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Transportation Problem With Exponential Random Variables

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ABSTRACT

In this paper we have considered single objective and multi-objective stochastic transportation problem involving an inequality type of constraints in which the source and destination parameters are exponential random variables with known means but the objectives are non-commensurable and conflicting in nature. At first we convert the proposed multi-objective linear stochastic transportation problem into an equivalent deterministic problem under chance constrained programming technique. Then fuzzy programming technique is applied to solve this problem and obtained the compromise solution. A numerical example is illustrated to verify the solution procedure and the developed methodology.

Keywords: Transportation problem, Multi-objective decision making, Stochastic programming, exponential distribution, linear membership function, fuzzy programming technique.

1. Introduction

In most mathematical programming problems it is assumed that parameters occurring in the model are constant and known, such models are called deterministic. However, deterministic models are usually just an approximation of the reality. These models can be used to solve the problem in one situation (for one specific set of data values), but the solutions obtained from these models may become sub-optimal or infeasible if the situation changes. Sometimes variability of the parameters is so significant and their evaluation so uncertain (especially if the parameters will be assigned values in the future) that treating the deterministic model as a good approximation is not acceptable. Therefore Uncertainty in decision problems comes from, for example, weather changes, market-related uncertainty and competition etc.

2. Single objective stochastic transportation problem involving exponential random variables

Let us consider the single objective stochastic transportation problem involving exponential random variables as follows :

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \quad (1)$$

subject to

$$\Pr[\sum_{j=1}^{n} x_{ij} \le a_i] \ge 1-\alpha_i, i=1,2,\ldots,m \quad (2)$$

$$\Pr[\sum_{i=1}^{m} x_{ij} \ge b_j] \ge 1-\beta_j, j=1,2,\ldots,n \quad (3)$$

$$x_{ij} \ge 0, i=1,2,\ldots,m, j=1,2,\ldots,n \quad (4)$$

where $0<\alpha_i<1$, $0<\beta_j<1$ and a_i, b_j are exponential random variables. For simplicity, let us consider the decision variables x_{ij} to be deterministic.

We shall first consider special cases where only a_i or only b_j are exponential random variables before considering the general case in which both a_i and b_j are exponential random variables.

Case-I: Only a_i are exponential random variables

Let a_i are exponential random variables with known means

$$E[a_i] = \frac{1}{\lambda_{a_i}}, i=1,2,\ldots,m \quad (5)$$

Let
$$\sum_{j=1}^{n} x_{ij} = y_i, i=1,2,\ldots,m.$$

Now the constraint (2) can be represented as

$$\Pr[y_i \le a_i] \ge 1-\alpha_i, i=1,2,\ldots,m$$

which is same as

$$\Pr[a_i \ge y_i] \ge 1-\alpha_i, i=1,2,\ldots,m$$

$$\Rightarrow 1-\Pr[a_i \le y_i] \ge 1-\alpha_i, i=1,2,\ldots,m$$

$$\Rightarrow \Pr[a_i \le y_i] \le \alpha_i, i=1,2,\ldots,m$$

$$\Rightarrow \int_{-\infty}^{y_i} f(a_i)\,\mathrm{d}a_i \le \alpha_i, i=1,2,\ldots,m \quad (6)$$

Now

$$\int_{-\infty}^{y_i} f(a_i)\,\mathrm{d}a_i = \int_{0}^{y_i} f(a_i)\,\mathrm{d}a_i = \int_{0}^{y_i} \lambda_{a_i} e^{-\lambda_{a_i} a_i}\,\mathrm{d}a_i = [-e^{-\lambda_{a_i} a_i}]_0^{y_i} = 1-e^{-\lambda_{a_i} y_i}$$

Thus equation (6) becomes

$$1-e^{-\lambda_{a_i} y_i} \le \alpha_i, i=1,2,\ldots,m$$

$$\Rightarrow e^{-\lambda_{a_i} y_i} \ge 1-\alpha_i, i=1,2,\ldots,m$$

$$\Rightarrow -\lambda_{a_i} y_i \geq \ln(1-\alpha_i), i = 1,2,\ldots,m$$

$$\Rightarrow \sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m \tag{7}$$

Thus the solution of the single objective stochastic transportation problem stated in equations (1) - (4) can be obtained by solving the equivalent deterministic programming problem,

