Journal of Physical Sciences, Vol. 19, 2014, 91-102
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/journal
Published on 26 December 2014

A New Method for Ranking Exponential Fuzzy Numbers with Using Distance Between Upper and Lower Central Gravity

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Received 13 October 2014; accepted 30 November2014

ABSTRACT
In this paper, we want to propose a new method for ranking exponential trapezoidal fuzzy numbers with using distance between upper and lower central gravity. In this method, use of membership function (f_i^L, f_i^R) for calculating x_i and inverse functions (g^L, g^R) for calculating y_i in ranking exponential fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Central gravity, exponential trapezoidal fuzzy numbers, metric, ranking method

1. Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [30] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Most of the ranking procedures proposed so far in the literature cannot discriminate fuzzy quantities and some are counterintuitive. As fuzzy numbers are represented by possibility distributions, they may overlap with each other, and hence it is not possible to order them. Ranking fuzzy numbers were first proposed by Jain [11] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [6] reviewed some of these ranking methods [5, 10, 11, 16, 17] for ranking fuzzy subsets and fuzzy numbers. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [10] presented the mean value of a fuzzy number. Chu and Tsao [6] proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Deng and Liu [7] presented a centroid-index method for ranking fuzzy numbers. Liang et al. [13] and Wang and Lee [27] also used the centroid concept in developing their ranking index. Chen and Chen [4] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α–levels of trapezoidal fuzzy numbers. Chen and Chen [5] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads and Parandian [15] proposed a method for ranking numbers by using distance between convex combination of upper and lower central gravity of α-level and the origin of coordinate systems. Also Some of the interesting Approach Ranking Of Trapezoidal Fuzzy Number can be found in Amit Kumar [12]. Moreover, Rezvani [23] proposed a ranking trapezoidal fuzzy numbers based on apex ngles

In this paper, we want to propose a new method for ranking exponential trapezoidal fuzzy numbers with using distance between upper and lower central gravity. In this method, use of membership function (f_i^L, f_i^R) for calculating x_i and inverse functions (g^L, g^R) for calculating y_i in ranking exponential fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2. Preliminaries

Generally, a generalized fuzzy number *A* is described as any fuzzy subset of the real line *R*, whose membership function μ_A satisfies the following conditions,

i) μ_A is a continuous mapping from *R* to the closed interval $[0,1]$,

ii) $\mu_A(x) = 0$, $-\infty < x \le a$,

iii) $\mu_A(x) = L(x)$ is strictly increasing on $[a,b]$,

iv) $\mu_A(x) = w$, $b \le x \le c$,

v) $\mu_A(x) = R(x)$ is strictly increasing on $[c,d]$,

vi) $\mu_A(x) = 0$, $d \le x < \infty$

where $0 < w \le 1$ and *a,b,c* and *d* are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (a,b,c,d;w). \tag{1}$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by

$$A = (c,a,b,d) \tag{2}$$

Here, we define its general from as follows:

$$f_A(x) = \begin{cases} we^{-[(a-x)/\alpha]} & x \le a \ , \\ w & a \le x \le b \ , \\ we^{-[(x-b)/\beta]} & x \le a \ , \end{cases} \tag{3}$$

where $0 < w \le 1$ and a,b are real numbers, and α, β are positive real numbers. We denote this type of generalized exponential fuzzy number as

$$A = (a,b,\alpha,\beta;w)_E \tag{4}$$

Especially, when $w = 1$, we denote it as

$$A=(a,b,\alpha,\beta)_E \tag{5}$$

integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A=(a,b,\alpha,\beta)_E$, Where $0<w\le 1$ and α,β are positive real numbers, a,b are real numbers as in formula (3). Now, let two monotonic functions be

$$L(x)=we^{-[(a-x)/\alpha]},\ R(x)=we^{-[(x-b)/\beta]} \tag{6}$$

Then the inverse functions of function L and R are L^{-1} and R^{-1} respectively. the h-level graded mean value of generalized exponential fuzzy number $A=(a,b,\alpha,\beta;w)_E$ can be express as

$$h[L^{-1}(h)+R^{-1}(h)]/2 \tag{7}$$

Definition 1. Let $A=(a,b,\alpha,\beta;w)_E$, be generalized exponential number, then the graded mean integration representation of A is define by

