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On Intuitionistic Fuzzy T_1-Spaces

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ABSTRACT

The basic concepts of the theory of intuitionistic fuzzy topological spaces have been defined by D. Coker and his co-workers. In this paper, we define some new notions of T_1 −spaces using intuitionistic fuzzy sets. We also show all of these good extensions property. Furthermore, we study some relations among them. Moreover, some of their other properties are obtained.

Keywords: Intuitionistic topological space, intuitionisticfuzzy topological space, intuitionistic fuzzy T_1–spaces.

1. Introduction

After the introduction of fuzzy sets by Zadeh [16] in 1965 and fuzzy topology by Chang [6] in 1968, several researches were conducted on the generalization of the notions of the fuzzy sets and topology. The concepts of intuitionistic fuzzy sets was introducedby Atanassov [1, 2] as a generalization of fuzzy sets. Coker [3, 4, 5, 7, 8, 9, 10] and his colleagues introducedintuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we investigate the properties and features ofT_1 –spaces.

Definition 1.1. [10] An intuitionistic set A is an object having the form $A=(x, A_1, A_2)$ where $A_1$and A_2 are subsets of Xsatisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of$A$ while A_2 is called the set of non-member of A.

Throughoutthis paper, we use the simpler notation $A = (A_1, A_2)$ for an intuitionistic set.

Remark 1.2. [10] Every subset A on a nonempty set$X$ may obviously be regarded as an intuitionistic set having the form$A' = (A, A^C)$, where $A^C = X \setminus A$ is the complement of A in X.

Definition 1.3. [10] Let the intuitionistic sets A and B on X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j : j \in J\}$ be an arbitrary family of intuitionistic sets in X,where $A_j = (A_j^{(1)}, A_j^{(2)})$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$and $A_2 \supseteq B_2$.
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(c) $\bar{A} = (A_2,\ A_1)$, denotes the complement of A.
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$.
(e) $\cup A_j = (\cup A_j^{(1)},\ \cap A_j^{(2)})$.
(f) $\phi_\sim = (\phi,\ X)$ and $X_\sim = (X,\ \phi)$.

Definition: 1.4. (cf [7]) An intuitionistic topology on a set $X$is a family τ of intuitionistic sets in X satisfying the following axioms:
(1) $\phi_\sim, X_\sim \in \tau$.
(2) $G_1 \cap G_2 \in \tau$for any $G_1,\ G_2 \in \tau$.
(3) $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.
In this case, the pair (X, τ) is called an intuitionistic topological space (ITS, in short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS, in short) in X.

Definition 1.5. [2] Let X be a non empty set and I be the unit interval $[0,1]$. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form$A$=$\{(x, \mu_A(x), \nu_A(x)), x \in X\}$, where $\mu_A: X \to I$and $\nu_A: X \to I$ denote the degree of membership and the degree of non- membership respectively, and $\mu_A(x) + \nu_A(x) \leq 1$.

Let $I(X)$ denote the set of all intuitionistic fuzzy sets in X. Obviouslyevery fuzzy set $\mu_A$in X is an intuitionistic fuzzy set of the form ($\mu_A, 1 - \mu_A$).

Throughout this paper, we use the simpler notation A = (μ_A, ν_A) instead of A=$\{(x, \mu_A(x), \nu_A(x)), x \in X\}$.

Definition 1.6. [2] Let A=(μ_A, ν_A) and B= (μ_B, ν_B) be intuitionistic fuzzy sets in X. Then

(1) $A \subseteq B$if and only if $\mu_A \leq \mu_B$and $\nu_A \geq \nu_B$.
(2) $A = B$if and only if $A \subseteq B$ and $B \subseteq A$.
(3) A^c= (ν_A, μ_A).
(4) $A \cap B$= $(\mu_A \cap \mu_B; \nu_A \cup \nu_B)$.
(5) $A \cup B$= $(\mu_A \cup \mu_B; \nu_A \cap \nu_B)$.
(6) $0_\sim$ = $(0^\sim, 1^\sim)$ and $1_\sim$= $(1^\sim, 0^\sim)$.

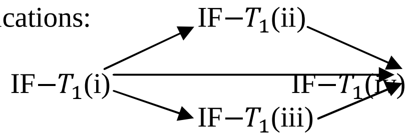
Definition 1.7. [8] An intuitionistic fuzzy topology (IFT, in short) on X is a family t of IFS's in X which satisfies the following axioms:
(1) $0_\sim, 1_\sim \in t$.
(2) if $A_1,\ A_2 \in t$, then $A_1 \cap A_2 \in t$.
(3) If $A_i \in t$ for each i, then $\cup A_i \in t$.
The pair (X, t) is called an intuitionistic fuzzy topological space (IFTS, in short).
Let (X, t) be an IFTS. Then any member of t is called an intuitionistic fuzzy open set (IFOS,in short) in X.The complement of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS, in short) in X.