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{8}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m \tag{9}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j;\ j = 1,2,\ldots,n \tag{10}$$

$$x_{ij} \geq 0; i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{11}$$

Case-II: Only b_j are exponential random variables

Let b_j are exponential random variables with known means

$$E[b_j] = \frac{1}{\lambda_{b_j}},\ j = 1,2,\ldots,n \tag{12}$$

Let $$\sum_{i=1}^{m} x_{ij} = y_j\ ,\ j = 1,2,\ldots,n.$$

Now the probabilistic constraint (3) can be rewritten as

$$\Pr[y_j \geq b_j] \geq 1-\beta_j, j = 1,2,\ldots,n$$

which is same as

$$\Pr[b_j \leq y_j] \geq 1-\beta_j, j = 1,2,\ldots,n$$

$$\Rightarrow \int_{-\infty}^{y_j} f(b_j)\,\mathrm{d}b_j \geq 1-\beta_j, j = 1,2,\ldots,n \tag{13}$$

Now

$$\int_{-\infty}^{y_j} f(b_j)\,\mathrm{d}b_j = \int_{0}^{y_j} f((b_j)\,\mathrm{d}b_j = \lambda_{b_j}\int_{0}^{y_j} e^{-\lambda_{b_j} b_j}\,\mathrm{d}b_j = [-e^{-\lambda_{b_j} b_j}]_0^{y_j} = 1-e^{-\lambda_{b_j} y_j}$$

Thus equation (13) becomes

$$1-e^{-\lambda_{b_j} y_j} \geq \beta b_j, j = 1,2,\ldots,n$$

$$\Rightarrow e^{-\lambda_{b_j} y_j} \le \beta_j,\ j = 1,2,\ldots,n$$

$$\Rightarrow -\lambda_{b_j} y_j \le \ln(\beta_j),\ j = 1,2,\ldots,n$$

$$\Rightarrow \sum_{i=1}^{m} x_{ij} \le \frac{-\ln(\beta_j)}{\lambda_{b_j}},\ j = 1,2,\ldots,n \qquad (14)$$

Therefore, we obtain the equivalent single objective deterministic transportation problem of the stochastic transportation problem (1) - (4) as follows:

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \qquad (15)$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i,\ i = 1,2,\ldots,m \qquad (16)$$

$$\sum_{i=1}^{m} x_{ij} \le \frac{-\ln(\beta_j)}{\lambda_{b_j}},\ j = 1,2,\ldots,n \qquad (17)$$

$$x_{ij} \ge 0;\ i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \qquad (18)$$

Case- III: Both a_i and b_j are exponential random variables

Let a_i and b_j are exponential random variables with known means defined as:

$$E[a_i] = \frac{1}{\lambda_{a_i}},\ i = 1,2,\ldots,m$$

$$E[b_j] = \frac{1}{\lambda_{b_j}},\ j = 1,2,\ldots,n$$

$$0 < \alpha_i < 1 \quad \text{and} \quad 0 < \beta_j < 1$$

As described in **Case- I** and **Case- II**, the equivalent deterministic constraints of the probabilistic constraints (2) and (3) are defined by

$$\sum_{j=1}^{n} x_{ij} \ge \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}},\ i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \le \frac{-\ln(\beta_j)}{\lambda_{b_j}},\ j = 1,2,\ldots,n$$

and hence the equivalent single objective deterministic transportation problem of the stochastic transportation problem (1) - (4) can be formulated as:

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{19}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \leq \frac{-\ln(\beta_j)}{\lambda_{b_j}}, j = 1,2,\ldots,n$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m; j = 1,2,\ldots,n \tag{20}$$

3. Multi-objective stochastic transportation problem involving exponential random variables

Let us consider the multiobjective stochastic unbalanced transportation problem with source and demand constraints involving exponential random variables as follows :

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \; ; k = 1,2,\ldots,K \tag{21}$$

subject to

$$\Pr[\sum_{j=1}^{n} x_{ij} \leq a_i] \geq 1-\alpha_i, i = 1,2,\ldots,m \tag{22}$$

$$\Pr[\sum_{i=1}^{m} x_{ij} \geq b_j] \geq 1-\beta_j, j = 1,2,\ldots,n \tag{23}$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m; j = 1,2,\ldots,n \tag{24}$$

where $0 < \alpha_i < 1$, $0 < \beta_j < 1$ and a_i, b_j are exponential random variables. For simplicity, let us consider the decision variables x_{ij} to be deterministic.