$$P(A)=\int_0^w h\ (\frac{L^{-1}(h)+R^{-1}(h)}{2}dh)/\int_0^w h\ dh \tag{8}$$

Theorem 1. Let $A=(a,b,\alpha,\beta;w)_E$, be generalized exponential number with $0<w\le 1$ and α,β are positive real numbers, a,b are real numbers. then the graded mean integration representation of A is

$$P(A)=\frac{a+b}{2}+\frac{\beta-\alpha}{4}\ . \tag{9}$$

Proof: $L^{-1}(h)=a-\alpha(\ln\frac{w}{h})$, $R^{-1}(h)=b+\beta(\ln\frac{w}{h})$.

$$P(A)=\frac{1}{2}\int_0^w h[a+b+\beta(\ln\frac{w}{h})-\alpha(\ln\frac{w}{h})]\ dh\,/\,\frac{1}{2}w^2$$

$$=\frac{a+b}{2}+\frac{\beta-\alpha}{2}\int_0^w h(\ln\frac{w}{h})\ dh=\frac{a+b}{2}+\frac{\beta-\alpha}{2}[\int_0^w h\ln(w)-\int_0^w h\ln(h)]\ dh$$

$$=\frac{a+b}{2}+\frac{\beta-\alpha}{2}\int_0^w h[\ln(w)-\int_0^w h\ln(h)]\ dh=\frac{a+b}{2}+\frac{\beta-\alpha}{4}\ .$$

3. Proposed approach

In this section first some important results, that are useful for the proposed approach, are proved. Granted that $A_1,A_2,\ldots,A_n$, n are positive fuzzy number. At first, we obtain the upper and lower central gravity of α-level for every fuzzy number. For each A_i, $i=1,\ldots,n$, the upper and lower central gravity of α-level should be indicated as (x_{i1},y_{i1}) and (x_{i2},y_{i2}) respectively and obtained as follows:

$$x_{i1}=\frac{\int_a^b x\; f_{i1}^l dx+\int_b^c x\; dx+\int_c^d x\; f_{i1}^r dx}{\int_a^b f_{i1}^l dx+\int_b^c dx+\int_c^d f_{i1}^r dx},\qquad x_{i2}=\frac{\int_a^b x\; f_{i2}^l dx+\int_b^c x\; dx+\int_c^d x\; f_{i2}^r dx}{\int_a^b f_{i2}^l dx+\int_b^c dx+\int_c^d f_{i2}^r dx}$$

$$y_{i1}=\frac{\int_o^{w_i} y\; g_{i1}^l dy+\int_0^{w_i} y\; g_{i1}^r dy}{\int_0^{w_i} g_{i1}^l dy+\int_0^{w_i} g_{i1}^r dy},\qquad y_{i2}=\frac{\int_o^{w_i} y\; g_{i2}^l dy+\int_0^{w_i} y\; g_{i2}^r dy}{\int_0^{w_i} g_{i2}^l dy+\int_0^{w_i} g_{i2}^r dy},$$

where

$$w_i\in[\alpha,1],\qquad \alpha\le\min\{w_i, i=1,...,n\},$$

Then

$$R(A_i)=\sqrt{(\lambda x_{i1}+(1-\lambda)x_{i2})^2+(\lambda y_{i1}+(1-\lambda)y_{i2})^2},\qquad \lambda\in[0,1]$$

Now, we can propose a new method for ranking exponential trapezoidal fuzzy numbers with using upper and lower central gravity.

As in formula (3), let two monotonic functions be

$$L(x)=we^{-[(a-x)/\alpha]}\ ,R(x)=we^{-[(x-b)/\beta]}$$

then the inverse functions of function L and R are L^{-1} and R^{-1} respectively.

