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Corrected Approximation of Fundamental Modal Field in Variational Analysis of Single Mode Fiber and Prediction of their Propagation Characteristics

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ABSTRACT

In variational analysis of single mode fiber, accurate prediction of propagation characteristics is possible if one chooses the trial approximation of the fundamental modal field accurately and optimizes it with respect to propagation constants. In this paper, we have corrected a recently approximate function and shown how our corrected function over the earlier one excellently predicts results in comparison to the previous report.

***Keywords**:* Fundamental mode, Single mode graded index fiber, Propagation characteristics.

1. Introduction

In recent times, optical fiber in the single mode region is now the only candidate because of its higher rate of data transmission without any appreciable power attenuation. Variational technique is one of the conventional methods for prediction of modal solution for step and graded index fibers for the fundamental mode (LP_{01}). Literatures show that different types of approximations of fundamental modal field are chosen so far in order to employ the variational technique for finding the fundamental modal field [1-3]. It is well-known that the accuracy of this method depends on the proper choice of the form of the trial field. In an earlier paper [4], the author had chosen a three-parameter approximate form of the wave function for the fundamental field, which is not in agreement with L' Hospital's rule for the trial field. Also, they had not matched the initial conditions for the wave function.

In this paper, we have corrected that wrong and inconsistent approximation and reduced the three-parameter form into two-parameter, which gives better accuracy than [4] in the measurement of waveguide parameters with much simpler calculation as well as in a short computational time.

2. Theory

Under weakly guiding approximation, the variation of refractive index profile of a graded index fiber can be described as

$$n^2(R) = n_1^2 \qquad \text{for } 0 \le R \le s$$

$$n^2(R) = n_2^2 + (n_1^2 - n_2^2)\left(\frac{1 - f(R)}{1 - s}\right) \qquad \text{for } s \le R \le 1$$

$$n^2(R) = n_2^2 \qquad \text{for } R > 1 \tag{1}$$

where, $R(= r/a)$ is the normalised radius; $a =$ core radius; n_1 and n_2 are the refractive indices of the core axis and cladding region; for graded index fiber with power-law profile, the profile shape function $f(R)$ is given by:

$f(R) = R^q$ for $R \le 1$

$= 1$, for $R > 1$

q is called the profile exponent, which determines the shape of the refractive index profile. For step index fiber (SIF), $q = \infty$ and for trapezoidal index fiber, $q = 1$. s is the aspect ratio and for SIF, $s = 0$ and for trapezoidal profile, $0 < s < 1$. Considering the above refractive index profile, one has to solve the Helmholtz scalar wave equation to obtain the modal field solution ψ for LP_{01} mode.

A. Variational Analysis

To predict the modal field solution of a graded index fiber for LP_{01} mode, we have considered the approximation, as in [4], stated earlier, which has three variational parameters. But, as stated earlier, the authors have not used the L' Hospital's theorem and boundary matching condition. These conditions will lead to two variational parameters and hence our approximation is as follows:

$$\psi_1(R) = \left(\frac{R_0}{\alpha}\right)\frac{\sin\left(\frac{\alpha R}{R_0}\right)}{R} \qquad \text{for } R \le R_0$$

$$\psi_2(R) = \left(\frac{R_0}{\alpha}\right)\frac{\sin\alpha}{R} e^{\mu\left(1 - \frac{R}{R_0}\right)} \sqrt{\frac{R_0}{R}} \qquad \text{for } R > R_0 \tag{2}$$

where, α and R_0 are the two independent variational parameters and μ is dependent and is related to α as:

$$\mu = -\alpha \cot\alpha - 0.5 \tag{3}$$

which is obtained from the following boundary matching condition at $R = R_0$:

$$\left.\frac{\psi_1'(R)}{\psi_1(R)}\right|_{R=R_0} = \left.\frac{\psi_2'(R)}{\psi_2(R)}\right|_{R=R_0} \tag{4}$$

We have kept α, R_0 and μ as in [4], but now we have two independent variational parameters, α and R_0.

The scalar variational expression for the propagation constant β is given as

$$\beta^2 = \frac{k_0{}^2 \int_0^\infty n^2(R)|\psi(R)|^2 R dR - \frac{1}{a^2}\langle \psi'^2 \rangle}{\langle \psi^2 \rangle} \tag{5}$$

where, k_0 is the free space wave number.

