. We define the delay time of car motion by δt as that in FVDM. Then we can estimate the kinematic wave speed at jam density $c_j = 7.4/\delta t$. The simulation result are showing in figure 5 and table 2 by adopting the same parameters as those in FVDM. From table 2, we can see that the observed δt is of the order of 1s, just as bando et al .[3] point out and c_j ranges between 17km/h~23km/h[16]. Therefore, OVFM is successful in anticipating the two parameters. Also the delay time δt and the kinematic wave speed c_j approach more exactly to the observed values with condition of τ and γ. We also obtain the acceleration in figure 6, we can see that the maximum value of acceleration in OVFM is not greater than that in FVDM (except the first following car). But for the following car, the car in OVFM accelerates more quickly than the car in FVDM because of the forecast effect in OVFM.

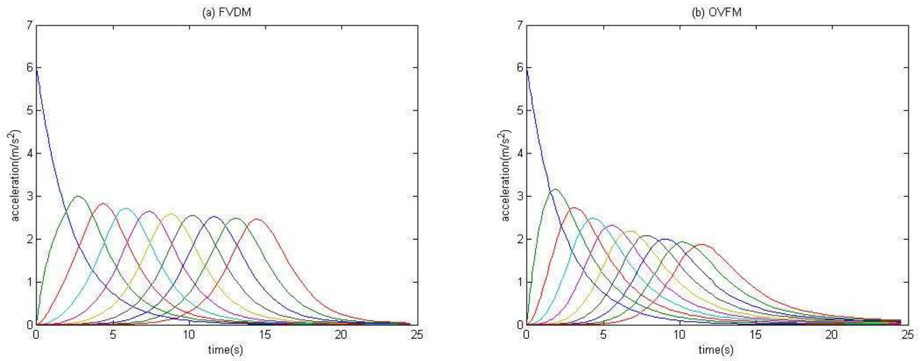

Figure 6: acceleration of leading car and its following vehicles for OVFM and FVDM

5. Conclusion

With the development of Intelligent Transportation Systems, drivers can forecast the future traffic situation. However, the existing traffic flow models can not be used to directly study the driver's forecast effect since they did not consider this factor. In this Letter, we develop a new car-following model with the consideration of the difference of optimal velocity at the Future time and the optimal velocity at the present time. The analytical and numerical results show that the OVFM can enhance the stability of traffic flow and that this stability will be improved with the increase of the parameters γ, τ .

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