2. Intuitionistic fuzzy T_1 −spaces
Definition 2.1. An intuitionistic fuzzy topological space (X, t) is called

(1) IF $-T_1$ (i) if $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A),\ B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x)$=1, $\nu_A(x)$=0;$\mu_A(y)$=0, $\nu_A(y)$=1 and$\mu_B(y)$=1, ν_B(y)=0;$\mu_B(x)$=0, $\nu_B(x)$=1.
(2) IF $-T_1$ (ii) if $\forall\ x,\ y\ \in X, x \neq y\ \exists$ A= $(\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x)$=1, $\nu_A(x)$=0;$\mu_A(y)$=0, $\nu_A(y)> 0$ and$\mu_B(y)$=1, $\nu_B(y)$=0; $\mu_B(x)$=0, $\nu_B(x)> 0$.
(3) IF $-T_1$ (iii) if $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) > 0$, $\nu_A(x)$=0;$\mu_A(y)$=0, $\nu_A(y) = 1$ and $\mu_B(y) > 0$, $\nu_B(y)$=0; $\mu_B(x)$=0, $\nu_B(x) = 1$.
(4) IF$-T_1$(iv) if$\forall x,\ y\ \in X, x \neq y\ \exists$ A= $(\mu_A, \nu_A), B$=$(\mu_B, \nu_B) \in t$ such that $\mu_A(x)> 0$, $\nu_A(x) = 0$;$\mu_A(y)$=0, $\nu_A(y)> 0$and$\mu_B(y)> 0$, $\nu_B(y)$ =0; $\mu_B(x)$=0, $\nu_B(x)> 0$.

Definition 2.2. Let $\alpha \in (0,1)$. An intuitionistic fuzzy topological space (X,t) is called
(a) $\alpha -$ IF $-T_1$ (i) if $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x)$=1, $\nu_A(x) = 0$;$\mu_A(y)= 0$,$\nu_A(y)\geq \alpha$and$\mu_B(y)$=1, $\nu_B(y)$=0; $\mu_B(x)= 0$, $\nu_B(x)\geq \alpha$.
(b) $\alpha -$ IF $-T_1$ (ii) if $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x)\geq \alpha$, $\nu_A(x) = 0$; $\mu_A(y)= 0$,$\nu_A(y)\geq \alpha$and$\mu_B(y)\geq \alpha$,$\nu_B(y) = 0$; $\mu_B(x)= 0$, $\nu_B(x)\geq \alpha$.
(c) $\alpha -$ IF $-T_1$ (iii) if $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A)$, , $B = (\mu_B, \nu_B) \in t$ such that$\mu_A(x)> 0$, $\nu_A(x) = 0$;$\mu_A(y) = 0$, $\nu_A(y)\geq \alpha$and$\mu_B(y)> 0$, $\nu_B(y) = 0$;$\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$.

Theorem 2.3. Let (X,t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose(X,t) is IF$-\ T_1$(i) space. We shall prove that (X,t) is IF$-\ T_1$(ii). Since (X,t) is IF$-T_1$(i), then $\forall x,\ y\ \in X, x \neq y \exists\ A = (\mu_A, \nu_A)$, $B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x)$ =1, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) = 1$and$\mu_B(y)$=1, $\nu_B(y) = 0$; $\mu_B(x)$= 0,$\nu_B(x) = 1$ $\Longrightarrow \mu_A(x)$= 1, $\nu_A(x) = 0$;$\mu_A(y) = 0$, $\nu_A(y) > 0$and$\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x)$= 0, $\nu_B(x) > 0$.Which is IF$-T_1$(ii).Hence IF$-T_1$(i) $\Longrightarrow$ IF$-T_1$(ii).Again, suppose(X,t) is IF$-T_1$(i) space. We shall prove that (X,t) is IF$-T_1$(iii). Since (X,t) is IF$-\ T_1$(i), then $\forall\ x,\ y\ \in X, x \neq y\ \exists\ A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$such that$\mu_A(x)$ =1, $\nu_A(x) = 0$;$\mu_A(y) = 0$, $\nu_A(y) = 1$ and$\mu_B(y)$ =1, $\nu_B(y) = 0$; $\mu_B(x)$= 0, $\nu_B(x) = 1$ $\Longrightarrow \mu_A(x)> 0$, $\nu_A(x) = 0$; $\mu_A(y)$ =0, $\nu_A(y)$ =1and$\mu_B(y)> 0$, $\nu_B(y) = 0$; $\mu_B(x)$=0, $\nu_B(x) = 1$.Which is IF$-T_1$(iii). Hence IF$-T_1$(i) $\Longrightarrow$ IF$-T_1$(iii).

