Journal of Physical Sciences, Vol. 17, 2013, 33-41
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on 26 December 2013

Inference for Lomax Distribution Based on Type-II Progressively Hybrid Censored Data

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Received 7 July 2013; accepted 11 November 2013

ABSTRACT
The mixture of Type-I and Type-II censoring schemes, called the hybrid censoring scheme is quite common in life-testing or reliability experiments. In this paper, we investigate the estimation of parameters from two-parameter Lomax distribution based on Type-II progressively hybrid censored samples. Maximum likelihood estimates for the distribution parameters and the reliability indices are obtained. Bayesian estimates of the unknown parameters and the reliability indices are obtained under square error loss function by using the important sample method and the Lindely Bayes approximation algorithm respectively. Different methods have been compared using Monte Carlo simulations.

Keywords: Maximum likelihood estimate; Bayesian inference; Lomax distribution; Importance sample; Lindely Bayes approximation; Type-II progressively hybrid

1. Introduction

The Lomax distribution was originally proposed by Lomax in the analysis of business failure. It is often used in economics, business, and actuarial modeling. It has received much attention from theoretical and applied statistics primarily due to its use in reliability and life testing studies. For example, Ref.[3] considered the competing risks model based on Lomax distribution under Type-II progressively censoring scheme (PCS); Ref.[4] discussed the parameter estimation of the hybrid censored Lomax distribution.

The two most common censoring schemes are termed as Type-I and Type-II censoring. There is also another common censoring scheme called the hybrid censoring scheme which was first introduced by Epstein. One of the drawbacks of these censoring schemes is that they do not allow for removal of units at points other than the terminal point of the experiment. The Type-I PCS or the Type-II PCS, however, has this advantage and has become very popular in the last few years. For example, Ref.[7]discussed the inference for Weibull distribution based on Type-II progressively hybrid censored schemes (PHCS). Ref.[8] considered parametric inference for Type-I PHCS on a simple step-stress accelerated life test model. Ref.[9] discussed on some exact distributional results based on Type-I PHC data from exponential distributions. Ref.[10]

discussed reliability analysis for accelerated life-test with progressive hybrid censored data using geometric process.

In this paper we consider the Type-II PHC lifetime data, when the lifetime follows the Lomax distribution. In section 3 we provide the maximum likelihood estimates (MLEs) of the unknown parameters and the reliability indices. It is observed that the MLEs do not have explicit forms. They can be obtained by a simple iterative scheme. We also obtained Bayesian estimates of the unknown parameters and the reliability indices using the important sample method and the Lindely Bayes approximation algorithm respectively in Section 4. Simulation results and data analysis are provided in Section 5. Finally, we conclude the paper in Section 6.

2. Model description

Type-II PHCS can be described as follows : n identical units are placed the life tested. Suppose integer $m < n$ and $R_1, ..., R_m$ satisfying $R_1 + ... + R_m + m = n$ are fixed at the beginning of the experiment. The time point T is also fixed before hand. At the time of the first failure, say $X_{1:m:n}$, R_1 surviving units of the remaining units are randomly removed. Similarly at the time of the second failure $X_{2:m:n}$, R_2 surviving units of the remaining units are randomly removed and so on. If the $m-th$ failure $X_{m:m:n}$ occurs before the time point T, the experiment stops at the time point $X_{m:m:n}$. On the other hand, if the $m-th$ failure does not occur before time point T and only J failures occur before the time point T, where $0 \le J < m$, then at the time point T all remaining R_J^* units are removed and the experiment terminates at the time point T. Note that $R_J^* = n-(R_1 + ... + R_J) - J$. We denote two cases as Case I and Case II respectively. Therefore in the presence of Type-II PHCS, we have one of the following types of observations:

Case I: $\{X_{1:m:n}, \cdots, X_{m:m:n}\}$, if $X_{m:m:n} < T$; Case II: $\{X_{1:m:n}, \cdots, X_{J:m:n}\}$, if $X_{J:m:n} < T < X_{J+1:m:n}$.

For simplicity, in the rest of this article we will use X_i to substitute $X_{i:m:n}, i = 12, ..., m$.