$$L^{-1}(h)=a-\alpha(\ln\frac{w}{h}),\ R^{-1}(h)=b+\beta(\ln\frac{w}{h}).$$

Theorem 2. Let $A=(a,b,\alpha,\beta;w)_E$ is a generalized exponential fuzzy number, then the upper and lower central gravity should be indicated as (x,y) respectively and obtained as follows:

$$x=\frac{w\alpha[e^{\frac{b-a}{\alpha}}(b-\alpha)-(a-\alpha)]+[\frac{\alpha^2}{2}-\frac{b^2}{2}]+w\beta[e^{\frac{b-\alpha}{\beta}}(\alpha-\beta)-2\beta e^{\frac{b-\beta}{\beta}}]}{w\alpha[e^{\frac{b-a}{\alpha}}-1]+[\alpha-b]+w\beta[e^{\frac{b-\alpha}{\beta}}-e^{\frac{b-\beta}{\beta}}]}\tag{10}$$

$$y=\frac{\int_0^w y\; g^L dy+\int_0^w y\; g^R dy}{\int_0^w g^L dy+\int_0^w g^R dy}=\frac{w[\frac{a}{2}-\frac{\alpha}{4}]+w[\frac{b}{2}+\frac{\beta}{4}]}{[a-\alpha]+[b+\beta]}\tag{11}$$

Proof:

$$x=\frac{\int_a^b x\ f^L dx+\int_b^\alpha x\ dx+\int_\alpha^\beta x\ f^R dx}{\int_a^b f^L dx+\int_b^\alpha dx+\int_\alpha^\beta f^R dx}=\frac{\int_a^b x\ we^{-[(a-x)/\alpha]}dx+\int_b^\alpha x\ dx+\int_\alpha^\beta x\ we^{-[(x-b)/\beta]}dx}{\int_a^b we^{-[(a-x)/\alpha]}dx+\int_b^\alpha dx+\int_\alpha^\beta we^{-[(x-b)/\beta]}dx}$$

$$=\frac{w\,\alpha[e^{\frac{b-a}{\alpha}}(b-\alpha)-(a-\alpha)]+[\frac{\alpha^2}{2}-\frac{b^2}{2}]+w\,\beta[e^{\frac{b-\alpha}{\beta}}(\alpha-\beta)-2\beta e^{\frac{b-\beta}{\beta}}]}{w\,\alpha[e^{\frac{b-a}{\alpha}}-1]+[\alpha-b]+w\,\beta[e^{\frac{b-\alpha}{\beta}}-e^{\frac{b-\beta}{\beta}}]}$$

and

$$y=\frac{\int_0^w y\ g^L dy+\int_0^w y\ g^R dy}{\int_0^w g^L dy+\int_0^w g^R dy}=\frac{\int_0^w y\ [a-\alpha(\ln\frac{w}{y})]dy+\int_0^w y\ [b+\beta(\ln\frac{w}{y})]dy}{\int_0^w [a-\alpha(\ln\frac{w}{y})]dy+\int_0^w [b+\beta(\ln\frac{w}{y})]dy}=\frac{w^2[\frac{a}{2}-\frac{\alpha}{4}]+w^2[\frac{b}{2}+\frac{\beta}{4}]}{[aw-\alpha w]+[bw+\beta w]}$$

$$=\frac{w\,[\frac{a}{2}-\frac{\alpha}{4}]+w\,[\frac{b}{2}+\frac{\beta}{4}]}{[a-\alpha]+[b+\beta]}\ .$$

Theorem 3. Let $A=(a_1,b_1,\alpha_1,\beta_1;w_A)_E$ be a generalized exponential fuzzy numbers, then the distance between upper and lower central gravity as follows

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2},\qquad \lambda\in[0,1] \tag{12}$$

Proof: $R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}$

$$=\sqrt{[\lambda[\frac{w\alpha[e^{\frac{b-a}{\alpha}}(b-\alpha)-(a-\alpha)]+[\frac{\alpha^2}{2}-\frac{b^2}{2}]+w\beta[e^{\frac{b-\alpha}{\beta}}(\alpha-\beta)-2\beta e^{\frac{b-\beta}{\beta}}]}{w\alpha[e^{\frac{b-a}{\alpha}}-1]+[\alpha-b]+w\beta[e^{\frac{b-\alpha}{\beta}}-e^{\frac{b-\beta}{\beta}}]}]+(1-\lambda)\frac{w[\frac{a}{2}-\frac{\alpha}{4}]+w[\frac{b}{2}+\frac{\beta}{4}]}{[a-\alpha]+[b+\beta]}]^2}$$

Theorem 4. Let $A=(a_1,b_1,\alpha_1,\beta_1;w_A)_E$ and $B=(a_2,b_2,\alpha_2,\beta_2;w_B)_E$ be two generalized exponential fuzzy numbers, where $0<w\le 1$ and α,β are positive real numbers, a,b are real numbers, then

i) If $R(A)>R(B)$, then $A>B$,

ii) If $R(A)<R(B)$, then $A<B$,

iii) If $R(A)\sim R(B)$, then $A\sim B$.

