Journal of Physical Sciences, Vol. 17, 2013, 117-130
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on 26 December 2013

Stochastic Transportation Problem with Cauchy Random Variables and Multi Choice Parameters

***M. P. Biswal*[1] *and H. K. Samal*[2]**

[1]Department of Mathematics, Indian Institute of Technology, Kharagpur - 721 302, India
[2]Department of Mathematics, Dinakrushna College, Jaleswar (Odisha) - 756084, India
Email: himansu68@gmail.com

Received 11 November 2013; accepted 12 December 2013

ABSTRACT

In this paper, single objective and multi-objective stochastic transportation problem with Cauchy random variables and their deterministic equivalents are presented. Multi-choice programming solves some optimization problems, where multiple information exists for a parameter. A stochastic transportation problem in which the source parameters are either Cauchy random variables or multi-choice type and the demand parameters are either multi-choice type or Cauchy random variables is presented in this paper. The proposed stochastic transportation problem is converted to an equivalent deterministic problem and solved by goal programming method. A Numerical example is presented to illustrate the solution procedure.

Keywords: transportation problem, multi-objective decision making, stochastic programming, cauchy distribution, goal programming technique, interpolating polynomial, multi-choice programming.

1. Introduction

The classical transportation problem is one of the many well-structured decision making problems in Operations Research. The basic transportation problem was originally stated by Hitchcock[7] and later discussed in detail by Koopmans[13]. The problem is called transportation problem because it involves the transportation or physical distribution of goods from several supply points to a number of demand points. When the market demands for a commodity are not known with certainty, the problem of scheduling shipments to a number of demand points from several supply points is a stochastic transportation problem (Williams[25]). Sometimes the coefficients of the transportation problem can be characterized by uncertain parameters such as, random, fuzzy and multi-choice parameters. These uncertainties occurs mainly due to scarcity of data or, incomplete information and knowledge regarding the data, or difficult to obtain the data, or to estimate or the system is subject to changes. This type situations mostly present in the real world. Transportation problems modeled in this situations are known as stochastic transportation problems or fuzzy transportation problems or multi-choice transportation problems.

Intensive investigations on stochastic transportation problem with random variables have been made by several researchers, namely Kataoka[14], Szwarc[23], Leclercq[15], Kall[12], Cooper[5], Isermann[11], Qi[18], Hassin[6], Roubens[21], Arunachalam[1], Holmberg[9], Verma et al.[24], Hulsurkar et al.[10], Mohan[17], Biswal

[3], Sahoo [22], Roghanian [19], Mahapatra et al. [16] and Romeijn [20].

The general model of an unbalanced stochastic, or fuzzy or multi-choice transportation problem is given as:

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i, i = 1,2,\ldots,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, j = 1,2,\ldots,n \tag{3}$$

$$x_{ij} \ge 0, \; forall\, i, j \tag{4}$$

$$\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} b_j, (\, feasibility\ condition) \tag{5}$$

where the decision variable x_{ij} represents the amount of the commodity to be shipped from i-th source to j-th destination. Some or all the coefficients c_{ij}, a_i and b_j are considered as random variables with known probability distributions, or known fuzzy numbers with membership functions, or multi-choice parameters.

2. Single objective stochastic transportation problem with cauchy random variable

Mathematical model of a single objective stochastic transportation problem with Cauchy random variable can be stated as :

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{6}$$

subject to

$$\Pr(\sum_{j=1}^{n} x_{ij} \le a_i) \ge 1 - \alpha_i, i = 1,2,\ldots,m \tag{7}$$

$$\Pr(\sum_{i=1}^{m} x_{ij} \ge b_j) \ge 1 - \beta_j, j = 1,2,\ldots,n \tag{8}$$

$$x_{ij} \ge 0, i = 1,2,\ldots,m, \; j = 1,2,\ldots,n \tag{9}$$

where $0 < \alpha_i < 1$, $0 < \beta_j < 1$ are specified probability levels and a_i , b_j are Cauchy random variables. Assume that the decision variables x_{ij} are deterministic.

a_i are Cauchy random variables.

