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Radiation Effect on Three Dimensional Flow Past a Vertical Porous Plate Through Porous Medium

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ABSTRACT

An analysis is made on the three dimensional flow past a vertical porous plate in the presence of radiation immersed in a porous medium. The solutions have been obtained for the velocity and temperature fields, shear stresses and rate of heat transfer using perturbation technique. It is found that the main flow velocity decreases with increase in radiation parameter for cooling of the plate and increases for heating of the plate. It is also found that with increase in radiation parameter, the main flow velocity increases for cooling of the plate and the reverse effect is observed for heating of the plate. The temperature distribution decreases with the increase of the radiation parameter. The shear stresses and the rate of heat transfer, which are of physical interest are presented in the form of tables.

Keywords: Three-dimension; radiation; incompressible; permeability; periodic-suction.

1. Introduction

The laminar flow is important for its application in engineering, particularly in Aeronautical engineering. The important application is the calculation of friction of bodies in a flow, for examples, the drag of a plate at zero incidence, the friction drag of ship, an airfoil. It is also important the heat transfer between a body and the fluid around it. Singh et al. [1] studied the effect of buoyancy forces on the three dimensional flow and heat transfer along a porous vertical plate. Singh et al. [2] also studied the flow of viscous incompressible fluid along an infinite porous plate subject to the sinusoidal suction velocity distribution fluctuating with time. Sing [3] extended this idea by applying transverse sinusoidal suction velocity in the presence of viscous dissipative heat. Sing [4] also discussed the effect of magnetic field on the three dimensional flow past a porous plate. In the above studies the radiation effect is ignored. It has important application in space vehicle re-entry problems. Many processes in engineering areas occur at high temperatures and it is important for the design of pertinent equipment. Nuclear power plants, gas turbines, and the various propulsion devices for aircraft missiles, satellites and space vehicles are example of such engineering areas. At high temperature radiation effect can be quite significant. The heating of rooms and buildings by the use of radiators is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens etc surrounded by cooler air, are at least in part due to free convection. Hassan [5] and Raptis and Perdikis [6] studied the effect of radiation on the flow of micropolar and viscoelastic fluid respectively. Seddeek [7] also studied the effect of radiation past a moving plate with variable viscosity. The effect of radiation on the flow past a vertical plate was discussed by Takhar et al. [8]. Rapits [9] also studied the effect of radiation and free convection on steady flow past a vertical porous plate through porous medium. Sharma et al. [10] studied the effect of radiation on temperature distribution in three-dimensional Coutte flow subjected to a periodic suction velocity distribution. Recently Guria et al. [11] investigated the effect of radiation on three dimensional flow in a vertical channel subjected to a periodic suction. Guria et al. [12] studied the effect of radiation on three dimensional flow past a vertical porous plate in the presence of magnetic field. The aim of this paper is to study the effect of radiation on three dimensional flow past a vertical porous plate subject to the periodic suction velocity distribution through porous medium.

2. Formulation of the problem

Consider the unsteady flow of viscous, incompressible fluid past a semi- infinite vertical porous plate through porous medium. Here the x^{Σ}-axis is chosen along the vertical plate, that is, in the direction of the flow, y^{Σ}- axis is perpendicular to the plate and z^{Σ}- axis is normal to the $x^{\Sigma}y^{\Sigma}$- plane (Fig.1). All the fluid properties are considered constant except the influence of the density variation with temperature is considered only in the body force term. The plate is considered to be infinite length, all derivatives with respect to x^* vanish and so the physical variables are functions of $y^*, z^*,$ and t^* only.

The plate is subjected to a periodic suction velocity distribution of the form

$$v^* = -V_0\left[1+\varepsilon\cos\left(\frac{\pi u_\infty z^*}{\nu}-ct^*\right)\right], \tag{1}$$

where $\varepsilon(=1)$ is the amplitude of the suction velocity. V_0 is the constant suction, u_∞ is the free stream velocity, ν is the kinematic coefficient of viscosity and t^{Σ} is the time. Denoting velocity components $u^{\Sigma}, v^{\Sigma}, w^{\Sigma}$ in the directions $x^{\Sigma}-, y^{\Sigma}-$ and $z^{\Sigma}-$ axes respectively, under Bousinesq approximation, the flow is governed by the following equations.

$$\frac{\partial v^{\Sigma}}{\partial y^{\Sigma}}+\frac{\partial w^{\Sigma}}{\partial z^{\Sigma}}=0, \tag{2}$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=\nu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)+g\beta(T-T_\infty)-\frac{\nu\left(u^*-u_\infty\right)}{K^*}, \tag{3}$$

$$\frac{\partial v^*}{\partial t^*}+v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-\frac{\nu v^*}{K^*}, \tag{4}$$

$$\frac{\partial w^*}{\partial t^*}+v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z^*}+\nu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\nu w^*}{K^*}, \tag{5}$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right)-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}, \tag{6}$$

where ρ is the density of the fluid, p^{Σ} is the fluid pressure, g is the acceleration due to gravity, β is the coefficient of thermal expansion, k is the coefficient of heat conduction, C_p is the specific heat at constant pressure K^* is the permeability of the porous medium. The equation of conservation of radiative heat transfer per unit volume for all wavelength is