Furthermore, it can easily verify that IF$-T_1$(i) $\Longrightarrow$IF$-T_1$(iv), IF$-T_1$(ii) $\Longrightarrow$ IF$-T_1$(iv) andIF$-T_1$(iii) $\Longrightarrow$ IF$-T_1$(iv).

Noneofthereverse implications is true in generalwhich can be seen from the following examples.

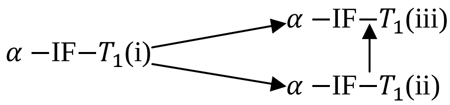
Example 2.3.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A= {(x, 1, 0), (y, 0, 0.5)} and B ={(x, 0, 0.7), (y, 1, 0)}.We see that the IFTS (X,t) is IF- T_1(ii) but not IF- T_1(i).

Example 2.3.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$where A = {(x, o.2, 0), (y, 0, 1)} and B={(x, 0, 1), (y, 0.6, 0)}.We see that the IFTS (X, t) is IF- T_1(iii) but not IF- T_1(i).

Example 2.3.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$where A={(x, 1, 0), (y, 0, 0.4)} and B = {(x, 0, 0.6), (y, 1, 0)}.We see that the IFTS (X, t) is IF- T_1(ii) but not IF- T_1(iii).

Example 2.3.4. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$where A = {(x, o.3, 0), (y, 0, 1)} and B={(x, 0, 1), (y, 0.5, 0)}.We see that the IFTS (X, t) is IF- T_1(iii) but not IF- T_1(ii).

Theorem 2.4. Let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, t) is $\alpha -$ IF $-T_1$ (i) space. We shall prove that (X, t) is $\alpha -$IF$-T_1$(ii).Let $\alpha \in$ (0, 1). Since (X, t) is $\alpha -$IF$-T_1$(i), then $\forall\, x,\ y \in X, x \neq y \exists A = (\mu_A, \nu_A), B= (\mu_B, \nu_B) \in t$ such that $\mu_A(x)$ =1, $\nu_A(x)$= 0; $\mu_A(y)$ = 0, $\nu_A(y) \geq \alpha$ and$\mu_B(y)$=1,$\nu_B(y)$ = 0;$\mu_B(x)$= 0, $\nu_B(x) \geq \alpha$
$\Longrightarrow \mu_A(x) \geq \alpha$,$\nu_A(x)$ =0;$\mu_A(y)$ = 0, $\nu_A(y) \geq \alpha$and $\mu_B(y) \geq \alpha$,$\nu_B(y)$ = 0;$\mu_B(x)$= 0, $\nu_B(x) \geq \alpha$for any $\alpha \in$ (0, 1), which is $\alpha -$IF$-T_1$(ii). Hence $\alpha -$IF$-T_1$(i) $\Longrightarrow \alpha -$IF$-T_1$(ii).
Again, suppose(X, t) is IF$-T_1$(ii) space. We shall prove that (X, t) is $\alpha -$IF$-T_1$(iii).Let $\alpha \in$ (0, 1). Since (X, t) is $\alpha -$IF$-T_1$(ii), then $\forall\ x,\ y \in X,\ x \neq y \exists A = (\mu_A, \nu_A)$,$B= (\mu_B, \nu_B) \in t$ such that $\mu_A(x) \geq \alpha$,$\nu_A(x)$ = 0;$\mu_A(y)$ = 0, $\nu_A(y) \geq \alpha$and$\mu_B(y) \geq \alpha$,$\nu_B(y)$ = 0;$\mu_B(x)$= 0, $\nu_B(x) \geq \alpha$ $\Longrightarrow \mu_A(x) > 0$,$\nu_A(x)$ = 0;$\mu_A(y)$ = 0,$\nu_A(y) \geq \alpha$ and $\mu_B(y) > 0$,$\nu_B(y)$ = 0;$\mu_B(x)$= 0, $\nu_B(x) \geq \alpha$for any $\alpha \in$ (0, 1). Which is $\alpha -$IF$-T_1$(iii). Hence $\alpha -$IF$-T_1$(ii) $\Longrightarrow \alpha -$IF$-T_1$(iii).