Applying variational technique and using (5) in the following expression of normalised fiber parameter

$$U^2 = a^2(k_0^2 n^2 - \beta^2) \tag{6}$$

We get (for $R_0 < 1$ and $R_0 > s$)

$$U^2 = \frac{V^2(I_1 + I_2) + \frac{V^2}{(1-s)}(I_3 + I_4 - I_1 - I_5) + (I_6 + I_7)}{(I_8 + I_2)} \tag{7}$$

In the above equation, the integrals are defined as follows:

$$I_1 = \int_s^{R_0} \psi_1^2 R dR \qquad I_5 = \int_{R_0}^{1} \psi_2^2 R dR$$

$$I_2 = \int_{R_0}^{\infty} \psi_2^2 R dR \qquad I_6 = \int_0^{R_0} \psi_1'^2 R dR$$

$$I_3 = \int_s^{R_0} f(R)\psi_1^2 R dR \qquad I_7 = \int_{R_0}^{\infty} \psi_2'^2 R dR$$

$$I_4 = \int_{R_0}^{1} f(R)\psi_2^2 R dR \qquad I_8 = \int_0^{R_0} \psi_1^2 R dR \tag{8}$$

Similarly, we have studied other two cases: for $R_0 < 1$ and $R_0 < s$; $R_0 > 1$ and obtained the variational parameters and hence ψ. The variational parameters α and R_0 are obtained by minimizing U^2 for a fixed value of the normalised frequency V. These values are then used in (2) to find the modal field as well as different propagation constants defined below.

B. Propagation Characteristics

Out of the two Petermann spot sizes I and II, we present Petermann II (WP_2) spot size for comparison of our result as a typical example; WP_2 is defined as [5]

$$WP_2{}^2 = \frac{2\langle \psi^2 \rangle}{\langle \psi'^2 \rangle} \tag{9}$$

$$\langle \psi^2 \rangle = \int_0^{R_0} |\psi_1|^2 R dR + \int_{R_0}^{\infty} |\psi_2|^2 R dR$$

$$\langle \psi'^2 \rangle = \int_0^{R_0} |\psi_1'|^2 R dR + \int_{R_0}^{\infty} |\psi_2'|^2 R dR$$

The above parameter can be expressed in terms of integrals as

$$WP_2{}^2 = \frac{2(I_8 + I_2)}{(I_6 + I_7)} \tag{10}$$

The normalized propagation constant is defined as:

$$b = W^2 / V^2 \tag{11}$$

3. Results and Discussions

In this section, using variational technique we have minimized the scalar variational expression of U^2 given in (7) with respect to α and R_0. For the present study, the value of the aspect ratio (s) has been chosen as, $s = 0$ for step index fiber (SIF) and

$s = 0.25$ for trapezoidal profile. Then we obtain the third parameter μ [4] using (3). Thus using these optimised parameters we obtain the modal field solution ψ for LP_{01} mode for SIF and trapezoidal index fiber. To check the accuracy of our corrected approximation, we have shown the modal field distribution of LP_{01} mode obtained using our approximation, the approximation used in [4] and an analytical results in terms of Bessel and modified Bessel functions [5] for step and exact numerical results for trapezoidal fiber, for a particular normalised frequency (V). In figure 1a and figure 1b, we have shown that, our corrected approximation is more accurate than that in [4], as it matches excellently with the exact values both for step and trapezoidal profiles.

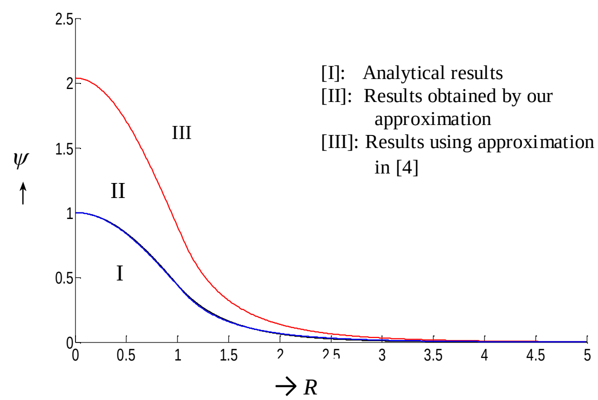

Figure 1a: ψ Vs. R plot for step profile for $V = 2.3$

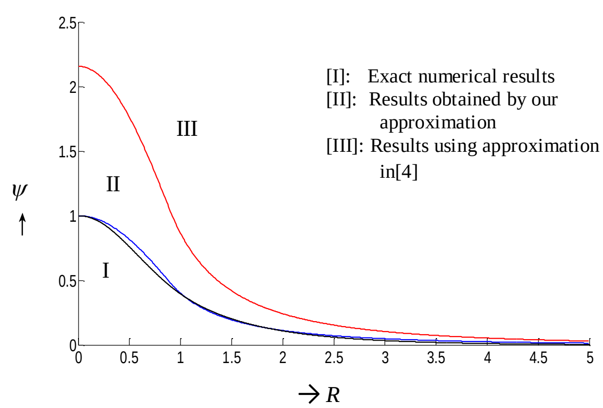

Figure 1b: ψ Vs. R plot for trapezoidal profile for $V = 2.3$

In figure 2, we have presented a comparison of the values of Petermann II spot size, for trapezoidal index profiles obtained from numerical calculation, our approximation and that in [4]. It is clear from figure 2 that, our results match closely with the available exact values than those in [4].