$$\nabla.q_r^*=\int_0^\infty K_\lambda(T^*)(4e_{\lambda h}(T^*)-G_\lambda)d\lambda,$$

where $e_{\lambda h}$ is the Plank's function and the incident radiation G_λ is defined as

$$G_\lambda=\frac{1}{\pi}\int_{\Omega=4\pi}e_\lambda(\Omega)d\Omega,$$

$\nabla.q_r^*$ is the radiative flux divergence and Ω is the solid angle. Now, for an optically thin fluid exchanging radiation with an isothermal flat plate at temperature T_0 and according to the above definition for the radiative flux divergence and Kirchhoffs law, the incident radiation is given by $G_\lambda=4e_{\lambda h}(T_0)$ then,

$$\nabla.q_r^*=4\int_0^\infty K_\lambda(T^*)(e_{\lambda h}(T^*)-e_{\lambda h}(T_0))d\lambda,$$

Expanding $K_\lambda(T^*)$ and $e_{\lambda h}(T_0)$ in a Taylor series around T_0, for small (T^*-T_0), we can rewrite the radiative flux divergence as

$$\nabla.q_r^*=4(T^*-T_0)\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda,$$

where $K_{\lambda_0}=K_{\lambda(T_0)}$.

Hence an optical thin limit for a non-gray gas near equilibrium, the following relation holds

$$\nabla.q_r^*=4(T^*-T_0)I,$$

and hence

$$\frac{\partial q_r^*}{\partial y^*}=4(T^*-T_0)I,$$

where

$$I=\int_0^\infty K_{\lambda_0}\left(\frac{\partial e_{\lambda h}}{\partial T}\right)_0 d\lambda.$$

The boundary conditions of the problem are

$$u^*=0, v^*=-V_0\left[1+\varepsilon\cos\left(\frac{\pi u_\infty z^*}{\nu}-ct^*\right)\right], w^*=0, T^*=T_0 \text{ at } y^*=0,$$

$$u^{\Sigma} = u_{\infty}, v^{\Sigma} = -V_0, w^{\Sigma} = 0, p^{\Sigma} = p_{\infty}, T^{\Sigma} = T_{\infty} \quad \text{as} \quad y^{\Sigma} \to \infty. \tag{7}$$

Introduce the non-dimensional variables

$$y = \frac{u_{\infty} y^{\Sigma}}{\nu}, z = \frac{u_{\infty} z^{\Sigma}}{\nu}, t = ct^{\Sigma}, p = \frac{p^{\Sigma}}{\rho u_{\infty}^2},$$

$$u = \frac{u^*}{u_{\infty}}, v = \frac{v^*}{u_{\infty}}, w = \frac{w^*}{u_{\infty}}, \theta = \frac{\left(T^* - T_{\infty}\right)}{T_0 - T_{\infty}}. \tag{8}$$

Using (8), equations (2)-(6) become

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{9}$$

$$\omega \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + Gr\theta - \frac{(u-1)}{K}, \tag{10}$$

$$\omega \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{v}{K}, \tag{11}$$

$$\omega \frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{w}{K} \tag{12}$$

$$\omega \frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial y} + w \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right) - F\theta, \tag{13}$$

where $Gr = \frac{g\beta(T_0 - T_{\infty})\nu}{u_{\infty}^3}$, the Grashof number, $\omega = \frac{c\nu}{u_{\infty}^2}$, the frequency parameter and $Pr = \frac{\rho\nu C_p}{k}$, the Prandtl number, $K = \frac{u_{\infty}^2 K^*}{\nu^2}$, the permeability parameter, $F = \frac{4I\nu}{\rho C_p u_{\infty}^2}$, the radiation parameter, T_{∞} and p_{∞} are the temperature and pressure outside the boundary layer. The boundary conditions (7) become

$$u = 0, w = 0, v = -S\left[1 + \varepsilon\cos\left(\pi z - t\right)\right], \theta = 1 \quad \text{at} \quad y = 0,$$

$$u = 1, w = 0, v = -S, \theta = 0 \quad \text{as} \quad y \to \infty, \tag{14}$$

where $S = V_0/u_{\infty}$ is the suction parameter.

3. Solution of the problem

To solve the equations (9)-(13), we assume the solution of the following form

$$u(y,z,t) = u_0(y) + \varepsilon u_1(y,z,t) + \varepsilon^2 u_2(y,z,t) + \cdots,$$

$$v(y,z,t) = v_0(y) + \varepsilon v_1(y,z,t) + \varepsilon^2 v_2(y,z,t) + \cdots,$$

$$w(y,z,t) = w_0(y) + \varepsilon w_1(y,z,t) + \varepsilon^2 w_2(y,z,t) + \cdots,$$

$$p(y,z,t) = p_0(y) + \varepsilon p_1(y,z,t) + \varepsilon^2 p_2(y,z,t) + \cdots, \tag{15}$$

$\theta(y,z,t) = \theta_0(y) + \varepsilon\theta_1(y,z,t) + \varepsilon^2\theta_2(y,z,t) + \cdots.$

Substituting (15) in equations (9) to (13), comparing the term free from ε and the coefficients of ε from both sides and neglecting those of ε^2. The term free from ε are

$$v_0' = 0, \tag{16}$$

$$u_0'' - v_0 u_0' - \frac{u_0}{K} = -Gr\theta_0 - \frac{1}{K}, \tag{17}$$

$$\theta_0'' - v_0 Pr\theta_0' - FPr\theta_0 = 0, \tag{18}$$

where the primes denote differentiation with respect to y.