Furthermore, it can easily verify that $\alpha -$IF$-T_1$(i) $\Longrightarrow \alpha -$IF$-T_1$(iii).

None of thereverse implications is true in general which can be seen from the following examples.

Example 2.4.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$where A= {(x, 0.5, 0), (y, 0, 0.5)} and B= {(x, 0, 0.4), (y, 0.4, 0)}. For α =0.3, we see that the IFTS (X, t) is $\alpha -$IF$-T_1$(ii) but not $\alpha -$IF$-T_1$(i).

Examples 2.4.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$where A= {(x, 0.2, 0), (y, 0, 0.4)} and B= {(x, 0, 0.5), (y, 0.3, 0)}.For α =0.4, we see that the IFTS (X, t) is $\alpha -$IF$-T_1$(iii) but not $\alpha -$IF$-T_1$(ii).

Theorem 2.5. Let (X, t) be an intuitionistic fuzzy topological space and 0$< \alpha \leq \beta < 1$, then

(a) β −IF−T_1(i) $\Longrightarrow$ α −IF−T_1(i).
(b) β −IF−T_1(ii) $\Longrightarrow$ α −IF−T_1(ii).
(c) β −IF−T_1(iii) $\Longrightarrow$ α −IF−T_1(iii).

Proof (a): Suppose the intuitionistic fuzzy topological space (X,t) is β −IF−T_1(i). We shall prove that (X,t) is α −IF−T_1(i). Since (X,t) is β −IF−T_1(i), then $\forall x,\ y \in X, x \neq y$ with $\beta \in (0,\ 1)$ $\exists A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in t$ such that $\mu_A(x) = 1$, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \beta$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \beta$ $\Longrightarrow \mu_A(x) = 1 \nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y) = 1$, $\nu_B(y) = 0$; $\mu_B(x) = 0$, $\nu_B(x) \geq \alpha$ as $0 < \alpha \leq \beta < 1$, which is α −IF−T_1(i). Hence β −IF−T_1(i) $\Longrightarrow$ α −IF−T_1(i).

Furthermore, it can easily verify that β −IF−T_1 (ii) $\Longrightarrow$ α −IF−T_1(ii) and β −IF−T_1(iii) $\Longrightarrow$ α −IF−T_1(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example 2.5.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 1, 0), (y, 0, 0.6)} and B= {(x, 0, 0.5), (y, 1, 0)}. For α =0.5 and β =0.7, it is clear that the IFTS (X,t) is α −IF−T_1(i) but not β −IF−T_1(i).

Example 2.5.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 0.5, 0), (y, 0, 0.5)} and B= {(x, 0, 0.6), (y, 0.7, 0)}. For $\alpha = 0.5$ and β =0.8, it is clear that the IFTS (X,t) is α − IF −T_1 (ii) but not β −IF−T_1(ii).

Example 2.5.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 0.2, 0), (y, 0, 0.4)} and B= {(x, 0, 0.5), (y, 0.3, 0)}. For $\alpha = 0.4$ and β =0.6, it is clear that the IFTS (X,t) is α − IF −T_1 (iii) but not β −IF−T_1(iii).

Theorem 2.6. Let (X,t) be an intuitionistic fuzzy topological space, $U \subseteq X$ and t_U ={ $A|U : A \in t$} and $\alpha \in (0,1)$, then

(a) (X,t) is IF−T_1(i) ⇒ (U,t_U) is IF−T_1(i).
(b) (X,t) is IF−T_1(ii) ⇒ (U,t_U) is IF−T_1(ii).
(c) (X,t) is IF−T_1(iii) ⇒ (U,t_U) is IF−T_1(iii).
(d) (X,t) is IF−T_1(iv) ⇒ (U,t_U) is IF−T_1(iv).
(e) (X,t) is α −IF−T_1(i) ⇒ (U,t_U) is α −IF−T_1(i).
(f) (X,t) is α −IF−T_1(ii) ⇒ (U,t_U) is α −IF−T_1(ii).
(g) (X,t) is α −IF−T_1(iii) ⇒ (U,t_U) is α −IF−T_1(iii).