It is given that the i-th Cauchy random variable a_i has two known parameters l_{a_i} and s_{a_i}, where the location parameter l_{a_i} is the median and s_{a_i} is the scale parameter of a_i.

The probability density function of the random variable a_i is given by

$$f(a_i) = \frac{s_{a_i}}{\pi[s_{a_i}^2 + (a_i - l_{a_i})^2]}, -\infty < a_i < \infty, s_{a_i} > 0 \tag{10}$$

Now the probabilistic constraint (7) can be rewritten as

$$\Pr(a_i \geq \sum_{j=1}^{n} x_{ij}) \geq 1 - \alpha_i, i = 1,2,\ldots,m \tag{11}$$

Let $\sum_{j=1}^{n} x_{ij} = u_i$. Hence (11) reduced to

$$\Pr(a_i \geq u_i) \geq 1 - \alpha_i, i = 1,2,\ldots,m \tag{12}$$

which can further be stated as

$$\int_{u_i}^{\infty} \frac{s_{a_i}}{\pi[s_{a_i}^2 + (a_i - l_{a_i})^2]} \mathrm{d}a_i \geq 1 - \alpha_i, i = 1,2,\ldots,m \tag{13}$$

Integrating we obtain

$$\frac{1}{\pi}[tan^{-1}(\frac{a_i - l_{a_i}}{s_{a_i}})]_{u_i}^{\infty} \geq 1 - \alpha_i, i = 1,2,\ldots,m \tag{14}$$

After substituting the limit of the integration, we get

$$\frac{\pi}{2} - tan^{-1}(\frac{u_i - l_{a_i}}{s_{a_i}}) \geq (1 - \alpha_i)\pi, i = 1,2,\ldots,m \tag{15}$$

which can be further simplified to

$$-tan^{-1}(\frac{u_i - l_{a_i}}{s_{a_i}}) \geq (\frac{\pi}{2} - \pi\alpha_i), i = 1,2,\ldots,m \tag{16}$$

Taking the tangent of both the sides, it becomes

$$(\frac{u_i - l_{a_i}}{s_{a_i}}) \leq \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{17}$$

which is same as

$$u_i \leq l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{18}$$

Finally, this can be expressed as a linear constraint of the form

$$\sum_{j=1}^{n} x_{ij} \leq l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{19}$$

b_j are Cauchy random variables.

It is given that b_j is a Cauchy random variable with known parameters l_{b_j} and s_{b_j}, where the location parameter l_{b_j} is the median and s_{b_j} is the scale parameter of b_j.

The probability density function of the random variable b_j is given by

$$f(b_j) = \frac{s_{b_j}}{\pi[s_{b_j}^2 + (b_j - l_{b_j})^2]}, -\infty < b_j < \infty, s_{b_j} > 0 \quad (20)$$

Now the probabilistic constraint (8) reduces to

$$1 - \Pr(b_j \geq \sum_{i=1}^{m} x_{ij}) \geq 1 - \beta_j, \ j = 1,2,\dots,n \quad (21)$$

which is same as

$$\Pr(b_j \geq \sum_{i=1}^{m} x_{ij}) \leq \beta_j, \ j = 1,2,\dots,n \quad (22)$$

Let $\sum_{i=1}^{m} x_{ij} = v_j$. Hence the probabilistic constraint (22) reduced to

$$\Pr(b_j \geq v_j) \leq \beta_j, \ j = 1,2,\dots,n \quad (23)$$

which can be written as

$$\int_{v_j}^{\infty} \frac{s_{b_j}}{\pi[s_{b_j}^2 + (b_j - l_{b_j})^2]} \, \mathrm{d}b_j \leq \beta_j, \ j = 1,2,\dots,n \quad (24)$$