The boundary conditions become

$$u_0 = 0, v_0 = -S, \theta_0 = 1 \quad \text{at } y = 0, \text{ and } u_0 = 1, v_0 = -S, \theta_0 = 0 \quad \text{as } y \to \infty. \tag{19}$$

The solutions of (16)- (18) under the boundary conditions (19) are

$$v_0(y) = -S, \ \theta_0(y) = e^{-\lambda_1 y}, \tag{20}$$

$$u_0(y) = 1 - Ae^{-\lambda_1 y} + Be^{-m_1 y}$$

$$\text{where, } \lambda_1 = \frac{SP_r + \sqrt{S^2Pr^2 + 4FPr}}{2}, \quad m_1 = \frac{S + \sqrt{S^2 + 4/K}}{2} \tag{21}$$

$$A = \frac{Gr}{\lambda_1^2 - S\lambda_1 - 1/K}, \quad B = \frac{Gr}{\lambda_1^2 - S\lambda_1 - 1/K} - 1$$

If $F = 0$, $K \to \infty$. Then the solution (20) coincides with equation (3.6) of Guria and Jana [13]. Equating the coefficient of ε from both sides, we get

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \tag{22}$$

$$\omega\frac{\partial u_1}{\partial t} + v_1\frac{du_0}{dy} - S\frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} + Gr\theta_1 - \frac{u_1}{K}, \tag{23}$$

$$\omega\frac{\partial v_1}{\partial t} - S\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} - \frac{v_1}{K}, \tag{24}$$

$$\omega\frac{\partial w_1}{\partial t} - S\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} - \frac{w_1}{K}, \tag{25}$$

$$\omega\frac{\partial \theta_1}{\partial t} + v_1\frac{d\theta_0}{dy} - S\frac{\partial \theta_1}{\partial y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - F\theta_1. \tag{26}$$

The boundary conditions become

$$u_1 = 0, v_1 = -S\cos(\pi z - t), w_1 = 0, \theta_1 = 0 \quad \text{at } y = 0,$$

$$u_1 = 0, v_1 = 0, w_1 = 0, \theta_1 = 0, p_1 = 0. \quad \text{as } y \to \infty. \tag{27}$$

These are the linear partial differential equations describing the three-dimensional flow. We assume the velocity components, pressure and temperature distribution in the following form

$$u_1(y,z,t) = u_{11}(y)e^{i(\pi z - t)},$$
$$v_1(y,z,t) = v_{11}(y)e^{i(\pi z - t)},$$
$$w_1(y,z,t) = \frac{i}{\pi}v_{11}'(y)e^{i(\pi z - t)}, \tag{28}$$
$$p_1(y,z,t) = p_{11}(y)e^{i(\pi z - t)},$$
$$\theta_1(y,z,t) = \theta_{11}(y)e^{i(\pi z - t)}.$$

Substituting (28) in (23)-(26), we get the following set of differential equations

$$v_{11}'' + Sv_{11}' - \left(\pi^2 - i\omega + 1/K\right)v_{11} = p_{11}', \tag{29}$$

$$v_{11}''' + Sv_{11}'' - \left(\pi^2 - i\omega + 1/K\right)v_{11}' = \pi^2 p_{11}, \tag{30}$$

$$\theta_{11}'' + SPr\theta_{11}' - \left(\pi^2 - i\,\mathrm{Pr}\,\omega + F\,\mathrm{Pr}\right)\theta_{11} = Prv_{11}\theta_0', \tag{31}$$

$$u_{11}'' + Su_{11}' - \left(\pi^2 - i\omega + 1/K\right)u_{11} = v_{11}u_0' - Gr\theta_{11}. \tag{32}$$

The boundary conditions become

$$u_{11} = 0,\ v_{11} = -S,\ v_{11}' = 0,\ \theta_{11} = 0 \quad \text{at} \quad y = 0,$$
$$u_{11} = 0,\ v_{11} = 0,\ w_{11} = 0,\ \theta_{11} = 0 \quad \text{as} \quad y \to \infty. \tag{33}$$