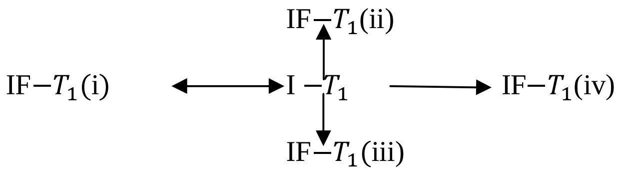
The proofs (a), (b), (c), (d), (e), (f), (g) are similar. As an example we proved (e).

Proof (e): Suppose (X,t) is an intuitionistic fuzzy topological space and is also α −IF−T_1(i). We shall prove that (U,t_U) is α −IF−T_1(i). Let $x,\ y \in U, x \neq y$ then $x,\ y \in X, x \neq y$ as $U \subseteq X$. Since (X,t) is α −IF−T_1(i), then $\exists$ A =(μ_A, ν_A), B= $(\mu_B, \nu_B) \in t$ such that $\mu_A(x)$ =1, $\nu_A(x) = 0$; $\mu_A(y) = 0$, $\nu_A(y) \geq \alpha$ and $\mu_B(y)$ =1, $\nu_B(y) = 0$; $\mu_B(x)$=

$0, \nu_B(x) \geq \alpha \Rightarrow \mu_A|U(x)$ =1, $\nu_A|U(x) = 0; \mu_A|U(y) = 0$, $\nu_A|U(y) \geq \alpha$ and $\mu_B|U(y)$ =1, $\nu_B|U(y) = 0; \mu_B|U(x) = 0$, $\nu_B|U(x) \geq \alpha$. Since $\{(\mu_A|U, \nu_A|U), (\mu_B|U, \nu_B|U)\} \in t_U \Rightarrow \{(B|U, C|U)\} \in t_U$. Hence, it is clear that the intuitionistic fuzzy topological space (U, t_U) is α −IF−T_1(i).

Definition 2.7. An intuitionistic topological space (ITS, in short) (X, τ) is called intuitionistic T_1 −space (I −T_1 space) if $\forall x, y \in X, x \neq y \exists C = (C_1, C_2), D = (D_1, D_2) \in \tau$ such that $x \in C_1, y \in C_2$ and $y \in D_1, x \in D_2$.

Theorem 2.8. Let (X, τ) be an intuitionistic topological space and let (X, t) be an intuitionistic fuzzy topological space. Then we have the following implications:



Proof: Suppose (X, τ) is I −T_1 space. We shall prove that (X, t) is IF−T_1(i). Since (X, τ) is I −T_1, then $\forall x, y \in X, x \neq y \exists C = (C_1, C_2), D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in C_2$ and $y \in D_1$, $x \in D_2 \Rightarrow 1_{C_1}(x) = 1$, $1_{C_2}(y) = 1$ and $1_{D_1}(y) = 1$, $1_{D_2}(x) = 1$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$, $1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$ then $\mu_A(x)$ =1, $\upsilon_A(x)$=0;$\mu_A(y)$ =0, $\upsilon_A(y)$ =1 and $\mu_B(y)$ =1, $\upsilon_B(y)$=0; $\mu_B(x) = 0$, $\upsilon_B(x)$ =1. Since $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \Rightarrow (X, t)$ is IF−T_1(i). Hence I −T_1 $\Rightarrow$IF−T_1(i).

Conversely, suppose (X, t) is IF−T_1(i). We shall show that $(X, \tau)$is I −T_1. Since (X, t) is IF−T_1(i), then $\forall x, y \in X, x \neq y \exists A = (\mu_A, \upsilon_A), B = (\mu_B, \upsilon_B) \in t$ such that $\mu_A(x)$ =1, $\upsilon_A(x)$ =0; $\mu_A(y)$ =0, $\upsilon_A(y)$=1 and $\mu_B(y)$ =1, $\upsilon_B(y)$ =0; $\mu_B(x) = 0$, $\upsilon_B(x)$ =1. Let $C_1 = \mu_A^{-1}\{1\}$, $C_2 = \upsilon_A^{-1}\{1\}$, $D_1 = \mu_B^{-1}\{1\}$, $D_2 = \upsilon_B^{-1}\{1\} \Rightarrow x \in C_1, y \in D_2$ and $y \in D_1$, $x \in D_2$. Since $\{(C_1, C_2), (D_1, D_2)\} \in \tau \Rightarrow (X, \tau)$ is I −T_1. Hence IF−T_1(i) $\Rightarrow$ I −T_1. Therefore I −T_1 $\Leftrightarrow$IF−T_1(i).

Furthermore, it caneasily verify that I− $T_1 \Rightarrow$ IF− T_1(ii), I− $T_1 \Rightarrow$IF− T_1(iii) andI− $T_1 \Rightarrow$ IF− T_1(iv).