After integration, we get

$$\frac{1}{\pi}[tan^{-1}(\frac{b_j - l_{b_j}}{s_{b_j}})]_{v_j}^{\infty} \leq \beta_j, \ j = 1,2,\dots,n \quad (25)$$

Substituting the limit of integration, we find

$$\frac{\pi}{2} - tan^{-1}(\frac{v_j - l_{b_j}}{s_{b_j}}) \leq \pi\beta_j, \ j = 1,2,\dots,n \quad (26)$$

which can be further simplified as

$$-tan^{-1}(\frac{v_j - l_{b_j}}{s_{b_j}}) \leq -(\frac{\pi}{2} - \pi\beta_j), \ j = 1,2,\dots,n \quad (27)$$

Taking the tangent of both the sides, we get

$$\frac{v_j - l_{b_j}}{s_{b_j}} \geq \tan(\frac{\pi}{2} - \pi\beta_j), \ j = 1,2,\dots,n \quad (28)$$

which is same as

$$v_j \geq l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j),\ j = 1,2,\ldots,n \tag{29}$$

Now, the probabilistic constraint (8) can be expressed as a linear constraint of the form

$$\sum_{i=1}^{m} x_{ij} \geq l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j),\ j = 1,2,\ldots,n \tag{30}$$

Hence the equivalent single objective deterministic transportation problem of the stochastic problem (6) - (9) can be formulated as:

$$\min : Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{31}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{32}$$

$$\sum_{i=1}^{m} x_{ij} \geq l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j),\ j = 1,2,\ldots,n \tag{33}$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{34}$$

where,

$$\sum_{i=1}^{m} l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}) \geq \sum_{j=1}^{n} l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j)\ (\textit{feasibility condition}) \tag{35}$$

3. Multi-objective stochastic transportation problem with cauchy random variables

A multi-objective stochastic transportation problem with source and demand constraints involving Cauchy random variables can be defined as:

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}\ ; k = 1,2,\ldots,K \tag{36}$$

subject to

$$\Pr(\sum_{j=1}^{n} x_{ij} \leq a_i) \geq 1 - \alpha_i, i = 1,2,\ldots,m \tag{37}$$

$$\Pr(\sum_{i=1}^{m} x_{ij} \geq b_j) \geq 1 - \beta_j,\ j = 1,2,\ldots,n \tag{38}$$

$$x_{ij} \geq 0, i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{39}$$

where $0 < \alpha_i < 1$, $0 < \beta_j < 1$ are specified probability levels and a_i, b_j are

Cauchy random variables. In the model, the decision variables x_{ij} are considered as deterministic variables.

As discussed in case of single objective transportation problem case, the equivalent multi-objective deterministic transportation problem of the stochastic transportation problem (36) - (39) is given as:

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}\ ; k = 1,2,\ldots,K \tag{40}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{41}$$

$$\sum_{i=1}^{m} x_{ij} \ge l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j), j = 1,2,\ldots,n \tag{42}$$

$$x_{ij} \ge 0, i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{43}$$

where,

$$\sum_{i=1}^{m} l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}) \ge \sum_{j=1}^{n} l_{b_j} + s_{b_j} \tan(\frac{\pi}{2} - \pi\beta_j)\ (\, feasibility\ condition) \tag{44}$$

4. Multi-objective stochastic transportation problem with cauchy random variables and multi-choice parameters

We shall consider two special cases for this transportation problem where

(i) a_i are Cauchy random variables and b_j are multi-choice parameters and

(ii) a_i are multi-choice parameters and b_j are Cauchy random variables.

(i) Source parameters, a_i are Cauchy random variables and destination parameters, b_j are multi-choice type.

Let us consider a multi-objective stochastic transportation problem with source constraints involving Cauchy random variables where demand parameters are multi-choice type.

