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Variational Analysis of Propagation Characteristics of Single Mode Graded Index Fiber using Corrected Approximation of the Fundamental Mode

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ABSTRACT

In this paper, a corrected form of a trial approximation of the fundamental modal field for single mode fiber is presented. Using variational analysis, propagation characteristics for triangular and parabolic index profiles for the fundamental mode have been predicted accurately. Also, by employing proper boundary matching condition, the previously reported three-parameter approximation form has been reduced to two parameter form, resulting in better accuracy of the results in comparison to the available exact results.

Keywords: Single mode graded index fiber, Fundamental mode, Propagation characteristics

1. Introduction

Optical fibers in the single mode region is now of tremendous importance in communication system. Due to absence of intermodal dispersion, higher data rate transmission occurs without any appreciable power attenuation. Earlier works using variational technique for finding the fundamental modal field [1-3] have shown that proper choice of the approximate form of the field leads to the actual trial field. In this article, a corrected form of the approximation, used in [4] has been presented and with this form applying variational technique we have studied the modal field distribution and different propagation characteristics of the fundamental (LP_{01}) mode and have compared our results with those obtained numerically and other already established methods. These comparisons show an excellent matching of our results with the exact values compared to those obtained from [4].

In section II, we have shown the necessary expressions obtained using variational technique. In this paper we have considered cases of triangular index fiber (TIF) and parabolic index fiber (PIF). The results are given in section III where we have compared our data with those obtained numerically and with those in [4]. These comparisons show that a correct choice of the approximate form of the function improves the accuracy of the results as well as reduces the computational time whereas the function in [4] is incorrect since it does not consider L'Hospital's theorem and the boundary matching condition.

2. Theory

The refractive index profile of a weakly guiding graded index fiber is given by:

$$n^2(R) = n_1^2 - \left(n_1^2 - n_2^2\right) f(R) \qquad \text{for } 0 \le R \le 1$$

$$n^2(R) = n_2^2 \qquad \text{for } R > 1 \tag{1}$$

where, n_1 and n_2 represent the values of the refractive indices of the core axis and cladding region; $R(=r/a)$ is called the normalised radius, a is the core radius and $f(R)$ is the profile shape function and for power-law profile $f(R) = R^q$; q is the profile exponent. For TIF, $q = 1$ and for PIF, $q = 2$.

To obtain the modal field solution ψ and the propagation characteristics for the fundamental mode (LP01) one has to solve the Helmholtz scalar wave equation considering the above index variation (1).

A. Variational Analysis

In order to obtain the modal field solution of a graded index fiber for LP_{01} mode, we have used the three parameter form as mentioned in a recent work [4] but they haven't mentioned any use of the L' Hospital's theorem and the following boundary matching condition:

$$\left.\frac{\psi_1'(R)}{\psi_1(R)}\right|_{R=R_0} = \left.\frac{\psi_2'(R)}{\psi_2(R)}\right|_{R=R_0} \tag{2}$$

which lead to the following corrected approximation:

$$\psi_1(R) = \left(\frac{R_0}{\alpha}\right) \frac{\sin\left(\frac{\alpha R}{R_0}\right)}{R} \qquad \text{for } R \le R_0$$

$$\psi_2(R) = \left(\frac{R_0}{\alpha}\right) \frac{e^{\mu} \sin\alpha}{R} \exp\left(-\frac{\mu R}{R_0}\right) \sqrt{\frac{R_0}{R}} \qquad \text{for } R > R_0 \tag{3}$$

where, α and R_0 are the two independent variational parameters and μ is dependent and is related to α as:

$$\mu = -\alpha \cot\alpha - 0.5 \tag{4}$$

The scalar variational expression for the propagation constant β is given as

$$\beta^2 = \frac{k_0^2 \int_0^\infty n^2(R) |\psi(R)|^2 R dR - \frac{1}{a^2}\left[\int_0^{R_0} |\psi_1'|^2 R dR + \int_{R_0}^\infty |\psi_2'|^2 R dR\right]}{\int_0^{R_0} |\psi_1|^2 R dR + \int_{R_0}^\infty |\psi_2|^2 R dR} \tag{5}$$

where, k_0 is the free space wave number; by applying variational technique and using (5) in the following expression of normalised fiber parameter U :

$$U^2 = a^2 (k_0^2 n^2 - \beta^2) \tag{6}$$

we arrive at:

$$U^2 = \frac{V^2 (I_1 + I_2 + I_3) + (I_4 + I_5)}{(I_6 + I_7)} \quad (\text{for } R_0 < 1) \tag{7}$$

where V is the normalised frequency.