Case-I: Only a_i are exponential random variables

Let a_i are exponential random variables with known means

$$E[a_i] = \frac{1}{\lambda_{a_i}}, i = 1,2,\ldots,m$$

As discussed earlier, the probabilistic constraint (22) reduced to

$$\sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m$$

Hence the equivalent deterministic transportation problem for the multiobjective stochastic transportation problem (21) - (24)becomes

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \; ; k = 1,2,\ldots,K \tag{25}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m \tag{26}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j;\ j = 1,2,\ldots,n \tag{27}$$

$$x_{ij} \geq 0; i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{28}$$

Case-II: Only b_j are exponential random variables

Given b_j are exponential random variables with means

$$E[b_j] = \frac{1}{\lambda_{b_j}},\ j = 1,2,\ldots,n$$

Now we obtain the equivalent deterministic constraint for (23) as

$$\sum_{i=1}^{m} x_{ij} \leq \frac{-\ln(\beta_j)}{\lambda_{b_j}},\ j = 1,2,\ldots,n$$

and hence the multiobjective deterministic transportation problem of the multiobjective stochastic transportation problem (21) - (24) is formulated as:

$$\min: Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}\ ; k = 1,2,\ldots,K \tag{29}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i, i = 1,2,\ldots,m \tag{30}$$

$$\sum_{i=1}^{m} x_{ij} \leq \frac{-\ln(\beta_j)}{\lambda_{b_j}},\ j = 1,2,\ldots,n \tag{31}$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \tag{32}$$

Case-III: Both a_i and b_j are exponential random variables

Let a_i and b_j are both exponential random variables with known means

$$E[a_i] = \frac{1}{\lambda_{a_i}}, i = 1,2,\ldots,m$$

$$E[b_j] = \frac{1}{\lambda_{b_j}},\ j = 1,2,\ldots,n$$

$$0 < \alpha_i < 1 \quad \text{and} \quad 0 < \beta_j < 1$$

In this case the equivalent multi-objective deterministic transportation problem of the stochastic transportation problem (21) - (24) is given as:

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \; ; k = 1,2,\ldots,K \tag{33}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq \frac{-\ln(1-\alpha_i)}{\lambda_{a_i}}, i = 1,2,\ldots,m \tag{34}$$

$$\sum_{i=1}^{m} x_{ij} \leq \frac{-\ln(\beta_j)}{\lambda_{b_j}}, \; j = 1,2,\ldots,n \tag{35}$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m; \; j = 1,2,\ldots,n \tag{36}$$

4. Numerical example

The numerical problem is related to a multi-objective stochastic transportation problem in which the source and destination parameters are exponential random variables with known means given by

$$E[a_i] = \frac{1}{\lambda_{a_i}}, i = 1,2,3,4$$

and

$$E[b_j] = \frac{1}{\lambda_{b_j}}, \; j = 1,2,3,4,5.$$

The objectives are non-commensurable and conflicting in nature.

$$\begin{aligned} \min : Z_1 = {} & 9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25} \\ & + 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} + 6x_{41} + 9x_{42} + 11x_{43} + 2x_{44} + 2x_{45} \end{aligned} \tag{37}$$

$$\begin{aligned} \min : Z_2 = {} & 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25} \\ & + 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45} \end{aligned} \tag{38}$$

$$\begin{aligned} \min : Z_3 = {} & 2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25} \\ & + 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45} \end{aligned} \tag{39}$$