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij}\ ; k = 1,2,\ldots,K \tag{45}$$

subject to

$$\Pr(\sum_{j=1}^{n} x_{ij} \le a_i) \ge 1 - \alpha_i, i = 1,2,\ldots,m \tag{46}$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, b_j \in \{b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, \ldots, b_j^{(q_j)}\},\ j = 1,2,\ldots,n \tag{47}$$

$$x_{ij} \ge 0, i = 1,2,\ldots,m;\ j = 1,2,\ldots,n \tag{48}$$

where $0 < \alpha_i < 1$ are specified probability levels and a_i are Cauchy random variables with two known parameters l_{a_i} and s_{a_i} where the location parameters l_{a_i} are the medians and s_{a_i} are the scale parameters of a_i and b_j are multi-choice parameters. Let us consider the decision variables x_{ij} are deterministic.

As discussed earlier an equivalent deterministic constraint of the probabilistic constraint (46) has been obtained as

$$\sum_{j=1}^{n} x_{ij} \le l_{a_i} + s_{a_i} \tan(\pi\alpha_i - \frac{\pi}{2}), i = 1,2,\ldots,m \tag{49}$$

Let us consider the constraint (47) of the stochastic transportation problem (45) - (48) as

$$\sum_{i=1}^{m} x_{ij} \ge \{b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, \ldots, b_j^{(q_j)}\},\ j = 1,2,\ldots,n \tag{50}$$

Right hand side of j-th constraint (50) has a set of q_j number of goals where only one goal is to be selected.

Let $0,1,2,\ldots(q_j - 1)$ be q_j number of node points, where $b_j^{(1)}, b_j^{(2)}, b_j^{(3)}, \ldots, b_j^{(q_j)}$ are the associated functional values of the interpolating polynomial at q_j different node points as shown in Table 1.

Table 1: Node Points

$\delta^{(j)}$	0	1	2	...	q_{j-1}
$f(\delta^{(j)}) = b_j$	$b_j^{(1)}$	$b_j^{(2)}$	$b_j^{(3)}$	...	$b_j^{(q_j)}$

We derive a polynomial $P_{q_j - 1}(\delta^{(j)})$ of degree $(q_j - 1)$ which interpolates the given data:

$$P_{q_j-1}(\delta^{(j)}) = b^{(\delta^{(j)}+1)}, \delta^{(j)} = 0,1,2,\ldots,(q_j - 1), j = 1,2,3,\ldots,n\,.$$

Let us formulate an interpolating polynomial for j-th multi-choice parameters by the Lagrange interpolation formula as

$$P_{q_j-1}(\delta^{(j)}) = \frac{(\delta^{(j)} - 1)(\delta^{(j)} - 2)\ldots(\delta^{(j)} - q_j + 1)}{(-1)^{(q_j-1)}(q_j - 1)!} b_j^{(1)} + \frac{\delta^{(j)}(\delta^{(j)} - 2)\ldots(\delta^{(j)} - q_j + 1)}{(-1)^{(q_j-2)}(q_j - 2)!} b_j^{(2)}$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)(\delta^{(j)}-3)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-3)}(q_j-3)!2!}b_j^{(3)}+\ldots$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)\ldots(\delta^{(j)}-q_j+2)}{(q_j-1)!}b_j^{(q_j)},\ j=1,2,...,n. \quad (51)$$

Thus the transformed form of the constraint (50) becomes

$$\sum_{i=1}^{m}x_{ij}\geq\frac{(\delta^{(j)}-1)(\delta^{(j)}-2)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-1)}(q_j-1)!}b_j^{(1)}+\frac{\delta^{(j)}(\delta^{(j)}-2)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-2)}(q_j-2)!}b_j^{(2)}$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)(\delta^{(j)}-3)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-3)}(q_j-3)!2!}b_j^{(3)}+\ldots$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)\ldots(\delta^{(j)}-q_j+2)}{(q_j-1)!}b_j^{(q_j)},\ j=1,2,...,n. \quad (52)$$

