Journal of Physical Sciences, Vol. 19, 2014, 143-155
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 26 December 2014

Squeezing and Entanglement in Quadratically-Coupled Optomechanical System

Kousik Mukherjee and Paresh Chandra Jana

Department of Physics and Technophysics
Vidyasagar University, Midnapore -- 721102, INDIA
e-mail: kousikmukherjee89@gmail.com, pareshjana@rediffmail.com

Received 11 October 2014; accepted 14 November 2014

ABSTRACT

An operator analytic solution of quadratically-coupled optomechanical system using Hisenberg equation of motion and short-time dynamics to investigate the nonclassical phenomena i.e. squeezing and entanglement for different modes of the system. These effect on strong coupling interaction between macroscopic membrane and cavity resonator neglecting the effect of dissipation and environmental noise have been studied. Phonon properties of the system under that conditions are also investigated. Temporal variation of these nonclassical properties for different strength of coupling are also reported.

Keywords: nonclassical, squeezing,entanglement.

1. Introduction

In optomechanical system (OMS) the interaction between high quality mechanical oscillators and high-finesse optical resonators via radiation pressure,optical gradient is of great interest in both theory and experiment recently [1, 2, 3, 4, 5]. There are several potential application of OMS in high-precession measurements and quantum information processing 6,7,8], optomechanical transducers [9], sensitive detectors for gravitational waves [10], microdisk-coupled nanomechanical beam waveguides [11], optomechanical sensor [12], optomechanical induced transparency [13, 14].

Observations like photon induced tunneling [15], single-photon optomechanics [2] are related to linear optomechanical coupling. Again, optomechanical coupling varies quadratically with mechanical displacement has been reported in a number of configuration, strong dispersive coupling optomechanical system[5], single-photon emission and scattering spectra [16], ultracold atoms [17]. Quadratic coupling switch over linear coupling by explanning optical springs [18], dressed states and nondemolition [1], photon transport [19]. Nonclassical properties like squeezing,entanglement are also observed in the dynamics of OMS [20, 21]. These nonclassical features has application in optical waveguide tap [22], nano-displacement measurement [23], optical storage [24], interferometric technique [25] , optical communication [26] , quantum cryptography [27], quantum dense coding [28].

The earlier studies are based on weak coupling so motivated by recent proposal [29] and experiment [30] we are interested to study these nonclassical features under single photon strong coupling regime. Starting from a seperable quantum state of cavity and mechanical mode, the optomechanical coupling can entangle them to get an entangled quantum state. In this article using analytic solution and short time perturbative solutions of Heisenberg's equations of motion for various field modes, we investigate the existence of single mode and intermodal squeezing, intermodal entanglement of quadratic coupled OMS. We also studied phononic properties under strong coupling conditions.

. The article is organised as follows at first we describe the model Hamiltonian of quadratically coupled OMS and its perturbative solutions for equation of motions corresponds to various field modes then the criteria of squeezing and entanglement and investigate the existence of them of various field modes and phonon properties.

2. The Model

A schematic diagram of the quadratically-coupled optomechanical system is shown in figure 1. One can easily observe that the optomechanical system with a'membrane-in-the-middle' (a thin dielectric) is of our interest. The coupling between the membrane and the optical cavity depends on the position of membrane relative to a node (or antinode) of the intracavity standing wave.

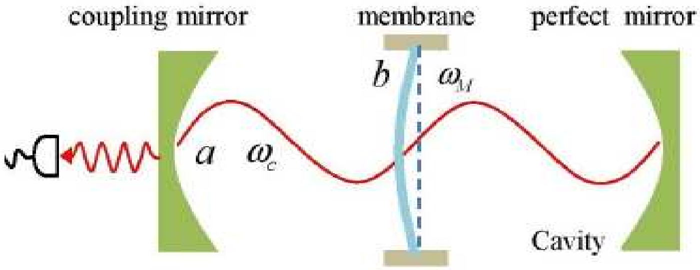

Figure 1: system configuration

The single mode cavity field characterized by the annihilation(creation) operators a ($a^\dagger$)and the field operators b ($b^\dagger$) corresponds to the mechanical motion of the membrane having resonant frequencies ω_c and frequency of mechanical mode ω_m respectively.The Hamiltonian of the system is given by[5]

$$H = \omega_c a^\dagger a + \omega_m b^\dagger b + g a^\dagger a (b^\dagger + b)^2 \qquad (1)$$

where g is the coupling strength between cavity field and the membrane.These operator satisfy bosonic commutation relation $[a, a^\dagger] = [b, b^\dagger] = 1$

(we consider $\hbar = 1$) The photon number operator $N_a = a^\dagger a$ in Hamiltonian H is a conserved quantity and for stability of the membrane the coupling strength g should satisfy the condition $\omega_m > -4ng$, where n is the number of photon inside the cavity[16].Consideration of this Hamiltonian neglecting optical loss,environmental effects and also in absence of driving due to analysis of optomechanical system zero drive Hamiltonian used by some of us[31, 32],theory[33] and experiment[34] both reported optomechanical systems in which environmental effects are greately reduced.