None of the reverse implications is true in general which can be seen from the following examples.

Examples 2.8.1. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$where A ={(x, 1, 0), (y, 0, 0.4)} and B={(x, 0, 0.5), (y, 1, 0)}, it is clear that the IFTS (X, t) is IF−T_1(ii) but not corresponding I− T_1.

Examples 2.8.2. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A={(x, o.5, 0), (y, 0, 1)} and B={(x, 0, 1), (y, 0.6, 0)}, it is clear that the IFTS (X, t) is IF−T_1(iii) but not correspondingI− T_1.

Examples 2.8.3. Let $X = \{x, y\}$ and t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A={(x, o.2, 0), (y, 0, 0.3)} and B={(x, 0, 0.4), (y, 0.6, 0)}, it is clear that the IFTS (X, t) is IF$-T_1$(iv) but not corresponding I$-$ T_1.

Theorem 2.9. Let (X, τ) be an intuitionistic topological space and let (X, t) be the intuitionistic fuzzy topological space. Then we have the following implications:

$$\mathrm{I}-T_1 \Rightarrow \alpha-\mathrm{IF}-T_1(\mathrm{ii})$$
$$\mathrm{I}-T_1 \Rightarrow \alpha-\mathrm{IF}-T_1(\mathrm{i})$$
$$\mathrm{I}-T_1 \Rightarrow \alpha-\mathrm{IF}-T_1(\mathrm{iii})$$

Proof: Let $\alpha \in (0, 1)$. Suppose (X, τ) is I $-T_1$ space. We shall prove that (X, t) is α $-$IF$-T_1$(i). Since (X, τ) is I $-T_1$, then$\forall x, y \in X, x \neq y \exists C = (C_1, C_2), D = (D_1, D_2) \in \tau$ such that $x \in C_1$, $y \in C_2$ and $y \in D_1$, $x \in D_2 \Longrightarrow 1_{C_1}(x) = 1$, $1_{C_2}(y) = 1$and$1_{D_1}(y) = 1$, $1_{D_2}(x) = 1 \Longrightarrow 1_{C_1}(x) = 1$, $1_{C_2}(y) \geq \alpha$ and$1_{D_1}(y) = 1$, $1_{D_2}(x) \geq \alpha$ for any $\alpha \in (0, 1)$. Let $1_{C_1} = \mu_A$, $1_{C_2} = \upsilon_A$,$1_{D_1} = \mu_B$, $1_{D_2} = \upsilon_B$ then $\Longrightarrow \mu_A(x)$=1, $\upsilon_A(x)$=0; $\mu_A(y)$=0, $\upsilon_A(y) \geq \alpha$ and $\mu_B(y)$ =1, $\upsilon_B(y)$=0; $\mu_B(x)$ = 0, $\upsilon_B(x) \geq \alpha$ for any $\alpha \in (0, 1)$. Since $\{(\mu_A, \upsilon_A), (\mu_B, \upsilon_B)\} \in t \Longrightarrow (X, t)$ is α $-$IF$-T_1$(i). Hence I $-T_1 \Longrightarrow \alpha$ $-$IF$-T_1$(i). Furthermore, it can be shown that I$-$ $T_1 \Longrightarrow \alpha$ $-$IF$-T_1$(ii) and I$-$ $T_1 \Longrightarrow \alpha$ $-$IF$-T_1$(iii).

None of the reverse implications is true in general which can be seen from the following examples.

Example 2.9.1. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by $\{A, B\}$ where A= {(x, 1, 0), (y, 0, 0.7)} and B= {(x, 0, 0.8), (y, 1, 0)}. For α =0.7, it is clear that the IFTS (X, t) isα $-$IF$-T_1$(i)but not corresponding I$-$ T_1 .

Example 2.9.2. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A= {(x, 0.5, 0), (y, 0, 0.5)} and B= {(x, 0, 0.6), (y, 0.6, 0)}. For α =0.4, it is clear that the IFTS (X, t) is α $-$IF$-T_1$(ii) but not corresponding I$-$ T_1.

Example 2.9.3. Let $X = \{x, y\}$ and let t be the intuitionistic fuzzy topology on X generated by$\{A, B\}$ where A= {(x, 0.3, 0), (y, 0, 0.5)} and B= {(x, 0, 0.5), (y, 0.4, 0)}. For α =0.4, it is clear that the IFTS (X, t) is α $-$IF$-T_1$(iii) but not corresponding I$-$ T_1.

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