Solving (29)- (32), under the boundary conditions (33), and on using (28), we get

$$v_1(y,z,t) = \frac{S}{(\pi - m_2)}\left[m_2 e^{-\pi y} - \pi e^{-m_2 y}\right]e^{i(\pi z - t)},$$

$$w_1(y,z,t) = \frac{iSm_2}{(\pi - m_2)}\left[e^{-m_2 y} - e^{-\pi y}\right]e^{i(\pi z - t)},$$

$$p_1(y,z,t) = \frac{Sm_2}{\pi(\pi - m_2)}\left(S\pi - i\omega + 1/k\right)e^{-\pi y}e^{i(\pi z - t)},$$

$$\theta_1(y,z,t) = \frac{SPr\lambda_1}{(\pi - m_2)}\left[B_1 e^{-\lambda_2 y} - B_2 e^{-(\lambda_1 + \pi)y} - B_3 e^{-(\lambda_1 + m_2)y}\right]e^{i(\pi z - t)},$$

$$u_1(y,z,t) = \frac{S}{(\pi - m_2)}\left[C_1 e^{-m_2 y} + C_2 e^{-(\lambda_1 + \pi)y} + C_3 e^{-(\lambda_1 + m_2)y}\right. \tag{34}$$
$$\left. + C_4 e^{-(m_1 + \pi)y} + C_5 e^{-(m_1 + m_2)y} + C_6 e^{-(\lambda_2 y)}\right]e^{i(\pi z - t)},$$

where

$$m_2 = \frac{S + \sqrt{S^2 + 4(\pi^2 - i\omega + i/k)}}{2}, \quad \lambda_2 = \frac{SP_r + \sqrt{S^2 P_r^2 + 4(\pi^2 - iP_r\omega + FP_r)}}{2}$$

$$B_2 = \frac{m_2}{P_r(i\omega - \pi S) + 2\lambda_1\pi}, \quad B_3 = \frac{-\pi}{m_2(m_2 + 2\lambda_1 - SP_r) - \pi^2 + iPr\omega}$$

$$B_1 = B_2 + B_3 \qquad C_2 = \frac{\lambda_1(Am_2 + GrPrB_2)}{(\lambda_1 + \pi)^2 - S(\lambda_1 + \pi) - (\pi^2 - i\omega + 1/K)},$$

$$C_3 = \frac{-(A\pi - GrPrB_3)}{(\lambda_1 + 2m_2 - S)} \qquad C_4 = \frac{-Bm_1 m_2}{(2m_1\pi - S\pi + i\omega)},$$
$$C_5 = \frac{Bm_1\pi}{(2m_1 m_2 + 1/K)}, \qquad C_6 = \frac{-GrPr\lambda_1 B_1}{\lambda_2^2 - S\lambda_2 - (\pi^2 - i\omega + 1/K)},$$
$$C_1 = -(C_2 + C_3 + C_4 + C_5 + C_6). \tag{35}$$

Particular cases:-

Case I: When $K \to \infty$, the problem is reduced to the three dimensional flow subject to periodic suction in the presence of radiation. In this case

$$m_1 = S, \quad m_2 = \frac{S + \sqrt{S^2 + 4(\pi^2 - i\omega)}}{2}.$$

Hence the solution becomes

$$v_1(y,z,t) = \frac{S}{(\pi - m_2)}\left[m_2 e^{-\pi y} - \pi e^{-m_2 y}\right]e^{i(\pi z - t)},$$
$$w_1(y,z,t) = \frac{iSm_2}{(\pi - m_2)}\left[e^{-m_2 y} - e^{-\pi y}\right]e^{i(\pi z - t)},$$
$$p_1(y,z,t) = \frac{Sm_2}{\pi(\pi - m_2)}\left(S\pi - i\omega\right)e^{-\pi y}e^{i(\pi z - t)},$$
$$\theta_1(y,z,t) = \frac{SPr\lambda_1}{(\pi - m_2)}\left[B_1 e^{-\lambda_2 y} - B_2 e^{-(\lambda_1 + \pi)y} - B_3 e^{-(\lambda_1 + m_2)y}\right]e^{i(\pi z - t)},$$
$$u_1(y,z,t) = \frac{S}{(\pi - m_2)}\Big[C_1 e^{-m_2 y} + C_2 e^{-(\lambda 1 + \pi)y} + C_3 e^{-(\lambda_1 + m_2)y}$$
$$+ C_4 e^{-(m_1 + \pi)y} + C_5 e^{-(m_1 + m_2)y} + C_6 e^{-(\lambda_2 y)}\Big]e^{i(\pi z - t)}. \tag{36}$$

Case II: When $F = 0$, the problem is reduced to the three dimensional flow subject to periodic suction through porous medium. Then

$$\lambda_1 = SPr, \quad \lambda_2 = \frac{SPr + \sqrt{S^2 Pr^2 + 4(\pi^2 - iPr\omega)}}{2}.$$

Hence the solution becomes

$$\theta_1(y,z,t) = \frac{S^2 Pr}{(\pi - m_2)}\left[B_1 e^{-\lambda_2 y} - B_2 e^{-(SPr + \pi)y} - B_3 e^{-(SPr + m_2)y}\right]e^{i(\pi z - t)},$$
$$u_1(y,z,t) = \frac{S}{(\pi - m_2)}\Big[C_1 e^{-m_2 y} + C_2 e^{-(SPr + \pi)y} + C_3 e^{-(SPr + m_2)y}$$
$$+ C_4 e^{-(m_1 + \pi)y} + C_5 e^{-(m_1 + m_2)y} + C_6 e^{-(\lambda_2 y)}\Big]e^{i(\pi z - t)}. \tag{37}$$

Case III: When $F = 0$ and $K \to \infty$, the solution (34) coincide with the solution (3.19) of Guria and Jana $[13]$.