Suppose the random variable X follows a Lomax distribution with shape and scale parameters α and λ, respectively, and with probability density function (pdf) and cumulative distribution function (cdf) as

$$f(x;\alpha,\lambda) = \alpha / \lambda(1 + x/\lambda)^{-(\alpha+1)}, \quad x > 0, \alpha, \lambda > 0 \tag{1}$$

$$F(x) = 1-(1 + x/\lambda)^{-\alpha} \qquad x > 0, \alpha, \lambda > 0 \tag{2}$$

Then the corresponding reliability function can be obtained:

$$r(x) = (1 + x/\lambda)^{-\alpha}, \qquad x > 0, \alpha, \lambda > 0 \tag{3}$$

Let D be the number of failures, where $D = m$ for Case I and $D = J$ for Case II. Let $Y = (X_1, \cdots, X_D)$ denote the type-II progressively hybrid censored data from a population with pdf and cdf given in (1) and (2) respectively.

3. Maximum likelihood estimates

In this section, we will discuss the MLE of the unknown parameters and the reliability function. The likelihood function based on the Type-II PHC samples Y, see Z. H. Li[1] and M. Z. Raqab[2], is given by

$$L(\alpha,\lambda) \propto \prod_{i=1}^{D} f(X_i;\alpha,\lambda)[1-F(X_i;\alpha,\lambda)]^{R_i}[1-F(T^*)]^{R_T} \ . \tag{4}$$

where $R_T = 0$, $T^* = X_m$ for Case I and $R_T = R_J^*$, $T^* = T$ for Case II. From (1), (2) and (4), we write the likelihood function of α and λ based on the Type-II PHC samples Y as follows:

For Case I

$$L(\alpha,\lambda) \propto (\alpha/\lambda)^m \exp\{-(\alpha+1)D_\lambda^1(Y) - \alpha T_\lambda^1(Y)\} \ , \tag{5}$$

while for Case II, it is

$$L(\alpha,\lambda) \propto (\alpha/\lambda)^J \exp\{-(\alpha+1)D_\lambda^2(Y) - \alpha T_\lambda^2(Y)\} \ , \tag{6}$$

where $D_\lambda^1(Y) = \sum_{i=1}^{m} \ln(1+X_i/\lambda)$, $T_\lambda^1(Y) = \sum_{i=1}^{m} R_i \ln(1+X_i/\lambda)$, $D_\lambda^2(Y) = \sum_{i=1}^{J} \ln(1+X_i/\lambda)$,

$T_\lambda^2(Y) = \sum_{i=1}^{J} R_i \ln(1+X_i/\lambda) + R_J^* \ln(1+T/\lambda)$.

Note that (5) and (6) can be combined as follows:

$$L(\alpha,\lambda) \propto (\alpha/\lambda)^D \exp\{-(\alpha+1)D_\lambda(Y) - \alpha T_\lambda(Y)\} \ , \tag{7}$$

where $D = m, D_\lambda(Y) = D_\lambda^1(Y), T_\lambda(Y) = T_\lambda^1(Y)$ for Case I and $D = J$, $D_\lambda(Y) = D_\lambda^2(Y)$, $T_\lambda(Y) = T_\lambda^2(Y)$ for Case II. The corresponding log-likelihood function is obtained from (7) as

$$l(\alpha,\lambda) = \log L(\alpha,\lambda) \propto D\ln\alpha - D\ln\lambda - (\alpha+1)D_\lambda(Y) - \alpha T_\lambda(Y) \ . \tag{8}$$

Taking derivatives with respect to α in (8) and equating it to zero, we obtain the likelihood equation as

$$\partial l/\partial\alpha = D/\alpha - D_\lambda(Y) - T_\lambda(Y) = 0 \ . \tag{9}$$

From (9), we obtain

$$\hat{\alpha}(\lambda) = D/[D_\lambda(Y) + T_\lambda(Y)] \ . \tag{10}$$

For Case I $D = m, D_\lambda(Y) = D_\lambda^1(Y), T_\lambda(Y) = T_\lambda^1(Y)$, taking derivatives with respect to λ in (8) and equating it to zero, we obtain the likelihood equation as

$$-m/\lambda + (\alpha+1)\sum_{i=1}^{m} X_i/[\lambda(\lambda+X_i)] + \alpha\sum_{i=1}^{m} R_i X_i/[\lambda(\lambda+X_i)] = 0 \ . \tag{11}$$

Upon using (10) in (11), it becomes

$$-m/\lambda + (\hat{\alpha}_1(\lambda)+1)\sum_{i=1}^{m} X_i/[\lambda(\lambda+X_i)] + \hat{\alpha}_1(\lambda)\sum_{i=1}^{m} R_i X_i/[\lambda(\lambda+X_i)] = 0 \ , \tag{12}$$

where $\hat{\alpha}_1(\lambda) = m/[D_\lambda^1(Y) + T_\lambda^1(Y)]$.