Hence the equivalent deterministic problem (45) - (48) is presented as:

$$\min: Z_k=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{k}x_{ij};\ k=1,2,\ldots,K \quad (53)$$

subject to

$$\sum_{j=1}^{n}x_{ij}\leq l_{a_i}+s_{a_i}\tan(\pi\alpha_i-\frac{\pi}{2}), i=1,2,\ldots,m \quad (54)$$

$$\sum_{i=1}^{m}x_{ij}\geq\frac{(\delta^{(j)}-1)(\delta^{(j)}-2)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-1)}(q_j-1)!}b_j^{(1)}+\frac{\delta^{(j)}(\delta^{(j)}-2)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-2)}(q_j-2)!}b_j^{(2)}$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)(\delta^{(j)}-3)\ldots(\delta^{(j)}-q_j+1)}{(-1)^{(q_j-3)}(q_j-3)!2!}b_j^{(3)}+\ldots$$

$$+\frac{\delta^{(j)}(\delta^{(j)}-1)\ldots(\delta^{(j)}-q_j+2)}{(q_j-1)!}b_j^{(q_j)},\ j=1,2,...,n. \quad (55)$$

$$\delta^{(j)}=0,1,2,\ldots,q_j-1,\ j=1,2,...,n \quad (56)$$

$$x_{ij}\geq 0, i=1,2,\ldots,m;\ j=1,2,\ldots,n \quad (57)$$

where

$$\sum_{i=1}^{m}l_{a_i}+s_{a_i}\tan(\pi\alpha_i-\frac{\pi}{2})\geq\sum_{j=1}^{n}max\{b_j^{(1)},b_j^{(2)},b_j^{(3)},\ldots,b_j^{(q_j)}\}\ (feasibility\ condition) \quad (58)$$

(ii) Source parameters, a_i are multi-choice type and destination parameters, b_j are

Cauchy random variables

Let us consider a multi-objective stochastic transportation problem in which the source parameters are multi-choice type and demand parameters are Cauchy random variables.

$$\min : Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \; ; k = 1,2,\ldots,K \tag{59}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le a_i , a_i \in \{a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, \ldots, a_i^{(q_i)}\}, i = 1,2,\ldots,m \tag{60}$$

$$\Pr[\sum_{i=1}^{m} x_{ij} \ge b_j] \ge 1-\beta_j , \; j = 1,2,\ldots,n \tag{61}$$

$$x_{ij} \ge 0, i = 1,2,\ldots,m; \; j = 1,2,\ldots,n \tag{62}$$

where, $0<\beta_j<1$ are specified probability levels and b_j are Cauchy random variables with two known parameters l_{b_j} and s_{b_j} where the location parameters l_{b_j} are the medians and s_{b_j} are the scale parameters of b_j, a_i are multi-choice parameters. Let us consider the decision variables x_{ij} are deterministic.

Let $0,1,2,\ldots(q_i-1)$ be q_i number of node points, where $a_i^{(1)}, a_i^{(2)}, a_i^{(3)}, \ldots, a_i^{(q_i)}$ are the associated functional values of the interpolating polynomial at q_i different node points as given in Table 2.

Table 2: Node Points

$\delta^{(i)}$	0	1	2	...	q_{i-1}
$f(\delta^{(i)}) = a_i$	$a_i^{(1)}$	$a_i^{(2)}$	$a_i^{(3)}$	...	$a_i^{(q_i)}$

We derive a polynomial $P_{q_i-1}(\delta^{(i)})$ of degree (q_i-1) which interpolates the given data:

$$P_{q_i-1}(\delta^{(i)}) = a^{(\delta^{(i)}+1)}, \delta^{(i)} = 0,1,2,\ldots,(q_i-1), i = 1,2,3,\ldots,m .$$