4. Results and discussion

We have computed the numerical value of the velocity, temperature, shear stresses, and rate of heat transfer for different values of the non dimensional parameters and plotted in the diagram.The value of dimensionless parameter Gr is taken positive and negative values. The positive value corresponds to an extremely cooled plate by the free convection currents and the negative value corresponds to the hotted plate. The value of Prandtl number is taken equal to 0.71 and this value corresponds to the air. The values of Grashof numbers (Gr) are taken to be large from the physical point of view. The large Grashof number values correspond to free convection problem.The effect of radiation parameter, Prandtl number, permeability parameter and suction parameter on main flow velocity is shown in Figs.2-5.

The effect of radiation parameter F on the main flow velocity is shown in Fig.2. for cooling and heating plate. This figure shows that velocity decreases with the increase of the radiation parameter for cooling of the plate and increases for heating of the plate.

Fig.3. shows the effect of Prandtl number on the main flow velocity profile for both cooling and heating of the plate. From Fig.3 we see that for cooling of the plate velocity profile decreases whereas these profile increases with the increase of Pr for heating of the plate.

Fig.4 shows the effects of permeability parameter on the main flow velocity for cooling and heating of the plate. For a cooling plate fluid velocity increases whereas for a heating plate it decreases with increase of K. Permeability parameter is the measurement of the porosity of the medium. As the porosity of the medium increases, the value of K increases. For large porosity of the medium fluid gets more space to flow as a consequence its velocity increases.

The effects of suction parameter S on the main flow velocity is shown in Fig.5. From Fig.5 we found that the velocity decreases with increase in suction for cooling of the plate and increases for heating of the plate. Thus suction stabilities the boundary layer growth.

The effects of permeability parameter K and suction parameter S on the cross flow velocity are shown in Fig.6. It is observed from this figure that both permeability parameter and suction parameter have decreasing effect on the cross velocity near to the plate while increasing effect away from the plate

Fig.7 shows the effect of F on the temperature profiles. It is clear that temperature decrease more rapidly with the increase of F. Therefore using radiation we can control the flow characteristic and temperature distribution.

The effect of suction parameter and Prandtl number on the temperature profile is shown in Fig.8. From Fig.8 we found that temperature decreases with the increase of both suction parameter and Prandtl number. The Prandtl number has decreasing effect on the temperature profile.It is also found that an increase in Pr decreases the temperature field indicating that the temperature field falls more rapidly for water in comparison to air. The phenomenon is that the temperature field remains almost stationary for mercury which is most sensible towards change in temperature. This leads to the conclusion that mercury is most effective for maintaining temperature differences and can be efficiently used in laboratory purposes. The air may replace mercury but the effectiveness of maintaining temperature changes is much less than mercury. However, air can be a better

and cheap replacement if the temperature is maintained for industrial purposes.
The non dimensional shear stress component due to main flow can be expressed as

$$\tau_x = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u_0'(0) + \varepsilon u_1'(0), = u_0'(0) + \varepsilon u_{11}'(0) e^{i(\pi z - t)},$$

$$= u_0'(0) + \varepsilon |R_1| \cos(\pi z - t + \phi_1), \tag{38}$$

where $R_1 = \sqrt{u_{11r}^2 + u_{11i}^2}$ and $\tan\phi_1 = \frac{u_{11i}}{u_{11r}}$.

The non dimensional shear stress component due to cross flow can be expressed as

$$\tau_z = \left(\frac{\partial \omega}{\partial y}\right)_{y=o} = \varepsilon \omega_1'(0) e^{i(\pi z - t)}, = \varepsilon \omega_{11}'(0),$$

$$= \varepsilon |R_2| \cos(\pi z - t + \phi_2), \tag{39}$$

where $R_2 = \sqrt{w_{11r}^2 + w_{11i}^2}$ and $\tan\phi_2 = \frac{w_{11i}}{w_{11r}}$.

The magnitude and the tangent of phase shift of the shear stress due to main flow is shown in Table 1 for $Pr = 0.025$.

	R_1			$-\tan\phi_1$		
F	$K=1$	$K=5$	$K=10$	$K=1$	$K=5$	$K=10$
2	7.07	8.83	9.40	5.21	4.15	3.97
3	6.20	7.72	8.15	5.89	4.84	4.63
4	5.97	7.26	7.59	6.95	5.73	6.80
5	5.93	6.98	7.24	8.41	6.80	6.44

Table 1: Variation of magnitude and tangent of phase shift due to main flow

The magnitude of the shear stress increases where the tangent of phase shift decreases with increase in permeability parameter. With increase in radiation parameter, the magnitude of the shear stress decreases where the tangent of phase shift increases. The effect of permeability parameter on magnitude and tangent of phase shift due to cross flow is shown in Table 2.