Note that (12) can be written in the form

$$\lambda = h(\lambda) \ , \tag{13}$$

where

$$h(\lambda) = m / [(\hat{\alpha}_1(\lambda)+1)\sum_{i=1}^{m} X_i / [\lambda(\lambda + X_i)] + \hat{\alpha}_1(\lambda)\sum_{i=1}^{m} R_i X_i / [\lambda(\lambda + X_i)]] \,. \tag{14}$$

Similarly for Case II $D = J, D_\lambda(Y) = D_\lambda^2(Y), T_\lambda(Y) = T_\lambda^2(Y)$, we can obtain

$$h(\lambda) = J / [(\hat{\alpha}_2(\lambda)+1)\sum_{i=1}^{J} X_i / [\lambda(\lambda + X_i)] + \hat{\alpha}_2(\lambda)\sum_{i=1}^{J} R_i X_i / [\lambda(\lambda + X_i)] + \hat{\alpha}_2(\lambda) R_J^* T / [\lambda(\lambda + T)]] \,. \tag{15}$$

From (13), we use a simple iterative scheme to solve for λ. It has been proposed in the literature by Kundu (2007). Start with an initial guess of λ, say $\lambda^{(0)}$, then obtain $\lambda^{(1)} = h(\lambda^{(0)})$ and proceed in this way iteratively to obtain $\lambda^{(n+1)} = h(\lambda^{(n)})$. Stop the iterative procedure, when $\left|\lambda^{(n+1)} - \lambda^{(n)}\right| < \varepsilon$, some pre-assigned tolerance limit.

Thus, from (3), the maximum likelihood estimation of the reliability function can be easily established:

$$\hat{r}(x) = (1 + x / \hat{\lambda})^{-\hat{\alpha}} \,. \tag{16}$$

4. Bayesian inferences

In this section we consider the Bayesian estimation of the unknown parameters and the reliability function under the squared error loss function. We assume that α and λ are independently distributed as $Ga(a_0, b_0)$ and $IGa(a_1, b_1)$ respectively, with the following densities:

$$g_{a_0, b_0}(\alpha) = \alpha^{a_0 - 1} \mathrm{e}^{-b_0 \alpha} b_0^{a_0} / \Gamma(a_0) \quad , \quad h_{a_1, b_1}(\lambda) = \lambda^{-(a_1+1)} \mathrm{e}^{-b_1/\lambda} b_1^{a_1} / \Gamma(a_1) \,, \tag{17}$$

Where $\alpha > 0$, $\lambda > 0$, $\Gamma(x) = \int_0^{+\infty} t^{x-1} e^{-t} dt$ and a_0, b_0, a_1, b_1 are assumed to be positive constants. By combining (17) and (7), we obtain the joint posterior density of α and λ:

$$p(\alpha, \lambda | Y) \propto g_{a_0+D, b_0+D_\lambda(Y)+T_\lambda(Y)}(\alpha) h_{a_1+D, b_1}(\lambda)(b_0 + D_\lambda(Y) + T_\lambda(Y))^{-(a_0+D)} \exp\{-D_\lambda(Y)\} \,, \tag{18}$$

The marginal posterior of any parameter is obtained by integrating the joint posterior distribution with respect to the other parameters. Thus, the posterior density function of λ can be obtained:

$$p(\lambda | Y) = \int_0^\infty p(\alpha, \lambda | Y) d\alpha \propto h_{a_1+D, b_1}(\lambda) Q_\lambda(Y) \,, \tag{19}$$

Where $Q_\lambda(Y) = (b_0 + D_\lambda(Y) + T_\lambda(Y))^{-(a_0+D)} \exp\{-D_\lambda(Y)\}$. Thus, the Bayesian estimation of λ is

$$\hat{\lambda}_{\mathrm{B}} = E(\lambda | Y) = \int_0^\infty \lambda h_{a_1+D, b_1}(\lambda) Q_\lambda(Y) d\lambda \Big/ \int_0^\infty h_{a_1+D, b_1}(\lambda) Q_\lambda(Y) d\lambda = E_\lambda(\lambda Q_\lambda(Y)) / E_\lambda(Q_\lambda(Y)) \tag{20}$$