subject to

$$\Pr[\sum_{j=1}^{5} x_{1j} \leq a_1] \geq 1 - \alpha_1 \tag{40}$$

$$\Pr[\sum_{j=1}^{5} x_{2j} \leq a_2] \geq 1 - \alpha_2 \tag{41}$$

$$\Pr[\sum_{j=1}^{5} x_{3j} \le a_3] \ge 1-\alpha_3 \quad (42)$$

$$\Pr[\sum_{j=1}^{5} x_{4j} \le a_4] \ge 1-\alpha_4 \quad (43)$$

$$\Pr[\sum_{i=1}^{4} x_{i1} \ge b_1] \ge 1-\beta_1 \quad (44)$$

$$\Pr[\sum_{i=1}^{4} x_{i2} \ge b_2] \ge 1-\beta_2 \quad (45)$$

$$\Pr[\sum_{i=1}^{4} x_{i3} \ge b_3] \ge 1-\beta_3 \quad (46)$$

$$\Pr[\sum_{i=1}^{4} x_{i4} \ge b_4] \ge 1-\beta_4 \quad (47)$$

$$\Pr[\sum_{i=1}^{4} x_{i5} \ge b_5] \ge 1-\beta_5 \quad (48)$$

$$x_{ij} \ge 0,\ i=1,2,3,4;\ j=1,2,3,4,5. \quad (49)$$

In the above numerical example, a_i and b_j are independent exponential random variables with known means

$E[a_1]=5,$ $E[a_2]=4,$ $E[a_3]=8,$ $E[a_4]=10,$

$E[b_1]=2,$ $E[b_2]=20,$ $E[b_3]=5,$ $E[b_4]=6,$ $E[b_5]=3$

And the specified probability levels

$\alpha_1=0.60,$ $\alpha_2=0.70,$ $\alpha_3=0.80,$ $\alpha_4=0.90,$

$\beta_1=0.10,$ $\beta_2=0.20,$ $\beta_3=0.30,$ $\beta_4=0.40,$ $\beta_5=0.50$

The deterministic model can be obtained using equations (33)-(35) as

$$\begin{aligned}\min: Z_1 &= 9x_{11}+12x_{12}+9x_{13}+6x_{14}+9x_{15}+7x_{21}+3x_{22}+7x_{23}+7x_{24}+5x_{25}\\ &+6x_{31}+5x_{32}+9x_{33}+11x_{34}+3x_{35}+6x_{41}+9x_{42}+11x_{43}+2x_{44}+2x_{45}\end{aligned} \quad (50)$$

$$\begin{aligned}\min: Z_2 &= 2x_{11}+9x_{12}+8x_{13}+x_{14}+4x_{15}+x_{21}+9x_{22}+9x_{23}+5x_{24}+2x_{25}\\ &+8x_{31}+x_{32}+8x_{33}+4x_{34}+5x_{35}+2x_{41}+8x_{42}+6x_{43}+9x_{44}+8x_{45}\end{aligned} \quad (51)$$

$$\begin{aligned}\min: Z_3 &= 2x_{11}+4x_{12}+6x_{13}+3x_{14}+6x_{15}+4x_{21}+8x_{22}+4x_{23}+9x_{24}+2x_{25}\\ &+5x_{31}+3x_{32}+5x_{33}+3x_{34}+6x_{35}+6x_{41}+9x_{42}+6x_{43}+3x_{44}+x_{45}\end{aligned} \quad (52)$$

subject to

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15} \ge 4.581454 \quad (53)$$

$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25} \geq 4.815891 \quad (54)$$

$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35} \geq 12.875503 \quad (55)$$

$$x_{41}+x_{42}+x_{43}+x_{44}+x_{45} \geq 23.025851 \quad (56)$$

$$x_{11}+x_{21}+x_{31}+x_{41} \leq 4.60517 \quad (57)$$

$$x_{12}+x_{22}+x_{32}+x_{42} \leq 32.188758 \quad (58)$$

$$x_{13}+x_{23}+x_{33}+x_{43} \leq 6.019864 \quad (59)$$

$$x_{14}+x_{24}+x_{34}+x_{44} \leq 5.497744 \quad (60)$$

$$x_{15}+x_{25}+x_{35}+x_{45} \leq 2.079442 \quad (61)$$

$$x_{ij} \geq 0,\ i=1,2,3;\ j=1,2,3,4. \quad (62)$$

As discussed in the solution procedure, we obtained three ideal solutions of the objective functions (50)-(52) with the set of constraints (53)-(62) and a pay off matrix is framed as shown in the following table.