3.1. Method to find the values of $R(A)$ and $R(B)$

Let $A=(a_1,b_1,\alpha_1,\beta_1;w_A)_E$ and $B=(a_2,b_2,\alpha_2,\beta_2;w_B)_E$ be two generalized exponential fuzzy numbers, then use the following steps to find the values of $R(A)$ and $R(B)$

$*$ Step 1: Find x_A and x_B

$*$ Step 2: Find y_A and y_B

$*$ Step 3: Calculation $R(A)$ and $R(B)$ and use theorem 4., for ranking this method.

Remark 1. In this article, in practical case, we assume that $\lambda = 0.2$.

4. Results

Example 1. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1: $x_A = 0.47$, $x_B = 0.24$

$*$ Step 2: $y_A = 0.15$, $y_B = 0.31$

$*$ Step 3:

$$R(A) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.47\lambda + (1-\lambda)0.15]^2} = 0.21$$

$$R(B) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.24\lambda + (1-\lambda)0.31]^2} = 0.3$$

Then, $P(A) < P(B) \Rightarrow A < B$.

Example 2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1: $x_A = 0.29$, $x_B = 0.27$

$*$ Step 2: $y_A = 0.44$, $y_B = 0.42$

$*$ Step 3:

$$R(A) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.29\lambda + (1-\lambda)0.44]^2} = 0.41$$

$$R(B) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.27\lambda + (1-\lambda)0.42]^2} = 0.88$$

Then, $P(A) < P(B) \Rightarrow A < B$.

Example 3. Let $A = (0.1, 0.2, 0.4, .5; 1)$ and $B = (1,1,1,1;1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1: $x_A = 0.29$, $x_B = \infty$

$*$ Step 2: $y_A = 0.44$, $y_B = 0.5$

$*$ Step 3:

$$R(A) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.29\lambda + (1-\lambda)0.44]^2} = 0.41$$

$$R(B) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[\infty\lambda + (1-\lambda)0.5]^2} = \infty$$

Then, $P(A) < P(B) \Rightarrow A < B$.

Example 4. Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$*$ Step 1: $x_A = 0.4$, $x_B = 0.27$

$*$ Step 2: $y_A = 0.87$, $y_B = 0.42$

$*$ Step 3:

$$R(A) = \sqrt{(\lambda x + (1-\lambda)y)^2} = \sqrt{[0.4\lambda + (1-\lambda)0.87]^2} = 0.8$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.27\lambda+(1-\lambda)0.42]^2}=0.39$$

Then, $P(A)>P(B)\Rightarrow A>B$.

Example 5. Let $A=(0.3,0.5,0.5,1;1)$ and $B=(0.1,0.6,0.6,0.8;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1: $x_A=0.61$, $x_B=0.44$
* Step 2: $y_A=0.4$, $y_B=0.44$
* Step 3:

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.61\lambda+(1-\lambda)0.4]^2}=0.44$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.44\lambda+(1-\lambda)0.44]^2}=0.44$$

Then, $P(A)\sim P(B)\Rightarrow A\sim B$.

Example 6. Let $A=(0,0.4,0.6,0.8;1)$ and $B=(0.2,0.5,0.5,0.9;1)$ and $C=(0.1,0.6,0.7,0.8;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1: $x_A=0.36$, $x_B=0.75$, $x_C=0.45$
* Step 2: $y_A=0.42$, $y_B=0.41$, $y_C=0.47$
* Step 3:

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.36\lambda+(1-\lambda)0.42]^2}=0.41$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.75\lambda+(1-\lambda)0.41]^2}=0.48$$

$$R(C)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.45\lambda+(1-\lambda)0.47]^2}=0.47$$

Then, $P(A)<P(C)<P(B)\Rightarrow A<C<B$.