Where E_λ denotes the expectation with respect to $IGa(a_1 + D, b_1)$. The marginal posterior density of α given λ and Y is

$$p(\alpha | \lambda, Y) \propto g_{a_0+D, b_0+D_\lambda(Y)+T_\lambda(Y)}(\alpha) \,. \tag{21}$$

Thus the Bayesian estimation of α under the squared error loss function is

$$\hat{\alpha}_B = (a_0 + D) / [b_0 + D_\lambda(Y) + T_\lambda(Y)] \,. \tag{22}$$

The Bayesian estimation of λ can be determined by the important sampling method as follows:

Step 1: Ten thousand λ values are generated from $IGa(a_1 + D, b_1)$.

Step 2: For each λ generated from the step1, compute $E_\lambda(\lambda Q_\lambda(Y))$ and $E_\lambda(Q_\lambda(Y))$ by averaging $\lambda Q_\lambda(Y)$ and $Q_\lambda(Y)$.

Obtaining the $\hat{\lambda}_B$, the Bayesian estimation of α can be established using (22) directly.

Let $g(\alpha,\lambda)=r(x)=(1+x/\lambda)^{-\alpha}$, $L(\alpha,\lambda,x)=L(\alpha,\lambda)\cdot g_{a_0,b_0}(\alpha)\cdot h_{a_1,b_1}(\lambda)$, where $L(\alpha,\lambda)$ and $g_{a_0,b_0}(\alpha)$, $h_{a_1,b_1}(\lambda)$ are given in (7) and (17). From (18), the Bayesian estimation of the $r(x)$ under squared error loss function can be obtained:

$$\hat{r}(x)=E_{\alpha,\lambda|Y}(g(\alpha,\lambda))=\int_0^\infty\int_0^\infty g(\alpha,\lambda)L(\alpha,\lambda,x)d\alpha d\lambda \Big/ \int_0^\infty\int_0^\infty L(\alpha,\lambda,x)d\alpha d\lambda \tag{23}$$

The ratio of the two integrals given by (23), generally, is obtained in a closed form. Therefore, in such situations, we can use numerical integration technique, which can be computationally intensive, especially in high-dimensional parameter space. Here we adopt the Lindley approximation to estimate the (23). This method has been used by some authors to obtain Bayesian estimates of the parameters. See for example, J. Zhao et al. (2013)[5]

Lindley developed an approximate procedure for evaluating the posterior expectation of $U(\theta)$ as $E(U(\theta)|x)=\int U(\theta)e^{l(\theta)+\rho(\theta)}d\theta \Big/ \int e^{l(\theta)+\rho(\theta)}d\theta$, which is the Bayesian estimate of $U(\theta)$ under squared error loss function, where $\rho(\theta)=\ln(p(\theta))$, $p(\theta)$ is arbitrary function of θ, and $l(\theta)$ is the logarithm of the likelihood function. In the two parameter case, when $\theta=(\theta_1,\theta_2)$, Lindley's approximately form reduces to the following form:

$$E(U(\theta)|x)=U(\theta)+A/2+\rho_1A_{12}+\rho_2A_{21}+(1/2)[l_{30}B_{12}+l_{21}C_{12}+l_{12}C_{21}+l_{03}B_{21}]. \tag{24}$$

where $A=\sum_{i=1}^2\sum_{j=1}^2 U_{ij}\sigma_{ij}$, $l_{\eta\varepsilon}=(\partial^{\eta+\varepsilon}l/\partial\theta_1^\eta\partial\theta_2^\varepsilon)$, η , ε =0,1,2,3, $\eta+\varepsilon=3$. For $i,j=1,2$,

$\rho_i=(\partial\rho/\partial\theta_i)$, $U_i=(\partial U/\partial\theta_i)$, $U_{ij}=(\partial^2U/\partial\theta_i\partial\theta_j)$, and for $i\neq j$,

$A_{ij}=U_i\sigma_{ii}+U_j\sigma_{ji}$, $B_{ij}=(U_i\sigma_{ii}+U_j\sigma_{ij})\sigma_{ii}$, $C_{ij}=3U_i\sigma_{ii}\sigma_{ij}+U_j(\sigma_{ii}\sigma_{jj}+2\sigma_{ij}^2)$.