K	R_2	$-\tan\varphi_2$
1	3.91	53.44
5	3.80	50.17
10	3.78	49.76

Table 2: Variation of magnitude of tangent of phase shift due to cross flow

It is seen from table that both the magnitude and tangent of phase shift decrease with increase in permeability parameter. Now we calculate the rate of heat transfer. The rate of heat transfer at the plate $y = 0$ is given by

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \theta_0'(0) + \varepsilon\theta_1'(0), = \theta_0'(0) + \varepsilon\theta_{11}'(0)e^{i(\pi z - t)},$$

$$= \theta_{0'}(0) + \varepsilon\,|\,R_3\,|\cos\left(\pi z - t + \phi_3\right), \qquad (40)$$

where $R_3 = \sqrt{\theta_{11r}^2 + \theta_{11i}^2}$ and $\tan\phi_3 = \dfrac{\theta_{11i}}{\theta_{11r}}$.

We have computed the magnitude and the tangent of phase shift for $S = 0.5$ and shown in Table 3. It is observed from table that both the magnitude and the tangent of phase shift decrease with increase in radiation parameter whereas both magnitude and tangent of phase shift increase with increase in permeability parameter.

	R_3				$-\tan\varphi_2$			
K \ F	1/10	1	5	10	1/10	1	5	10
2	0.83	1.02	1.03	1.04	12.98	26.48	31.21	31.97
3	0.77	0.95	0.97	0.97	9.92	16.29	17.85	18.08
4	0.73	0.90	0.91	0.92	8.30	12.43	13.30	13.43
5	0.70	0.85	0.87	0.88	7.26	10.34	10.93	11.01

Table 3: Variation of magnitude and the tangent of phase shift of the rate of heat transfer for S=1.0.

5. Conclusion

In this paper we have studied the effect of radiation, free convection and permeability of the medium on three dimensional flow past a vertical porous plate. The dimensionless governing partial differential equations are solved by perturbation technique. The effect of non dimensional parameters such as radiation parameter, Prandtl number, permeability parameter, Grashof number and suction parameter on velocity and temperature fields are studied. Conclusion of the study are follows.

(i) The momentum and thermal boundary layers are found to thicken when the radiation is present.

(ii) Suction stabilizes the boundary layer growth.

(iii) The velocity and temperature field falls owing to increase in the Prandtl number.

(iv) Permeability parameter are leads to the increase of the main velocity profile.

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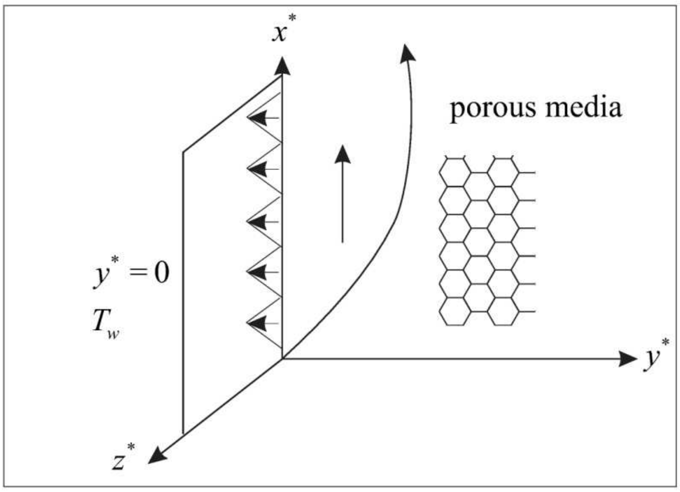

Figure 1: Physical model and Co-ordinates system

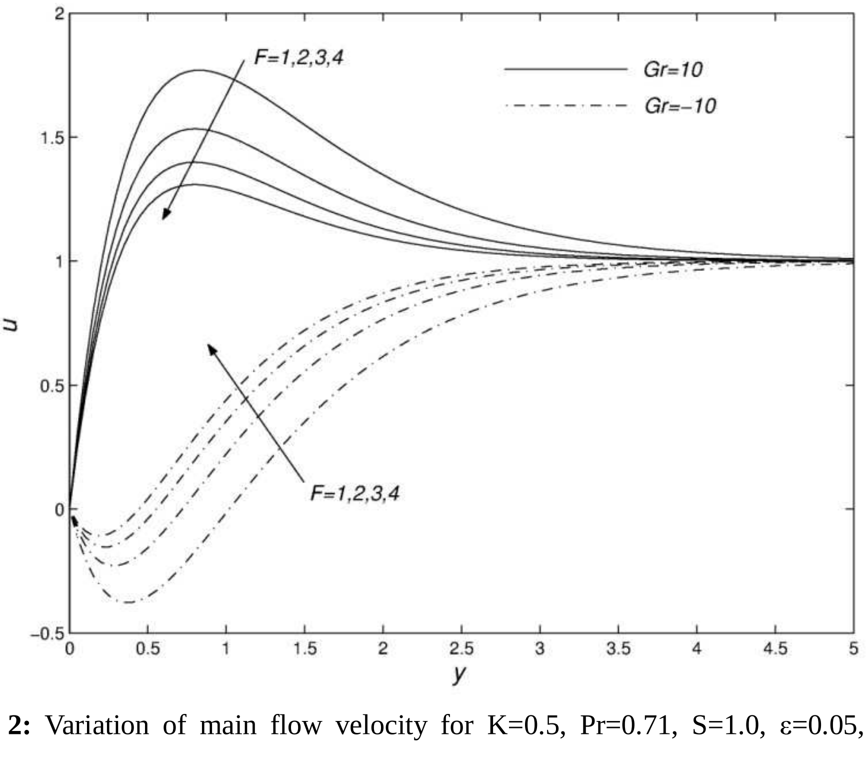

Figure 2: Variation of main flow velocity for K=0.5, Pr=0.71, S=1.0, ε=0.05, z=0.0