In the above equation, the integrals are defined as follows:

$$I_1 = \int_0^{R_0} f(R)\psi_1^2 RdR\,; \qquad I_4 = \int_0^{R_0} \psi_1'^2 \, RdR\,;$$

$$I_2 = \int_{R_0}^{1} f(R)\psi_2^2 RdR\,; \qquad I_5 = \int_{R_0}^{\infty} \psi_2'^2 \, RdR\,;$$

$$I_3 = \int_1^{\infty} \psi_2^2 RdR\,; \qquad I_6 = \int_0^{R_0} \psi_1^2 RdR\,;$$

$$I_7 = \int_{R_0}^{\infty} \psi_2^2 RdR \tag{8}$$

Now, for a fixed value of V, the optimised values of the variational parameters α and R_0 are obtained by minimizing U^2. The third parameter μ is obtained using (4). These values of the variational parameters are then used to get the modal field as well as different propagation constants as given below.

Similarly, we have studied the other case – for $R_0 > 1$ and obtained the variational parameters and hence ψ.

B. Propagation Characteristics

To compare our results we have shown the values of Petermann II (WP_2) spot size [5] as a typical example. It is defined as

$$WP_2^{\,2} = \frac{2\left[\int_0^{\infty} \psi^2 RdR\right]}{\int_0^{\infty} \psi'^2 \, RdR} \tag{9}$$

The above parameter can be expressed in terms of integrals as:

$$WP_2^{\,2} = \frac{2(I_6 + I_7)}{(I_4 + I_5)} \tag{10}$$

The normalized propagation constant is defined as:

$$b = W^2 / V^2 \tag{11}$$

3. Results and Discussions

In order to predict different propagation characteristics for the LP_{01} mode of TIF and PIF using variational technique, U^2 has been minimized first, as given in (7) with respect to α and R_0, for a fixed value of V. Then, using (4) the third parameter μ is obtained and hence using these parameters we obtain ψ.

In Figure 1 and Figure 2, the modal field distribution of LP_{01} mode has been shown for TIF and PIF respectively. In these graphs we have shown a comparison of the plot obtained using our corrected approximation (3), with the exact numerical results and with those obtained using the approximation mentioned in [4]. It is found that, our approximation is more accurate than that in [4], as it matches excellently with the exact values both for TIF and PIF.

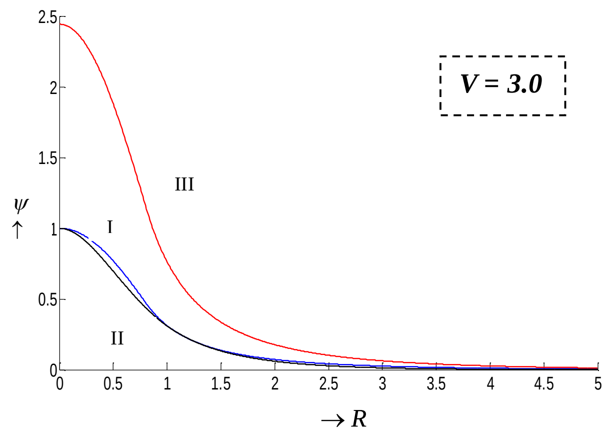

Fig. 1: Variation of ψ with R for LP_{01} mode for TIF

[I]: Values obtained by our corrected approximation
[II]: Exact numerical values
[III]: Values obtained using approximation in [4]

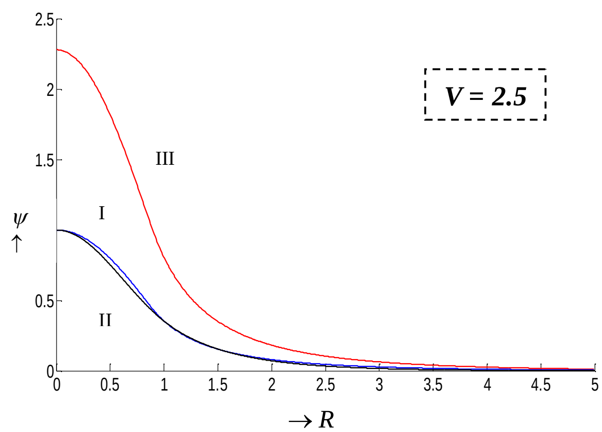

Fig. 2: Variation of ψ with R for LP_{01} mode for PIF

[I]: Values obtained by our corrected approximation
[II]: Exact numerical values
[III]: Values obtained using approximation in [4]