For $i,j=1,2$, $L_{ij}=(\partial^2 l/\partial\theta_i\partial\theta_j)$. $N=L_{11}L_{22}-L_{12}L_{21}$, $\sigma_{11}=-L_{22}/N$, $\sigma_{22}=-L_{11}/N$,

$\sigma_{12}=L_{12}/N=\sigma_{21}$.

In our case, $\theta=(\alpha,\lambda)$, $U(\theta)=U(\alpha,\lambda)=g(\alpha,\lambda)$, $p(\theta)=p(\alpha,\lambda)=g_{a_0,b_0}(\alpha)\cdot h_{a_1,b_1}(\lambda)$,

where $g_{a_0,b_0}(\alpha)$ and $h_{a_1,b_1}(\lambda)$ are given in (17). Thus, we observe that:

$\rho(\alpha,\beta)=\ln p(\alpha,\lambda)=\text{constant}+(a_0-1)\ln\alpha-b_0\alpha-(a_1+1)\ln\lambda-b_1/\lambda$

Therefore, $\rho_1=\partial\rho/\partial\alpha=(a_0-1)/\alpha-b_0$, $\rho_2=\partial\rho/\partial\lambda=-(a_1+1)/\lambda+b_1/\lambda^2$.

Further more, $U_1=(1+x/\lambda)^{-\alpha}\ln(1+x/\lambda)$, $U_2=\alpha x(1+x/\lambda)^{-1-\alpha}/\lambda^2$,

$U_{11}=(1+x/\lambda)^{-\alpha}[\ln(1+x/\lambda)]^2$, $U_{12}=-(1+x/\lambda)^{-\alpha}x/[\lambda(\lambda+x)]+\alpha x(1+x/\lambda)^{-\alpha-1}\ln(1+x/\lambda)/\lambda^2$

$U_{21} = (1 + x/\lambda)^{-\alpha-1} x/\lambda^2 - \alpha x(1 + x/\lambda)^{-\alpha-1} \ln(1 + x/\lambda)/\lambda^2$

$U_{22} = \alpha(\alpha+1)x^2(1 + x/\lambda)^{-2-\alpha}/\lambda^4 - 2x\alpha(1 + x/\lambda)^{-1-\alpha}/\lambda^3$.

For case I:

$L_{11} = \partial^2 l/\partial\alpha^2 = -m/\alpha^2$, $L_{12} = \partial^2 l/\partial\alpha\partial\lambda = \sum_{i=1}^{m}(1+R_i)X_i/[\lambda(\lambda + X_i)] = L_{21}$.

$L_{22} = \partial^2 l/\partial\lambda^2 = m/\lambda^2 - (\alpha+1)\sum_{i=1}^{m} X_i(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2 - \alpha\sum_{i=1}^{m} R_i X_i(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2$,

$l_{03} = \partial^3 l/\partial\lambda^3 = -2m/\lambda^3 + (\alpha+1)\sum_{i=1}^{m} 2X_i[3\lambda^2 + 3\lambda X_i + X_i^2]/[\lambda(\lambda + X_i)]^3 +$

$\alpha\sum_{i=1}^{m} 2X_i R_i[3\lambda^2 + 3\lambda X_i + X_i^2]/[\lambda(\lambda + X_i)]^3$, $l_{21} = \partial^3 l/\partial\alpha^2\partial\lambda = 0$,

$l_{30} = \partial^3 l/\partial\alpha^3 = 2m/\alpha^3$, $l_{12} = \partial^3 l/\partial\alpha\partial\lambda^2 = -\sum_{i=1}^{m}(1+R_i)X_i(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2$.

While for case II:

$L_{11} = \partial^2 l/\partial\alpha^2 = -J/\alpha^2$, $L_{12} = \partial^2 l/\partial\alpha\partial\lambda = \sum_{i=1}^{J}(1+R_i)X_i/[\lambda(\lambda + X_i)] + R_J^* T/[\lambda(\lambda + T)] = L_{21}$,

$L_{22} = \partial^2 l/\partial\lambda^2 = J/\lambda^2 - (\alpha+1)\sum_{i=1}^{J} X_i(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2 - \alpha\sum_{i=1}^{J} R_i X_i(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2$

$-\alpha R_J^* T(2\lambda + T)/[\lambda(\lambda + T)]^2$, $l_{21} = \partial^3 l/\partial\alpha^2\partial\lambda = 0$, $l_{30} = \partial^3 l/\partial\alpha^3 = 2J/\alpha^3$,