2.1. Solutions

To study various nonclassical properties in quadratically coupled OMS we consider the solutions of the following Heisenberg equations of motion for various field modes corresponds to Hamiltonian of equation (1).

$$\dot{a}(t) = -i[\omega_c a(t) + ga(t)\left(b^{\dagger 2}(t) + b^2(t) + 2b^\dagger(t)b(t) + 1\right)]$$
$$\dot{b}(t) = -i[\omega_m b(t) + 2ga^\dagger(t)a(t)(b^\dagger(t) + b(t))] \qquad (2)$$

In coventional short-time approach we may expand $a(t)$ in taylor series expansion

$$a(t) = a(0) + t\dot{a}(t)|_{t=0} + ... \qquad (3)$$

where $\dot{a}(t) = i[H, a(0)]$. In first order short-time approximated solution we do not obtain any term which arises from $\ddot{a}(t)$. The evalution of the commutator in equation (3) we get the time development of operator $a(t)$ interms of $a(0)$. In this we neglect the term beyond linear power of coupling coefficient g. This short-time approximated solution is very much powerful to get the existence of nonclassical properties as used in [35].

The solutions of equation (2) assumes the following from

$$a(t) = f_1 a(0) + f_2 a(0)b^{\dagger 2}(0) + f_3 b^2(0) + f_4 a(0)b^\dagger(0)b(0) + f_5 a(0)$$
$$b(t) = h_1 b(0) + h_2 a^\dagger(0)\, a(0)\, b(0) + h_3 a^\dagger(0)\, a(0)\, b^\dagger(0) \qquad (4)$$

where the parameters $f_i(i = 1,...,5)$ and $h_i(i = 1,2,3)$ are function of time and evaluated from initial boundary conditions $f_1(0) = h_1(0) = 0$ and $f_i(0) = h_i(0) = 0(i = 2,3,4,5)$

The solutions of $f_i(t)$ and $h_i(t)$ are given by

$$f_1(t) = e^{-i\omega_c t}$$
$$f_2(t) = \frac{g}{2\omega_m} f_1(t)F(t)$$
$$f_3(t) = -\frac{g}{2\omega_m} f_1(t)F^*(t)$$
$$f_4(t) = -2igtf_1(t)$$
$$f_5(t) = -igtf_1(t)$$
$$h_1(t) = e^{-i\omega_m t}$$
$$h_2(t) = 2igth_1(t)$$
$$h_3(t) = \frac{g}{\omega_m}[h_1(t) - h_1^*(t)] \qquad (5)$$

where $F(t) = 1 - e^{2i\omega_m t}$.

These solutions are valid up to first order in g such that $gt < 1$ and perturbation theory holds good. Equal time commutation relation (ETCR) to check the solutions

$$[a(t), a^{\dagger}(t)] = [b(t), b^{\dagger}(t)] = 1 \tag{6}$$

has been studied. For checking we use the fact that the operator $a(0)$ commutes with $b(0)$ and $b^{\dagger}(0)$. Similarly, $b(0)$ commutes with $a(0)$ and $a^{\dagger}(0)$.

2.2. Number operator

Number operators of the field modes a and b are given by

$$N_a = a^{\dagger}(t)a(t) = a^{\dagger}(0)a(0)\, f_1^* f_1 + [a^{\dagger}(0)a(0)b^2(0)\, f_2^* f_1 + a^{\dagger}(0)a(0)b^{\dagger 2}(0)\, f_3^* f_1 +$$
$$a^{\dagger}(0)\, a(0)\, b^{\dagger}(0)\, b(0)\, f_4^* f_1 + a^{\dagger}(0)\, a(0)\, f_5^* f_1 + h.c.]$$
$$N_b = b^{\dagger}(t)b(t) = b^{\dagger}(0)b(0)h_1^* h_1 + [a^{\dagger}(0)a(0)b^{\dagger}(0)b(0)h_2^* h_1 + a^{\dagger}(0)a(0)b^2(0)h_3^* h_1 + h.c.]$$

Here h.c. stands for hermitian conjugates. In short, here and also in remaining portion of the solutions we use $f_i(t) = f_i$ (for $i = 1,...,5$)and $h_i(t) = h_i$ (for $i = 1,2,3$).

3. Conditions for quadrature squeezing and entanglement

To investigate nonclassical properties (i.e.squeezing and entanglement) in quadratic coupled OMS we use the following conditions and consider the field modes are initially coherent.