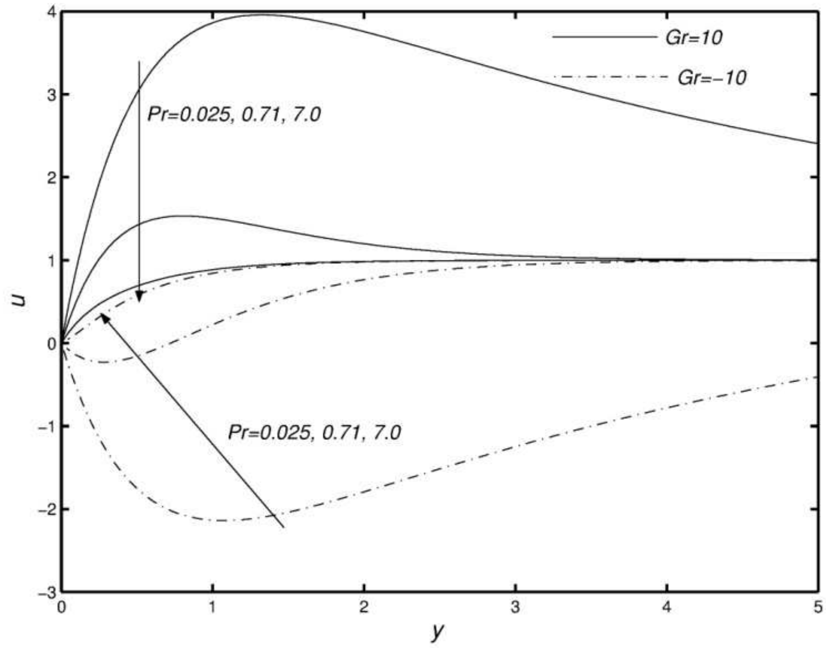

Figure 3: Variation of main flow velocity for F=2.0, S=1.0, K=0.5, ε=0.05, z=0.0.

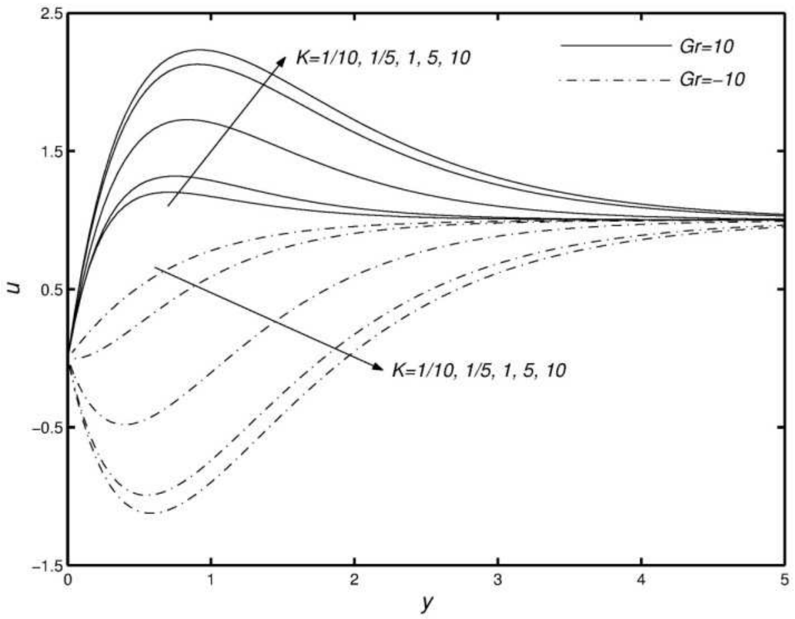

Figure 4: Variation of main flow velocity for F=2.0, Pr=0.71, S=1.0, ε=0.05,z=0.0.

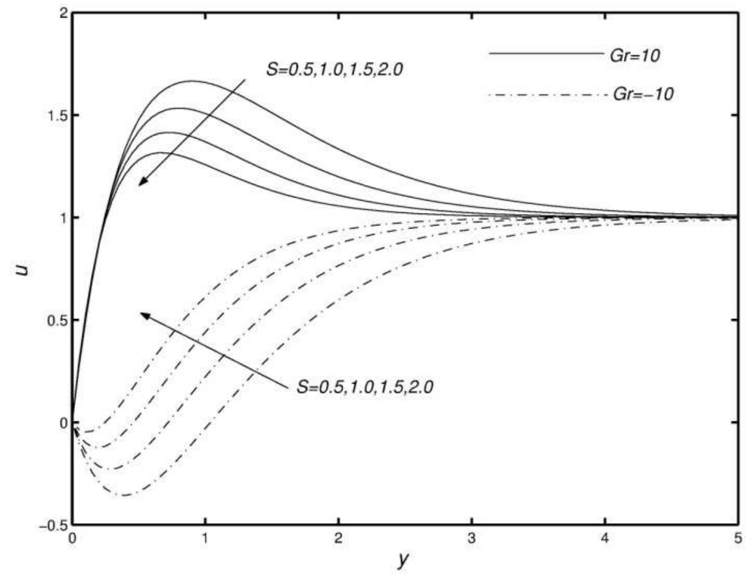

Figure 5: Variation of main flow velocity for F=2.0, Pr=0.71, K=0.5, ε=0.05, z=0.0