$l_{03} = \partial^3 l/\partial\lambda^3 = -2J/\lambda^3 + (\alpha+1)\sum_{i=1}^{J} 2X_i[3\lambda^2 + 3\lambda X_i + X_i^2]/[\lambda(\lambda + X_i)]^3 +$

$\alpha\sum_{i=1}^{J} 2X_i R_i[3\lambda^2 + 3\lambda X_i + X_i^2]/[\lambda(\lambda + X_i)]^3 + 2\alpha R_J^* T[3\lambda^2 + 3\lambda T + T^2]/[\lambda(\lambda + T)]^3$,

$l_{12} = \partial^3 l/\partial\alpha\partial\lambda^2 = -\sum_{i=1}^{J} X_i(1+R_i)(2\lambda + X_i)/[\lambda(\lambda + X_i)]^2 - R_J^* T(2\lambda + T)/[\lambda(\lambda + T)]^2$.

Substitution of the above values in Equation (24) yields the Bayesian estimate using Lindley's method relative to squared error loss function, of a function $U(\alpha,\lambda)$, denoted by $\hat{U}_B$

$$\hat{U}_B = E[U(\alpha,\lambda)|x] = U(\alpha,\lambda) + \Phi + \Psi_1 U_1 + \Psi_2 U_2 \qquad (25)$$

$\Phi = A/2 = (U_{11}\sigma_{11} + U_{12}\sigma_{12} + U_{21}\sigma_{21} + U_{22}\sigma_{22})/2$

$\Psi_1 = \rho_1\sigma_{11} + \rho_2\sigma_{12} + [l_{30}\sigma_{11}^2 + l_{12}(\sigma_{11}\sigma_{22} + 2\sigma_{21}^2) + l_{03}\sigma_{21}\sigma_{22}]/2$

$\Psi_2 = \rho_1\sigma_{21} + \rho_2\sigma_{22} + l_{30}\sigma_{12}\sigma_{11}/2 + 3l_{12}\sigma_{21}\sigma_{22}/2 + l_{03}\sigma_{22}^2/2$

All functions in Equation (25) are evaluated at the MLE of (α,λ).

5. Numerical results and discussions

In this section, we use Monte Carlo simulations to compare different methods for different parameter values and for different sampling schemes. The term different sampling schemes means different sets of R_i's and for different T values.

We consider different n , m and T . We have used three different samplings schemes, see Kundu[6], namely: Scheme 1: $R_1 = \ldots = R_{m-1} = 0$ and $R_m = n - m$. Scheme 2: $R_1 = n - m$ and $R_2 = \ldots = R_m = 0$. Scheme 3: $R_1 = \ldots = R_{m-1} = 1$ and $R_m = n - 2m + 1$. Without loss of generality we take α=0.3 and λ=0.2 in each case. Two measures, such as bias and MSE are used to assess the performance of the proposed methods. We replicate the process 1000 times and the results are shown in Tab 1. Each value represents the average bias, and the corresponding MSE is reported within brackets. From Tab 1, we can observe that the biases and MSEs of the parameters are generally smaller using the Bayesian method rather than the maximum likelihood estimation method. Due to the difference of MLE, the estimates for reliability function using the Bayesian method show relatively large differences. When the sample is large, we have more exact maximum likelihood estimations, and as a result, we have better Bayesian inference for reliability function. But overall, the estimates for reliability function using MLE are better than using the Bayesian method.

6. Conclusion

In this paper, we have discussed the classical and Bayesian inferential procedures for the Type-II progressively hybrid censored data from the Lomax distribution. It is shown that the maximum likelihood estimate of the parameters can be obtained by using an iterative procedure. Under the assumptions of independent gamma and Igamma priors, Bayesian estimates of the unknown parameters can be obtained using the important sampling method. We also employed the Lindely Bayes approximation to compute the Bayesian inference for reliability function. A comparison of the MLEs and Bayesian estimates in terms of the average biases and MSEs is made by Monte-Carlo simulation for different censoring scheme. It is observed that Bayesian estimates for parameters are generally better than MLEs. Bayesian estimates for reliability function display comparatively large differences because of MLE. As sample size increases, the biases and MSEs of Bayesian inferences for reliability function decrease. However, on the whole, the estimates for reliability function using MLE are better than using the Bayesian method.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No.71 171164, No.70471057)