Table 1: Pay-off Matrix

	$Z_1(X)$	$Z_2(X)$	$Z_3(X)$
$X^{(1)}$	260.435061	254.943643	248.437456
$X^{(2)}$	322.335967	183.474493	252.875062
$X^{(3)}$	318.876828	238.696511	216.39911

Using the linear membership function of fuzzy technique, we get

$$\mu_1(x)=\begin{cases}1, & z_1 \leq 260.435061\\ \dfrac{322.335967-z_1}{322.335967-260.435061}, & 260.435061 < z_1 < 322.335967\\ 0, & z_1 \geq 322.335967\end{cases} \quad (63)$$

$$\mu_2(x)=\begin{cases}1, & z_2 \leq 183.474493\\ \dfrac{254.943643-z_2}{254.943643-183.474493}, & 183.474493 < z_2 < 254.943643\\ 0, & z_2 \geq 254.943643\end{cases} \quad (64)$$

$$\mu_3(x)=\begin{cases}1, & z_3 \leq 216.39911\\ \dfrac{252.875062-z_3}{252.875062-216.39911}, & 216.39911 < z_3 < 252.875062\\ 0, & z_3 \geq 252.875062\end{cases} \quad (65)$$

Finally, the single objective transportation problem which is equivalent to the multiobjective problem (37)-(49) is derived as:

$$\max : \lambda \tag{66}$$

subject to

$$9x_{11} + 12x_{12} + 9x_{13} + 6x_{14} + 9x_{15} + 7x_{21} + 3x_{22} + 7x_{23} + 7x_{24} + 5x_{25}$$
$$+ 6x_{31} + 5x_{32} + 9x_{33} + 11x_{34} + 3x_{35} + 6x_{41} + 9x_{42} + 11x_{43} + 2x_{44} + 2x_{45}$$
$$+ 61.900906\lambda \leq 322.335967 \tag{67}$$

$$2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{15} + x_{21} + 9x_{22} + 9x_{23} + 5x_{24} + 2x_{25}$$
$$+ 8x_{31} + x_{32} + 8x_{33} + 4x_{34} + 5x_{35} + 2x_{41} + 8x_{42} + 6x_{43} + 9x_{44} + 8x_{45}$$
$$+ 71.469150\lambda \leq 254.943643 \tag{68}$$

$$2x_{11} + 4x_{12} + 6x_{13} + 3x_{14} + 6x_{15} + 4x_{21} + 8x_{22} + 4x_{23} + 9x_{24} + 2x_{25}$$
$$+ 5x_{31} + 3x_{32} + 5x_{33} + 3x_{34} + 6x_{35} + 6x_{41} + 9x_{42} + 6x_{43} + 3x_{44} + x_{45}$$
$$+ 36.475952\lambda \leq 252.875062 \tag{69}$$

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} \geq 4.581454 \tag{70}$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \geq 4.815891 \tag{71}$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \geq 12.875503 \tag{72}$$
$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} \geq 23.025851 \tag{73}$$
$$x_{11} + x_{21} + x_{31} + x_{41} \leq 4.60517 \tag{74}$$
$$x_{12} + x_{22} + x_{32} + x_{42} \leq 32.188758 \tag{75}$$
$$x_{13} + x_{23} + x_{33} + x_{43} \leq 6.019864 \tag{76}$$
$$x_{14} + x_{24} + x_{34} + x_{44} \leq 5.497744 \tag{77}$$
$$x_{15} + x_{25} + x_{35} + x_{45} \leq 2.079442 \tag{78}$$
$$x_{ij} \geq 0, \; i = 1,2,3; \; j = 1,2,3,4. \tag{79}$$

Solving the above linear programming problem (66) - (79), the compromise solution is obtained as :

$$\lambda = 0.4830865$$
$$x_{11} = 0, x_{12} = 1.679055, x_{13} = 0, x_{14} = 2.902399, x_{15} = 0$$
$$x_{21} = 0.4729809, x_{22} = 4.342910, x_{23} = x_{24} = x_{25} = 0$$
$$x_{31} = 0, x_{32} = 12.87550, x_{33} = x_{34} = x_{35} = 0$$
$$x_{41} = 4.132189, x_{42} = 8.199011, x_{43} = 6.019864, x_{44} = 2.595345, x_{45} = 2.079442$$

5. Conclusions

In this paper we have formulated single objective and multi-objective stochastic unbalanced transportation problems in which the objective functions are of minimization type and either the supply or the demand or both the supply and demand are considered as exponential random variables of known means. For simplicity we have considered the decision variables to be deterministic. Then using the probability density functions, the probabilistic linear programming model is transformed in to an equivalent deterministic linear programming model. We have applied the fuzzy programming technique for solving the given specified problem and to obtain an optimal compromise solution from the set of non dominated solutions.

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