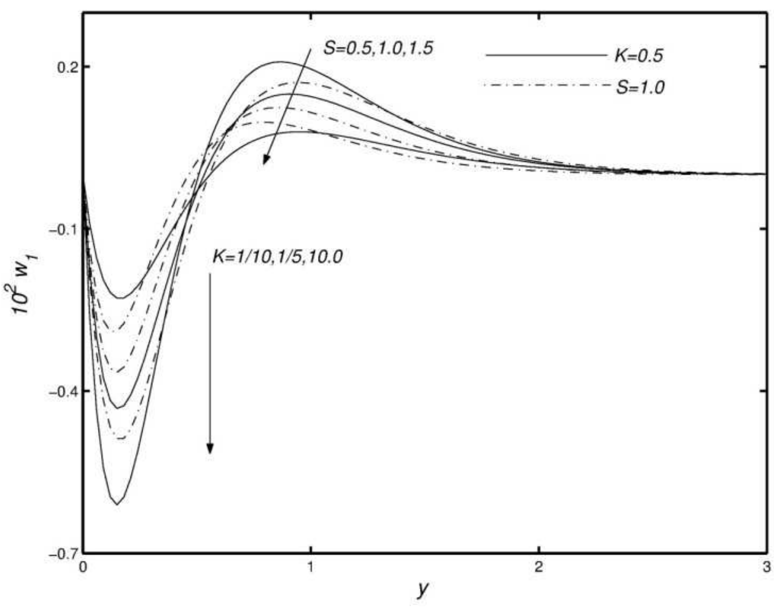

Figure 6: Variations of cross flow velocity.

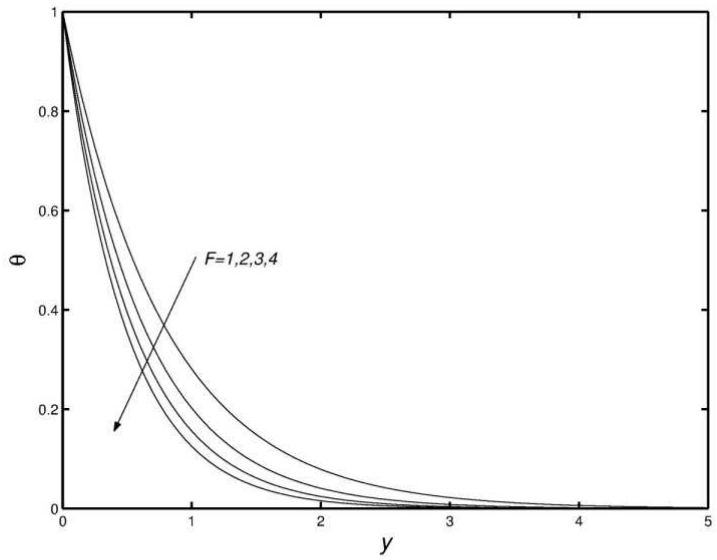

Figure 7: Variations of temperature profile θ for S=1.0, K=0.5, Pr=0.71, z=0.0.

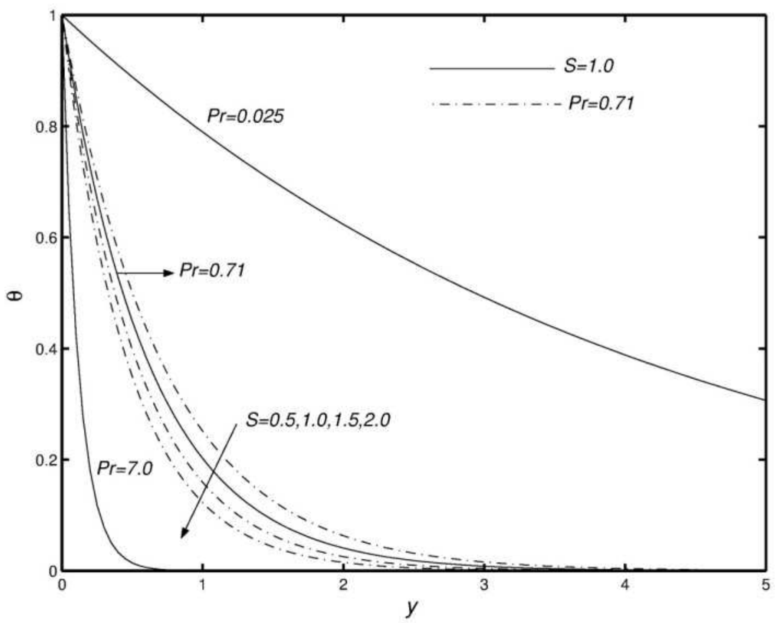

Figure 8: Variations of temperature profile θ for F=2.0, z=0.0, K=0.5.