Let us formulate an interpolating polynomial for $i-th$ multi-choice parameters by the Lagrange interpolation formula as

$$P_{q_i-1}(\delta^{(i)}) = \frac{(\delta^{(i)}-1)(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-1)}(q_i-1)!} a_i^{(1)} + \frac{\delta^{(i)}(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-2)}(q_i-2)!} a_i^{(2)}$$
$$+ \frac{\delta^{(i)}(\delta^{(i)}-1)(\delta^{(i)}-3)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-3)}(q_i-3)!2!} a_i^{(3)} + \ldots$$

$$+\frac{\delta^{(i)}(\delta^{(i)}-1)\ldots(\delta^{(i)}-q_i+2)}{(q_i-1)!}a_i^{(q_i)}, i=1,2,\ldots m. \quad (63)$$

Thus the transformation of the constraint (60) becomes

$$\sum_{j=1}^{n}x_{ij} \le \frac{(\delta^{(i)}-1)(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-1)}(q_i-1)!}a_i^{(1)} + \frac{\delta^{(i)}(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-2)}(q_i-2)!}a_i^{(2)}$$

$$+\frac{\delta^{(i)}(\delta^{(i)}-1)(\delta^{(i)}-3)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-3)}(q_i-3)!2!}a_i^{(3)}+\ldots+\frac{\delta^{(i)}(\delta^{(i)}-1)\ldots(\delta^{(i)}-q_i+2)}{(q_i-1)!}a_i^{(q_i)},$$

$$i=1,2,\ldots m. \quad (64)$$

As discussed earlier an equivalent deterministic constraint of the probabilistic constraint (61) has been obtained as

$$\sum_{i=1}^{m}x_{ij} \ge l_{b_j}+s_{b_j}\tan(\frac{\pi}{2}-\pi\beta_j), j=1,2,\ldots,n.$$

Thus, the stochastic transportation problem (59) - (62) reduced to the deterministic form as:

$$\min: Z_k = \sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}^{k}x_{ij}\, ; k=1,2,\ldots,K \quad (65)$$

subject to

$$\sum_{j=1}^{n}x_{ij} \le \frac{(\delta^{(i)}-1)(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-1)}(q_i-1)!}a_i^{(1)} + \frac{\delta^{(i)}(\delta^{(i)}-2)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-2)}(q_i-2)!}a_i^{(2)}$$

$$+\frac{\delta^{(i)}(\delta^{(i)}-1)(\delta^{(i)}-3)\ldots(\delta^{(i)}-q_i+1)}{(-1)^{(q_i-3)}(q_i-3)!2!}a_i^{(3)}+\ldots+\frac{\delta^{(i)}(\delta^{(i)}-1)\ldots(\delta^{(i)}-q_i+2)}{(q_i-1)!}a_i^{(q_i)},$$

$$i=1,2,\ldots m. \quad (66)$$

$$\sum_{i=1}^{m}x_{ij} \ge l_{b_j}+s_{b_j}\tan(\frac{\pi}{2}-\pi\beta_j), j=1,2,\ldots,n \quad (67)$$

$$x_{ij} \ge 0, i=1,2,\ldots,m;\ j=1,2,\ldots,n \quad (68)$$

where,

$$\sum_{i=1}^{m}min\,\{a_i^{(1)},a_i^{(2)},a_i^{(3)},\ldots,a_i^{(q_i)}\} \ge \sum_{j=1}^{n}l_{b_j}+s_{b_j}\tan(\frac{\pi}{2}-\pi\beta_j)\ (\mathit{feasibility\ condition}) \quad (69)$$

4.1. Numerical example

A numerical example is considered for the multi-objective transportation problem in which the supply parameters are multi choice type and the demand parameters are Cauchy random variables with known location parameters l_{b_j} and scale parameters s_{b_j}. The objectives are non-commensurable and conflicting in nature.