The criteria for quadrature squeezing in single mode(a or b) and compound mode (a, b) are[36]

$$(\Delta X_a)^2 < \frac{1}{4}, \ (\Delta Y_a)^2 < \frac{1}{4}$$
$$(\Delta X_{ab})^2 < \frac{1}{4} \quad , \quad (\Delta Y_{ab})^2 < \frac{1}{4} \tag{8}$$

The quadratic operators are defined as

$$X_a = \frac{1}{2}[a(t) + a^{\dagger}(t)] \ , Y_a = \frac{1}{2i}[a(t) - a^{\dagger}(t)]$$
$$X_{ab} = \frac{1}{2\sqrt{2}}[a(t) + a^{\dagger}(t) + b(t) + b^{\dagger}(t)]$$
$$Y_{ab} = \frac{1}{2\sqrt{2}i}[a(t) - a^{\dagger}(t) + b(t) - b^{\dagger}(t)] \tag{9}$$

To find out existence of intermodal entanglement here we use two inseparability criteria which are sufficient for characterization of entanglement. First one, to find the existence of entanglement of a given two mode state, Duan *et al.* derived an inequality interms of position and momentum linear combination. This inequality satisfied by any seperable state and violated by inseperable state. this violation is sufficient but not necessary condition for inseparability of arbitrary states. This can be expressed in terms of moments of creation and annihilation operators is given by [37]

$$(\Delta u)^2 + (\Delta v)^2 < 2 \tag{10}$$

is called as Duan *et al.* criterion, where

$$u = \frac{1}{\sqrt{2}}[(a(t) + a^{\dagger}(t)) + (b(t) + b^{\dagger}(t))]$$

$$v = \frac{1}{\sqrt{2}i}[(a(t) - a^{\dagger}(t)) + (b(t) - b^{\dagger}(t))] \tag{11}$$

Second, for a quadratic operator $F_1 = ab + a^{\dagger}b^{\dagger}$ and $F_2 = -i(ab - a^{\dagger}b^{\dagger})$, adding up the variances and assuming seperability provides an inequality which coincides with the uncertainty relation.So, it cannot be the criteria of inseparability. For a state to be seperable its uncertainty in one of these variables must greater than lower bound. Hillery-Zubairy shows that for F_1 and F_2, that lower bound is one and employed Schwarz inequality to find the variance lower bound for a general operator $F(\phi) = e^{i\phi}a^{\dagger}b^{\dagger} + e^{-i\phi}ab$. For any state to be satisfy $(\Delta F(\phi)) < 1$ which leads to the criterion for inseparability expressed in terms of expectation values of moments of the field operators such as Hillery-Zubairy criterion[38, 39]

For product state

$$|< ab >|> [< N_a >< N_b >]^{\frac{1}{2}} \tag{12}$$

3.1. Nonclassicality in OMS

Using above said criterion equation (8-12) and also by perturbative solutions equation (4,5) we now find out the existence of different nonclassical properties.Considering initial states of photon and phonon modes are coherent so product of two coherent states $|\alpha\rangle|\beta\rangle$ where $|\alpha\rangle$ and $|\beta\rangle$ are the eigenkets of field operators a and b respectively.

Thus, $$a(t)|\alpha\rangle|\beta\rangle = \alpha|\alpha\rangle|\beta\rangle \tag{13}$$

$|\alpha|^2$ and $|\beta|^2$ are number of photons and phonons respectively in the field mode a and b. For spontaneous process, $\beta = 0$ and $\alpha \neq 0$. But for stimulated process,these complex amplitudes are not necessarily zero and it seems to consider $\alpha > \beta$.

3.1.1. Single mode and intermodal quadrature squeezing

Using perturbative solutions equation (4,5) and squeezing conditions equation (8,9) we obtain analytic expressions for quadrature fluctuations in single mode as

$$\begin{pmatrix} (\Delta X_a)^2 \\ (\Delta Y_a)^2 \end{pmatrix} = \frac{1}{4}\left[1 + (\beta^{*2} f_2 f_1^* + \beta^2 f_3 f_1^* + \beta^* \beta f_4 f_1^* + f_5 f_1^* + c.c.)\right] \tag{14}$$

$$\begin{pmatrix} (\Delta X_b)^2 \\ (\Delta Y_b)^2 \end{pmatrix} = \frac{1}{4}\left[1 + (\alpha^* \alpha h_1 h_2^* + c.c.) \pm (\alpha^* \alpha h_1 h_3 + c.c.)\right] \tag{15}$$

where c.c. stands for complex conjugates. Whose time variation is shown in fig.(2-3).