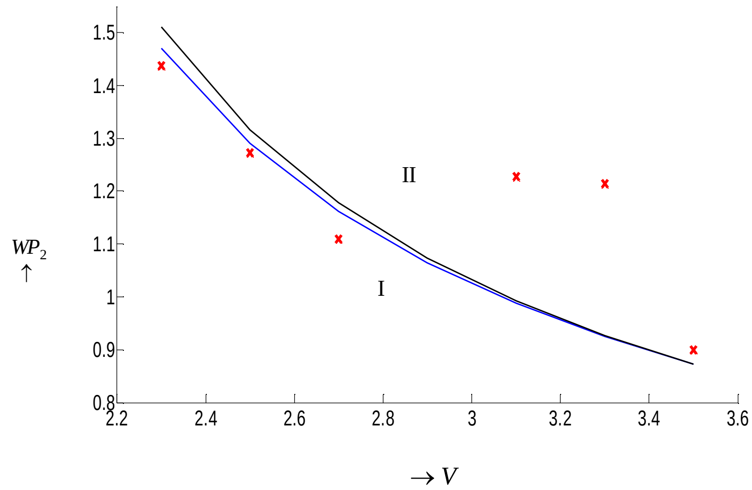

Fig. 3: Variation of WP_2 with V for LP_{01} mode for TIF
[I]: Values obtained by our corrected approximation
[II]: Exact numerical values
Cross (×):Values obtained from [4]

We have calculated the Petermann II spot size, for triangular index profile for the LP_{01} mode using the corrected approximation (3) for different V - values and have compared the results with the exact values and those obtained from [4]. It is clear from Figure 3, that, our results match closely with the exact values than those in[4]. Similar discrepancies have been observed in case of WP_2 for parabolic profile also. In the following Table I, a comparison of the values of α, R_0 and μ for different V - values for TIF obtained by the corrected approximation (3) and the approximation used in [4] have been presented.

TABLE I: Values of variational parameters for different normalised frequencies (V) for TIF

V	Our corrected approximation			[4]		
	α	R_0	μ	α	R_0	μ
2.9	1.9602	0.8251	0.3044	1.9547	0.8177	0.2902
3.3	2.0110	0.7647	0.4472	1.9416	0.7189	0.2551
3.5	2.0335	0.7404	0.5144	2.1504	0.7840	0.2782

In our paper we have also shown a comparison of the values of different propagation characteristics like U, W (cladding decay parameter), b and WP_2 for LP_{01} mode for PIF obtained by using our corrected approximation by variational analysis (VA) and exact numerical values which is given as Table II. These values are quite close to the exact values.

TABLE II: Comparison of the values of different propagation constants for PIF

V	*U*		*W*		*b*		WP_2	
	VA	EXACT	VA	EXACT	VA	EXACT	VA	EXACT
2	1.85302	1.84248	0.75253	0.77797	0.14157	0.15131	1.42066	1.46155
2.4	2.09603	2.08771	1.16902	1.18382	0.23726	0.24330	1.13026	1.14375
2.8	2.30650	2.29843	1.58746	1.59911	0.32143	0.32617	0.96890	0.97316

4. Conclusion

We have presented a corrected two parameter approximation of [4] for LP_{01} mode for single-mode triangular and parabolic index fibres, which provides a close approximation of the actual field; applying variational technique, the normalised fibres parameter has been minimised with respect to the two independent variational parameters and with their optimised values we have obtained the modal field and hence the propagation characteristics. We have compared these results with the exact values and it is clear that our approximation consisting of two independent parameters is in excellent agreement with exact results than those in [4]. Also, this proper choice of the two parameter approximation of the fundamental mode improves the accuracy of the results and also requires less computational time in comparison to the three parameter approximation in [4].

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