$$\min: Z_1 = 8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34} \quad (70)$$

$$\min: Z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + x_{34} \quad (71)$$

$$\min: Z_3 = 2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + 3x_{34} \quad (72)$$

subject to

$$\sum_{j=1}^{4} x_{1j} \le a_1,\ a_1 \in \{26, 27, 29, 30\} \quad (73)$$

$$\sum_{j=1}^{4} x_{2j} \le a_2,\ a_2 \in \{29, 30, 31, 32, 36\} \quad (74)$$

$$\sum_{j=1}^{4} x_{3j} \le a_3,\ a_3 \in \{40, 42, 45\} \quad (75)$$

$$\Pr[\sum_{i=1}^{3} x_{i1} \ge b_1] \ge 1 - \beta_1 \quad (76)$$

$$\Pr[\sum_{i=1}^{3} x_{i2} \ge b_2] \ge 1 - \beta_2 \quad (77)$$

$$\Pr[\sum_{i=1}^{3} x_{i3} \ge b_3] \ge 1 - \beta_3 \quad (78)$$

$$\Pr[\sum_{i=1}^{3} x_{i4} \ge b_4] \ge 1 - \beta_4 \quad (79)$$

$$x_{ij} \ge 0, i = 1,2,3; j = 1,2,3,4 \quad (80)$$

Let us take the the location parameters l_{b_j} , the scale parameters s_{b_j} of the Cauchy random variables b_j , $j = 1,2,3,4$ along with the specified probabilities $\beta_j, j = 1,2,3,4$ as given in the Table 3.

Table 3: Parameters with specified probabilities of b_j

LocationParameters	*ScaleParameters*	*Specified Probabilities*
$l_{b_1} = 8$	$s_{b_1} = 3$	$\beta_1 = 0.05$
$l_{b_2} = 4$	$s_{b_2} = 4$	$\beta_2 = 0.06$
$l_{b_3} = 5$	$s_{b_3} = 5$	$\beta_3 = 0.07$
$l_{b_4} = 7$	$s_{b_4} = 2$	$\beta_4 = 0.08$

Using the relations (66) and (67), the equivalent deterministic linear transportation problem of the stochastic transportation problem (70)-(80) can be derived as:

$$\min : Z_1 = 8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34} \quad (81)$$

$$\min : Z_2 = 2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + x_{34} \quad (82)$$

$$\min : Z_3 = 2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + 3x_{34} \quad (83)$$

subject to

$$x_{11} + x_{12} + x_{13} + x_{14} \le 26 - \frac{1}{6}\delta_1 + \frac{3}{2}\delta_1^2 - \frac{1}{3}\delta_1^3 \quad (84)$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le 29 + \frac{121}{4}\delta_2 + \frac{11}{8}\delta_2^2 - \frac{3}{4}\delta_2^3 + \frac{1}{8}\delta_2^4 \quad (85)$$

$$x_{31} + x_{32} + x_{33} + x_{34} \le 40 + \frac{3}{2}\delta_3 + \frac{1}{2}\delta_3^2 \quad (86)$$

$$x_{11} + x_{21} + x_{31} \ge 26.941256 \quad (87)$$

$$x_{12} + x_{22} + x_{32} \ge 24.968736 \quad (88)$$

$$x_{13} + x_{23} + x_{33} \ge 27.368715 \quad (89)$$

$$x_{14} + x_{24} + x_{34} \ge 14.789486 \quad (90)$$

$$x_{ij} \ge 0,\ i = 1,2,3;\ j = 1,2,3,4. \quad (91)$$

where, $\delta_1 = 0,1,2,3$; $\delta_2 = 0,1,2,3,4$ and $\delta_3 = 0,1,2$

Let us consider the goals of the three objectives as 500, 375 and 425 respectively. Using goal programming method, (81)-(91) can be derived as

$$\min : \rho_1 + \rho_2 + \rho_3 \quad (92)$$

subject to

$$8x_{11} + 9x_{12} + 7x_{13} + 2x_{14} + 5x_{21} + 6x_{22} + 4x_{23} + 7x_{24} + 3x_{31} + 7x_{32} + 7x_{33} + 5x_{34} + \eta_1 - \rho_1 = 500 \quad (93)$$