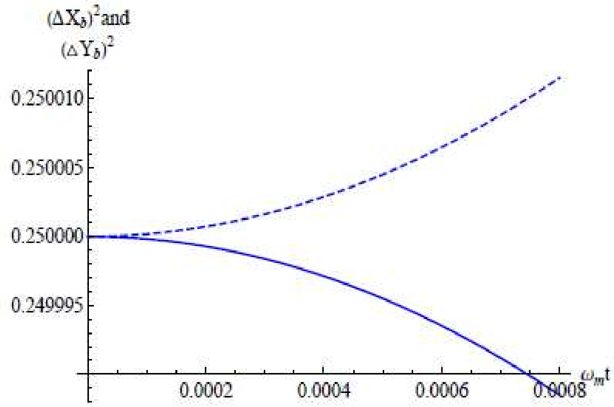

Figure 2: (color online) Plot of $(\Delta X_b{}^2)$ (solid curve) and $(\Delta Y_b{}^2)$ (dashed curve) of mode b with rescaled time $\omega_m t$ (dimensionless) for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2MHz$, $g/\omega_m = 2$ .These values of ω_c , ω_m and g are consisted with experimental values[30] and proposals [29].

From equation (14) it is clear that no squeezing is observed in mode a .

However, single mode squeezing is possible in mode b as shown in fig. (2,3). Here we observed that the X_b quadrature is always squeezed at the cost of the Y_b quadrature and the amount of squeezing is very small. The result in fig.(2,3) are also depends on relative phase angles of complex amplitude's α and β (not shown). From above two graphical variation we see that depth of squeezing increases with increasing coupling strength g.

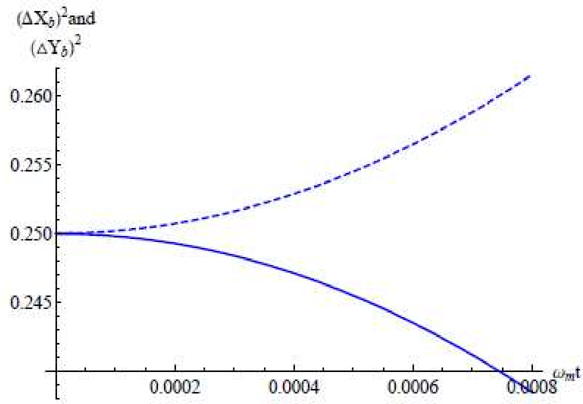

Figure 3: (color online) Plot of $(\Delta X_b{}^2)$ (solid curve) and $(\Delta Y_b{}^2)$ (dashed curve) of mode b with rescaled time $\omega_m t$ (dimensionless) for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2GHz$, $g/\omega_m = 2000$.

$$\begin{pmatrix}(\Delta X_{ab})^2\\(\Delta Y_{ab})^2\end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix}(\Delta X_a)^2\\(\Delta Y_a)^2\end{pmatrix} + \begin{pmatrix}(\Delta X_b)^2\\(\Delta Y_b)^2\end{pmatrix}\right]$$
$$+\frac{1}{8}\left((\alpha\beta\,(2\,f_3h_1^* + f_1h_3^*) + \alpha\beta^*(f_4h_1^* + f_1h_2^*)) + c.c.\right)$$
$$\pm\frac{1}{8}\left((\alpha\beta\,(f_1h_2 + f_4h_1) + \alpha\beta^*(f_1h_3 + 2\,f_2h_1)) + c.c.\right) \qquad (16)$$

where c.c. stands for complex conjugates. Whose time variation is shown in fig. (4-5). Intermodal squeezing is observed for compound mode (ab) as fig.(4,5). Here we see for compound mode the collapse and revival of squeezing effects in both quadrature components. The squeezing of X_{ab} quadrature automatically prohibits the squeezing of Y_{ab} quadrature or viceversa. It is also clear that depth of squeezing increases as coupling strength g increases but this increaseness is much greater than single mode b .

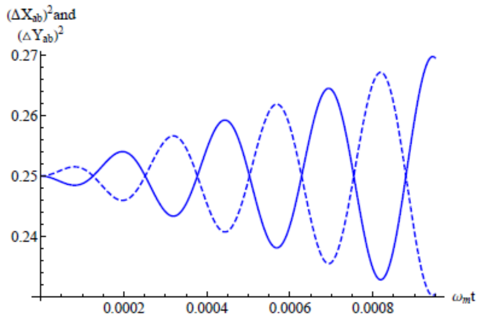

Figure 4: (color online) Plot of $(\Delta X_{ab}{}^2)$ (solid curve) and $(\Delta Y_{ab}{}^2)$ (dashed curve) of mode ab with rescaled time $\omega_m t$ (dimensionless) for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2MHz$, $g/\omega_m = 2$.