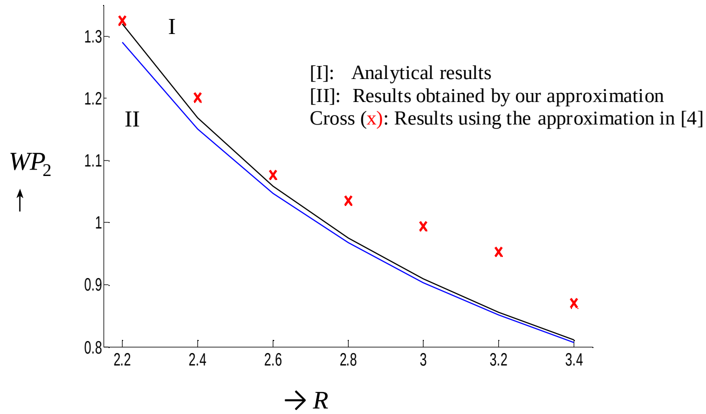

Figure 2: Variation of WP_2 with V for trapezoidal index profile

In Table I, we have presented the values of some waveguide parameters for step and trapezoidal profiles, like normalised propagation constant b, cladding decay parameter W, Petermann II spot size WP_2 and compared our results obtained by variational analysis (VA) with exact numerical values; then in the next table, the values of the optimised parameters from our corrected form and as reported in the earlier work [4] have been compared.

TABLE I: DIFFERENT WAVEGUIDE PARAMETERS FOR SIF FOR DIFFERENT NORMALISED FREQUENCY (V) AND COMPARISON WITH EXACT ANALYTICAL METHODS

V	U		W		b		WP_2	
	VA	**Exact**	**VA**	**Exact**	**VA**	**Exact**	**VA**	**Exact**
1.7	1.418494	1.412672	0.936950	0.945705	0.303763	0.309466	1.419469	1.452812
1.9	1.496541	1.492849	1.170626	1.175330	0.379603	0.382659	1.282486	1.298647
2.1	1.563289	1.560824	1.402186	1.404930	0.445834	0.447580	1.183220	1.191617
2.3	1.620896	1.619160	1.631777	1.633500	0.503345	0.504409	1.108566	1.113038

TABLE II: VALUES OF THREE VARIATIONAL PARAMETERS FOR BY OUR APPROXIMATION (2) AND (4) FOR TRAPEZOIDAL PROFILE

Our corrected approximation				**[4]**		
V	α	R_0	μ	α	R_0	μ
2.6	1.973125	0.825667	0.339645	1.954907	0.814689	0.289126
2.8	2.003656	0.792685	0.425863	1.987340	0.781444	0.277328
3.2	2.057912	0.741786	0.590053	2.207612	0.797867	0.283156

3.4	2.081339	0.721009	0.665698	1.930533	0.637208	0.226140

It is clear from Table I, that the values of different waveguide parameters obtained by considering our corrected approximation fairly match with those obtained by analytical method for SIF. Also, in Table II, we have presented a comparison between the set of optimised parameters for different values of normalised frequency (V) obtained from our calculation and those in [4]; It is observed that, in [4], the values of the third parameter μ do not satisfy the boundary matching condition at $R = R_0$, which leads to the inconsistent nature of the field distribution and hence WP_2.

4. Conclusion

In summary, we have proposed a novel alternative of two-parameter approximation of the fundamental modal field in corrected form for step and trapezoidal profile and shown that it is the actual approximation closely consistent with the exact field. Then with variational framework, we optimised the normalised field parameter and derived the field expression and other waveguide parameters and compared these results with the exact values. From these observations, it is clear that our correction and the use of proper boundary matching condition, in the approximation mentioned in [4] reduce the three-parameter approximation form into two-parameter form, that leads to the closest approximation of the actual modal solution; hence this results in the accurate prediction of the propagation characteristics with a simple calculation and less computational time.

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