(n, m)	T	S	$\hat{\alpha}_{\mathrm{M}}$	$\hat{\lambda}_M$	$\hat{\alpha}_{\mathrm{B}}$	$\hat{\lambda}_{\mathrm{B}}$	$\hat{r}_M$	$\hat{r}_B$
(50, 20)	5	1	0.1749(0.214)	0.1743(0.187)	0.0718(0.121)	0.0428(0.052)	0.0012(0.054)	0.5791(0.611)
		2	0.2037(0.216)	0.1703(0.186)	0.0067(0.054)	0.0020(0.006)	0.0675(0.054)	0.5699(0.775)
		3	0.1926(0.207)	0.1639(0.182)	0.0200(0.062)	0.0387(0.039)	0.0617(0.054)	0.4642(0.467)

	10	1	0.1535(0.248)	0.1287(0.142)	0.0885(0.130)	0.1013(0.101)	0.0524(0.056)	0.4547(0.516)
		2	0.1945(0.207)	0.1346(0.171)	0.0266(0.066)	0.0080(0.009)	0.0582(0.060)	0.4540(0.455)
		3	0.1603(0.194)	0.1661(0.183)	0.0031(0.068)	0.0013(0.001)	0.0615(0.070)	0.3810(0.411)
(70, 30)	5	1	0.1106(0.176)	0.1027(0.156)	0.1030(0.129)	0.1026(0.123)	0.0043(0.046)	0.3032(0.345)
		2	0.1293(0.181)	0.1342(0.171)	0.0313(0.053)	0.0685(0.071)	0.0665(0.083)	0.3403(0.542)
		3	0.1123(0.184)	0.1129(0.161)	0.1515(0.155)	0.1883(0.188)	0.0547(0.059)	0.2823(0.294)
	10	1	0.0783(0.197)	0.1128(0.161)	0.0418(0.054)	0.0736(0.086)	0.0363(0.040)	0.2042(0.219)
		2	0.0603(0.138)	0.1729(0.276)	0.0005(0.057)	0.1354(0.193)	0.0662(0.083)	0.2015(0.211)
		3	0.1455(0.175)	0.1209(0.164)	0.1138(0.118)	0.1130(0.129)	0.0398(0.042)	0.2667(0.271)
(90, 40)	5	1	0.1205(0.180)	0.1326(0.169)	0.0874(0.088)	0.1049(0.104)	0.0453(0.045)	0.2104(0.216)
		2	0.1086(0.175)	0.1187(0.164)	0.0658(0.072)	0.1233(0.130)	0.0768(0.089)	0.1827(0.188)
		3	0.0987(0.134)	0.0477(0.128)	0.0977(0.101)	0.1220(0.131)	0.0475(0.052)	0.2272(0.234)
	10	1	0.1116(0.168)	0.1155(0.157)	0.0875(0.092)	0.1112(0.111)	0.0374(0.042)	0.2118(0.255)
		2	0.1354(0.174)	0.1222(0.166)	0.0933(0.095)	0.1053(0.112)	0.0477(0.057)	0.2268(0.282)
		3	0.1520(0.176)	0.1245(0.166)	0.0828(0.085)	0.1200(0.132)	0.0627(0.065)	0.2402(0.242)
(100, 50)	5	1	0.1266(0.182)	0.1394(0.172)	0.1029(0.104)	0.1350(0.135)	0.0528(0.057)	0.2080(0.216)
		2	0.1030(0.178)	0.1462(0.175)	0.0803(0.085)	0.1353(0.147)	0.1187(0.125)	0.1929(0.217)
		3	0.1424(0.179)	0.1567(0.178)	0.0859(0.092)	0.1329(0.123)	0.0590(0.065)	0.2126(0.216)
	10	1	0.0910(0.172)	0.1127(0.161)	0.0909(0.102)	0.1050(0.136)	0.0596(0.061)	0.1704(0.173)
		2	0.0320(0.156)	0.0732(0.142)	0.0731(0.076)	0.1275(0.136)	0.0824(0.089)	0.1474(0.157)
		3	0.1005(0.168)	0.1247(0.166)	0.0994(0.101)	0.1158(0.135)	0.0862(0.088)	0.1844(0.188)

Table 1: Biases and MSEs of the MLE estimators and Bayesian estimates a_0=0.1, $b_0 = a_1 = b_1 = 0.5$ figures in brackets represent the MSEs

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