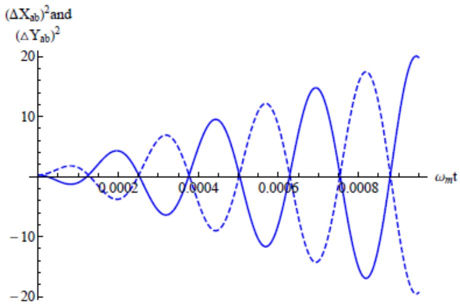

Figure 5: (color online) Plot of $(\Delta X_{ab}{}^2)$ (solid curve) and $(\Delta Y_{ab}{}^2)$ (dashed curve) of mode ab with rescaled time $\omega_m t$ (dimensionless) for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2GHz$, $g/\omega_m = 2000$.

3.1.2. Intermodal entanglement

We examine intermodal entanglement of compound mode ab by Duan *et.al.* inseparability criteria so, using equations (4,10,11)

$$(\Delta u)^2+(\Delta v)^2-2=[(\beta^{*2} f_2 f_1^* + \beta^2 f_3 f_1^* + \beta^* \beta f_4 f_1^* + f_5 f_1^* + \alpha^* \alpha h_1 h_2^*)+c.c.]$$
$$+[(2\alpha\beta f_3 h_1^* + \alpha\beta f_1 h_3^* + \alpha\beta^* f_4 h_1^* + \alpha\beta^* f_1 h_3^*)+c.c.] \quad (17)$$

Graphical variation of $(\Delta u)^2+(\Delta v)^2-2$ with rescaled time $\omega_m t$ is shown in fig.(6) which shows that result is negative i.e. intermodal entanglement exists. It also seen that amount of entanglement changes with variation of α.

From fig. (6,7) we say that depth of entanglement increases as the coupling strength g .

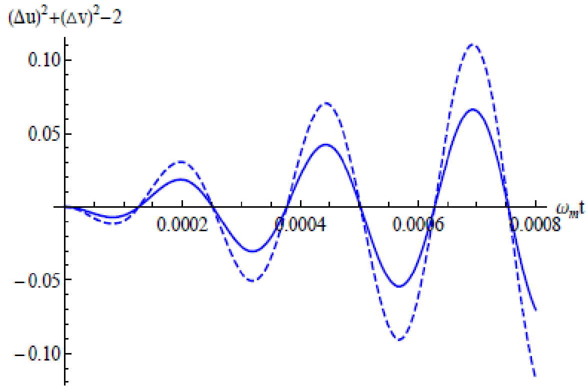

Figure 6: (color online) Plot of $(\Delta u)^2+(\Delta v)^2-2$ (solid curve)for $\alpha=3$ and (dashed curve) for $\alpha=5$ with rescaled time $\omega_m t$ (dimensionless) for $\beta=2$, $\omega_c=15GHz$, $g=1.2MHz$, $g/\omega_m=2$

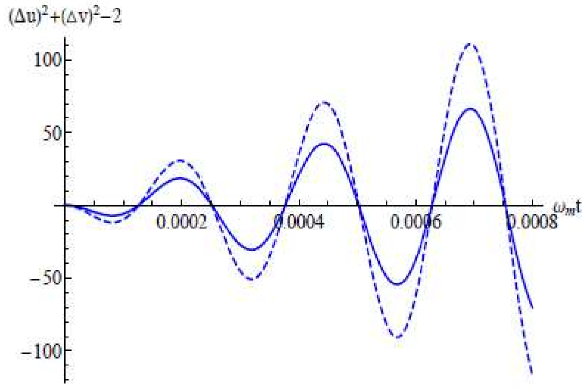

Figure 7: (color online) Plot of $(\Delta u)^2+(\Delta v)^2-2$ (solid curve)for $\alpha=3$ and (dashed curve) for $\alpha=5$ with rescaled time $\omega_m t$ (dimensionless) for $\beta=2$, $\omega_c=15GHz$, $g=1.2GHz$, $g/\omega_m=2000$

Now, to investigate existence of entanglement by Hilery-Zubairy criteria of intermodal entanglement using equation (4,7,12)

$$E_{ab} = (\langle N_a \rangle\langle N_b \rangle) - |\langle ab \rangle|^2$$
$$= -[\left(\alpha^* \alpha f_1^* f_1 (\beta^* \beta h_2 h_1^* + \beta^{*2} h_3 h_1^*)\right) + c.c.] \quad (18)$$

From fig.(8-9) we see entanglement exists all possible time according to Hilery-Zubairy criteria unlike criteria Duan *et al.* in which negativity arises almost periodic variation.It also seen that depth of entanglement increases with α as similar to Duan *et al.* criteria.

Again, from fig.(6-9) we can say that for both inseparability criteria the depth of entanglement increases with variation of coupling strength g . Although the optical and mechanical modes are initially seperable , but they are entangled.