Example 7. Let $A=(0.1,0.2,0.4,0.5;1)$ and $B=(-2,0,0,2;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1: $x_A=0.29$, $x_B=0.84$
* Step 2: $y_A=0.44$, $y_B=\infty$
* Step 3:

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.29\lambda+(1-\lambda)0.44]^2}=0.41$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[0.84\lambda+(1-\lambda)\infty]^2}=\infty$$

Then, $P(A)<P(B)\Rightarrow A<B$.

It is clear from Table 1. the results of the proposed approach are same as obtained by using the existing approach.

Approaches	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
Cheng [8]	A<B	A~B	Error	A~B	A>B	A<B<C	Error
Chu [9]	A<B	A<B	A<B	A<B	A>B	A<B<C	Error
Chen [4]	A<B	A<B	A<B	A<B	A>B	A<C<B	A>B
Abbasbandy[1]	Error	A~B	A<B	A~B	A<B	A<B<C	A>B
Chen [7]	A<B	A<B	A<B	A<B	A>B	A<B<C	A>B

Kumar [11]	A>B	A~B	A<B	A<B	A>B	A<B<C	A>B
Singh [15]	A~B	A>B	A>B	A~B	A~B	A>C>B	A<B
Rezvani [23]	A<B	A>B	A<B	A~B	A>B	A<C<B	A<B
Proposed approach	A<B	A<B	A<B	A>B	A~B	A<C<B	A<B

Table 1: A comparison of the ranking results for different approaches

Example 8. Consider the following sets, see Yao and Wu [29].
Set 1: $A=(0.4,0.5,0.5,1;1)$, $B=(0.4,0.7,0.7,1;1)$, $C=(0.4,0.9,0.9,1;1)$.
Set 2: $A=(0.3,0.4,0.7,0.9;1)$, $B=(0.3,0.7,0.7,0.9;1)$, $C=(0.5,0.7,0.7,0.9;1)$.
Set 3: $A=(0.3,0.5,0.8,0.9;1)$, $B=(0.2,0.5,0.5,0.9;1)$, $C=(0.1,0.6,0.6,0.8;1)$.
To compare other methods, researchers refer reader to Table 2.
Solution:
Set 1: $A=(0.4,0.5,0.5,1;1)$, $B=(0.4,0.7,0.7,1;1)$, $C=(0.4,0.9,0.9,1;1)$.
$*$ Step 1: $x_A=-2.38$, $x_B=-2.49$, $x_C=-1.97$
$*$ Step 2: $y_A=0.41$, $y_B=0.45$, $y_C=0.48$
$*$ Step 3

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-2.38\lambda+(1-\lambda)0.41]^2}=0.15$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-2.49\lambda+(1-\lambda)0.45]^2}=0.14$$

$$R(C)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-1.97\lambda+(1-\lambda)0.48]^2}=0.01$$

Then, $P(C)<P(B)<P(A)\Rightarrow C<B<A$.

Set 2: $A=(0.3,0.4,0.7,0.9;1)$, $B=(0.3,0.7,0.7,0.9;1)$, $C=(0.5,0.7,0.7,0.9;1)$
$*$ Step 1: $x_A=-1.54$, $x_B=-1.68$, $x_C=-3.43$
$*$ Step 2: $y_A=0.44$, $y_B=0.45$, $y_C=0.46$
$*$ Step 3

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-1.54\lambda+(1-\lambda)0.44]^2}=0.04$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-1.68\lambda+(1-\lambda)0.45]^2}=0.024$$

$$R(C)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-3.43\lambda+(1-\lambda)0.46]^2}=0.32$$

Then, $P(B)<P(A)<P(C)\Rightarrow B<A<C$.