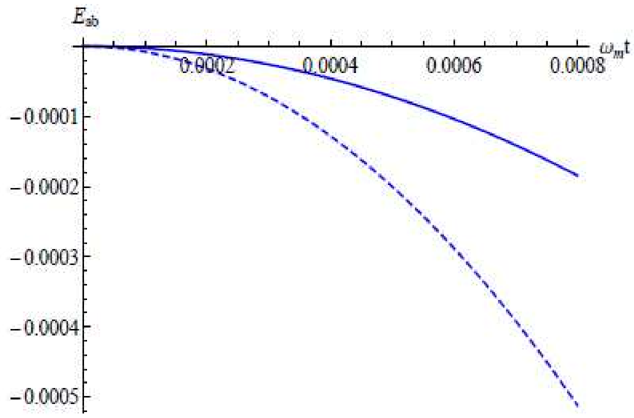

Figure 8: (color online) Plot of E_{ab} (solid curve)for $\alpha = 3$ and (dashed curve) for $\alpha = 5$ with rescaled time $\omega_m t$ (dimensionless) for $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2MHz$, $g/\omega_m = 2$

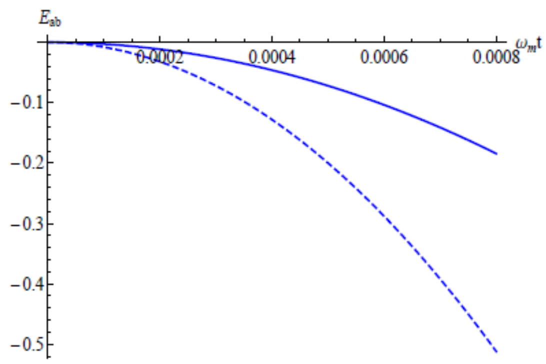

Figure 9: (color online)Plot of E_{ab} (solid curve)for $\alpha = 3$ and (dashed curve) for $\alpha = 5$ with rescaled time $\omega_m t$ (dimensionless) for $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2GHz$, $g/\omega_m = 2000$.

4. Phonon statistics

From the knowledge of $b(t)$ we investigate the physical behaviour of the mechanical mode of oscillation, such as the average phonon number $< b^{\dagger}(t)b(t) >$, whose time dependence is shown in fig.(10).

$$< N_b >= \beta^{*2} + [(\alpha^{*}\alpha(\beta^{*}\beta h_2^{*}h_1 + \beta^{2}h_3^{*}h_1)) + c.c] \tag{19}$$

which shows that phonon number increases with time or dimensionless parameter $\omega_m t$. So,phonon number is not conserved quantity.

We can also find analytically the phononic Mandel parameter defined by

$$Q_k = [\frac{< (b^{\dagger}(t)b(t))^2 > -(< b^{\dagger}(t)b(t) >)^2}{< b^{\dagger}(t)b(t) >} - 1] \tag{20}$$

This parameter indicates the nature of the phonon statistics. If $Q_k = 0$ i.e. the eigenstate phonon statistics are poissonian. $Q_k > 0$ and $Q_k < 0$ corresponds to super-poissonian and sub-poissonian statistics respectively.

Using equation (4, 20) we get

$$Q_k = (\alpha^{*}\alpha(\beta^{*}\beta h_1^{*}h_2 + \beta^{2}h_3^{*}h_1)) + c.c \tag{21}$$

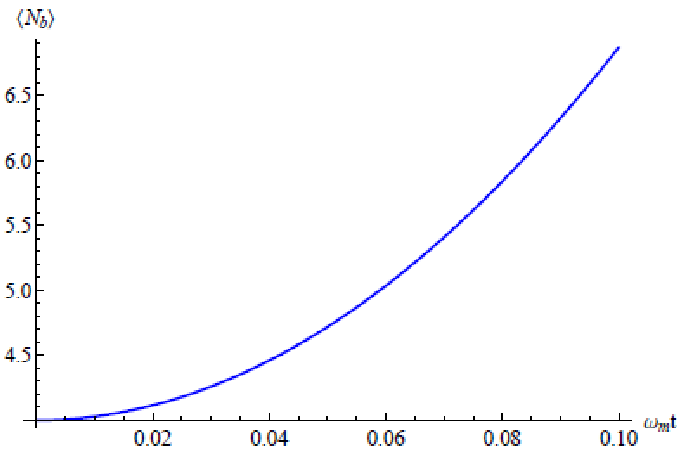

Figure 10: (color online) Plot of $< b^{\dagger}(t)b(t) >$ or $< N_b >$ with rescaled time $\omega_m t$ (dimensionless) for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2MHz$, $g/\omega_m = 2$

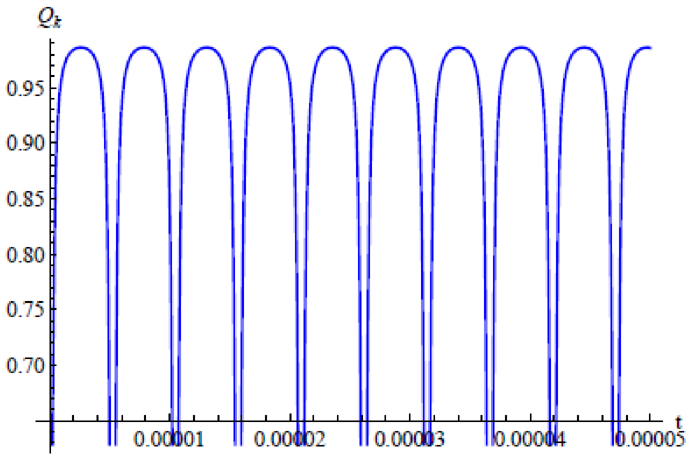

Figure 11: (color online) Plot of $Q_k(t)$ with time t for $\alpha = 3$, $\beta = 2$, $\omega_c = 15GHz$, $g = 1.2MHz$, $g/\omega_m = 2$.

The temporal behaviour of phononic Mandel parameter is shown in fig.(11). The oscillation in Q_k due to the periodic time dependence of $<(b^{\dagger}(t)b(t))^2>$ and $<b^{\dagger}(t)b(t)>$. The phonon statistics super-poissonian in nature.

5. Conclusions

We explore the nonclassical properties such as single mode and intermodal squeezing, intermodal entanglement for a quadratically-coupled optomechanical system. Investigations of these properties under strong coupling such as coupling strength is order of MHz and GHz. Under strong coupling condition single photon affect the cavity field. So, there is variation of depth of nonclassical properties due to change in coupling strength. This system yield superposition of squeezed state (intermodal squeezing for ab mode) rather than pure squeezed state (single mode for a). Indermodal entanglement (ab mode) also exists for higher coupling strength. From Hillery-Zubairy criterion entanglement exists for all time but Duan *et al.* criteria shows there is disentangled state in periodic variation. Our study should be specifically used to entanglement generation, optomechanical spectroscopy, photon-induced tunneling, optical and mechanical squeezing.

REFERENCES

1. H.Shi and M.Bhattacharya, Quantum mechanical study of a generic quadratically coupled optomechanical system, *Phys. Rev. A*, 87 (2013) 043829.
2. A.Nunnenkamp, K.Borkje and S.M.Girvin, Single-photon optomechanics, *Phys. Rev. Lett.*, 107 (2011) 063602.
3. T.J.Kippenberg and K.J.Vahala, Cavity optomechanics: Back-action at the mesoscale, *Science*, 321 (2008) 1172.

4. F.Marquardt and S.M.Grivin, Optomechanics, *Physics*, 2 (2009) 40.
5. J.D.Thompson, B.M.Zwickl, A.M.Jayich, F.Marquardt, S.M.Grivin and J.G.E.Harris, Strong dispersive coupling of a high-finesse cavity to a micromechanical membrane, *Nature* (London), 452 (2008) 72.
6. K.Stannigel, P.Komar, S.J.M.Habraken, S.D.Bennett, M.D.Lukin, P. Zoller and P.Rabl, Optomehchanical quantum information processing with photons and phonons, *Phys. Rev. Lett.*, 109 (2012) 013603.
7. M.Ludwig, A.H.Safavi-Naeini, O.Painter and F.Marquardt, Enhanced quantum nonlinearities in a two mode optomchanical system, *Phys. Rev. Lett.*, 109 (2012) 063601.
8. X.-Y.Lu, W.-M.Zhang, S.Ashhab, Y.Wu and F.Nori, Quantum crtically induced strongness linearities in optomechanical system, *Sci. Rep.*, 3 (2013) 2943.
9. K.Stannigel, P.Rabl, A.S.Sorensen, P.Zoller and M.D.Lukin, Optomechanical transducer for long-distance communication, *Phys. Rev. Lett.*, 105 (2010) 220501.
10. C.M.Caves et al, On the measurement of a weak classical force coupled to quantum mechanical oscillator issue of principle, *Rev. Mod. Phys.*, 52 (1980) 341.
11. Mo Li, Wolfram H.P.Pernice and Hong X.Tang, Reactive cavity optical force on Microdisk-coupled nanomechanical beam, *Phys. Rev. Lett.*, 103 (2009) 223901.
12. K.Zhang, F.Bariani, Y.Dong, W.Zhang and P.Meystre, Proposal for an optomechanical microwave sensor at the sub photon level., *arxiv*: 1401 (2014) 0070, quant-ph.
13. S.Weis et al., Optomechanically induced transparency , *Science*, 330 (2010) 1520.
14. J.D.Teufel et al., Circuit cavity electromechanics in the strong coupling regime, *Natue* (London), 471 (2011) 204.
15. X.-W Xu, Y.-J Li and Y.-x Liu, Photon-induced tunnelling in optomechanical system, *Phys. Rev. A*, 87 (2013) 025803.
16. J.-Q.Lio and F.Nori, Single-photon emission and scattering in quadratically-coupled optomechanical models, *arxiv*:1304 (2013) 6612, quant-ph.
17. T.P.Purdy, D.W.C.Brooks, T.Botter, N.Brahms, Z.-Y.Ma and D.M.Stamper-Kurn, Tunable cavity optomechanics with ultracold atoms, *Phys. Rev. Lett.*, 105 (2010) 133602.
18. A.Rai and G.S.Agarwal, Quantum optical spring , *Phys. Rev. A* 78 (2008) 013831.
19. G.Heinrich, J.G.E.Harris and F.Marquardt, Photon shuttle: Landaue zener stuckberg dynamics in an optomechanical system , *Phys. Rev. A*, 81(2010) 011801.
20. A.Nunnenkamp, K.Borkje, J.G.E.Harris and S.M.Girvin, Colling and squeezing via quadratic optomechanical coupling , *Phys. Rev. A*, 82 (2010) 021806(R).
21. D.Vitali, S.Gigan, A.Ferreira, H.R.Bhom, P.Tombesi and A.Guerreiro,
22. V.Vedral, A.Zeilinger and M.Aspelmeyer, Optomechanical entanglement between a movable mirror and a cavity , *Phys. Rev. Lett.*, 98 (2007) 030405.
23. J.H.Shapiro, Optical wave guide tap with infinitesimal insertion loss, *Opt. Lett.*, 5 (1980) 351.
24. N.Treps, N.Grosse, W.P.Bowen, M.T.L.Hsu, A.Maitre, C.Fabre, H.-A.Bachor and P.K.Lam, Nano displacement measurements using spatilly multimode squeezing, *J.Opt. B : Quantum Semiclass*, 6 (2004) S664.
25. M.T.L.Hsu, V.Delaubert, W.P.Bowen, C.Fabre, H.-A.Bachor and P.K.Lam, A quantum study of multi-bit phase coding for optical stroage, *IEEE J.Quantum*