Set 3: $A=(0.3,0.5,0.8,0.9;1)$, $B=(0.2,0.5,0.5,0.9;1)$, $C=(0.1,0.6,0.6,0.8;1)$.
$*$ Step 1: $x_A=-1.39$, $x_B=-1.7$, $x_C=-0.9$
$*$ Step 2: $y_A=0.47$, $y_B=0.41$, $y_C=0.44$
$*$ Step 3:

$$R(A)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-1.39\lambda+(1-\lambda)0.47]^2}=0.1$$

$$R(B)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-1.7\lambda+(1-\lambda)0.41]^2}=0.01$$

$$R(C)=\sqrt{(\lambda x+(1-\lambda)y)^2}=\sqrt{[-0.9\lambda+(1-\lambda)0.44]^2}=0.17$$

Then, $P(B) < P(A) < P(C) \Rightarrow B < A < C$.

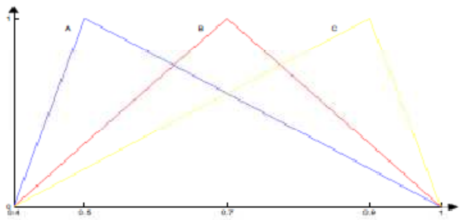

Fig. 1. Set 1.

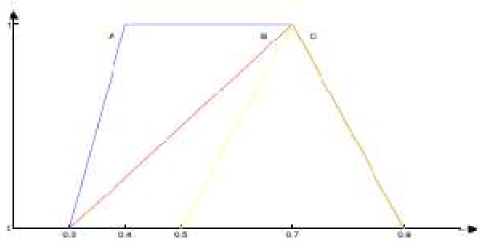

Fig. 2. Set 2.

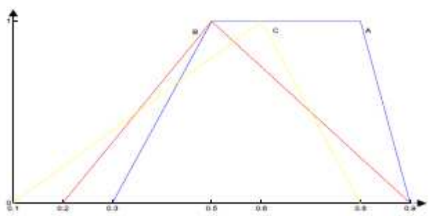

Fig. 3. Set 3.

Approaches	Fuzzy number	Set 1	Set 2	Set 3
Results		A<B<C	A<B<C	A<B~C
Sing Distance	A	1.2	1.15	0.095
method	B	1.4	1.3	1.05
With p=1	C	1.6	1.4	1.05
Results		A<B<C	A<B<C	A<B<C
Sing Distance	A	0.8869	0.8756	0.7853
method	B	1.0194	0.9522	0.7958
With p=2	C	1.1605	1.0033	0.8386

Results		A<B<C	A<B<C	A<B~C
Distance	A	0.6	0.575	0.475
Minimization	B	0.7	0.65	0.525
	C	0.9	0.7	0.525
Results		A<B<C	A<B<C	B<A<C
Abbasbandy	A	0.5334	0.5584	0.525
and	B	0.7	0.6334	0.5058
Hajjari	C	0.8666	0.7	0.575
Results		A<B<C	A<B<C	A<B<C
Choobineh	A	0.3333	0.548	0.5
and	B	0.5	0.583	0.5833
Li	C	0.667	0.667	0.6111
Results		A<B<C	A<B<C	B<A<C
	A	0.6	0.575	0.45
Yager	B	0.7	0.65	0.525
	C	0.8	0.7	0.55
Results		A<B<C	A<B<C	A<B<C
	A	0.3375	0.4315	0.52
Chen	B	0.5	0.5625	0.57
	C	0.667	0.625	0.625
Results		A<B<C	A~B<C	A<B~C
Baldwin	A	0.3	0.27	0.4
and	B	0.33	0.27	0.42
Guild	C	0.44	0.37	0.42
Results		A<B<C	A<B<C	A<C<B
Chu	A	0.299	0.2847	0.244
and	B	0.35	0.3247	0.2624
Tsao	C	0.3993	0.35	0.2619
Results		A<B<C	A<B<C	A<B~C
Yao	A	0.6	0.575	0.475
and	B	0.7	0.65	0.525
Wu	C	0.8	0.7	0.525
Results		A<B<C	A<B<C	A<C<B

Cheng	A	0.79	0.7577	0.7106
	B	0.8602	0.8149	0.7256
	C	0.9268	0.8602	0.7241
Results		C<B<A	B<A<C	B<A<C
Proposed method	A	0.15	0.04	0.1
	B	0.14	0.024	0.01
	C	0.01	0.3	0.17

Table 2: Comparative results of example 8

5. Conclusion

In this paper, we want to propose a new method for ranking exponential trapezoidal fuzzy numbers with using distance between upper and lower central gravity. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

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