Electron, 42 (2006) 1001.
26. C.M.Caves and B.L.Schumaker, New formalism for two photon quantum optics.l.quadrature phases and squeezed states, *Phys. Rev. A,* 31 (1985) 3068.
27. H.P.Yuen and J.H.Shapiro, Optical communication with two photon coherent states part-III :Quadrature measurement relaizable with photoemmissive detectors, *IEEE Trans. Inf. Theory,* 24 (1978) 657.
28. M.Hillery, Quantum cryptograph with squeezed state, *Phys. Rev. A,* 61(2009) 022309.
29. S.L.Braunstein and H.J.Kimble, Dense coding for continuous variables , *Phys. Rev. A*, 61 (2000) 042302.
30. H.K.Li, Y.C.Liu, X.Yi, C.L.Zou, X.X.Ren and Y.F.Xiao, Proposal for a near-field optomechanical system with enhanced linear and quadratic coupling, *Phys. Rev. A*, 85(2012) 053832.
31. N.E.Flowers-Jacobs, S.W.Hoch, J.C.Sanky, A.Kashkanova, A.M.Jayich, C.Deutsch, J.Reichel and J.G.E.Harris, Fibre cavity base optomechanical device, *Appl. Phys. Lett.*, 101 (2012) 221109.
32. S.Bose, K.Jacobs and P.L.Knight, Preparation of nonclassical states in cavities with a moving mirror, *Phys. Rev. A,* 56 (1997) 4175.
33. P.Rabl, Photon blockade effect in optomechanical systems, *Phys. Rev. Lett.,* 107 (2011) 063601.
34. S.Singh, G.A.Phelps, D.S.Goldbaum, E.M.Wright and P.Meystre, All optical optomechanics:An optical spring mirror , *Phys. Rev. Lett.*, 105 (2010) 213602.
35. J.Gieseler, B.Deutsch, R.Quidant and L.Novotny, Subkelvin parametric feedback cooling of a laser trapped nanoparticle , *Phys. Rev. Lett.,* 109 (2012) 103603.
36. B.Sen and S.Mondal, Squeezed state in spontaneous Ramman and in stimulated Ramman, *J. Mod .Opt.,* 52(13) (2005) 1789-1807.
37. R.Loudon and P.L.Night, Squeezed light, *J. Mod. Opt* 34 (1987) 709.
38. L.M.Duan, G.Giedke, J.I.Cirac and P.Zoller, Inseparability criterion for continuous variable systems, *Phys. Rev. Lett.,* 84 (2000) 2722.
39. M.Hillery and M.S.Zubairy, Entanglement conditions for two-mode states , *Phys. Rev. Lett.,* 96 (2006) 050503.
40. M.Hillery and M.S.Zubairy, Entanglement conditions for two-mode states: Applications , *Phys. Rev. A.,* 74 (2006) 032333.