$$2x_{11} + 9x_{12} + 8x_{13} + x_{14} + 4x_{21} + x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 2x_{32} + 8x_{33} + x_{34} + \eta_2 - \rho_2 = 325 \quad (94)$$

$$2x_{11} + 4x_{12} + 7x_{13} + 3x_{14} + 6x_{21} + 4x_{22} + 8x_{23} + 4x_{24} + 8x_{31} + 2x_{32} + 5x_{33} + 3x_{34} + \eta_3 - \rho_3 = 250 \quad (95)$$

$$x_{11} + x_{12} + x_{13} + x_{14} \le 26 - \frac{1}{6}\delta_1 + \frac{3}{2}\delta_1^2 - \frac{1}{3}\delta_1^3 \quad (96)$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le 29 + \frac{121}{4}\delta_2 + \frac{11}{8}\delta_2^2 - \frac{3}{4}\delta_2^3 + \frac{1}{8}\delta_2^4 \quad (97)$$

$$x_{31} + x_{32} + x_{33} + x_{34} \le 40 + \frac{3}{2}\delta_3 + \frac{1}{2}\delta_3^2 \quad (98)$$

$$x_{11} + x_{21} + x_{31} \ge 26.941256 \quad (99)$$

$$x_{12} + x_{22} + x_{32} \geq 24.968736 \quad (100)$$

$$x_{13} + x_{23} + x_{33} \geq 27.368715 \quad (101)$$

$$x_{14} + x_{24} + x_{34} \geq 14.789486 \quad (102)$$

$$x_{ij} \geq 0,\ i = 1,2,3;\ j = 1,2,3,4. \quad (103)$$

where, $\delta_1 = 0,1,2,3$; $\delta_2 = 0,1,2,3,4$ and $\delta_3 = 0,1,2$.

Solving the above linear programming problem (92) - (103) by LINGO 10 package, the Optimal compromise solution is obtained as given in Table 4.

Table 4: Optimal Solution

Decision Variables	*Optimal Objective Values*	*Deviational Variables*
$x_{11} = 5.6710$		$\rho_1 = 0$
$x_{12} = 1.8066$		
$x_{14} = 14.7895$		$\rho_2 = 0$
$x_{21} = 14.3340$	$Z_1 = 499.9999$	
$x_{23} = 12.4671$		$\rho_3 = 0$
$x_{31} = 6.9363$	$Z_2 = 374.7482$	
$x_{32} = 23.1621$		$\eta_1 = 0$
$x_{33} = 14.9016$	$Z_3 = 395.1967$	
$x_{13} = x_{22} =$		$\eta_2 = 0.2517$
$x_{24} = x_{34} = 0$		
$\delta_1 = 3, \delta_2 = 0, \delta_3 = 2$		$\eta_3 = 0$

5. Conclusions

Solution procedure for single objective and and multi-objective stochastic transportation problems considering source and demand parameters as Cauchy random variables are presented in this chapter. Equivalent deterministic models of the stochastic transportation problem are established using chance constrained programming technique. After establishing the deterministic model, the goal programming model is formulated. The problem is solved by standard non-linear programming software. Further, stochastic transportation problems with source and demand parameters as Cauchy random variables and multi-choice type is also presented in this chapter. Lagrange interpolating polynomials of corresponding multi-choice parameters are established.

REFERENCES

1. G.Aruna Chalam, Fuzzy Goal Programming Approach to a Stochastic Transportation Problem under budgetary constraint, Fuzzy Sets and Systems, 66 (1994) 293-299.

2. S.Acharya, M.P.Biswal, Transformation of a multi-choice linear programming problem, Applied Mathematics and Computation, 210 (2009) 182–188.
3. M.P. Biswal and S. Acharya, Some modifications on sequential linear programming problem, Journal of Interdisciplinary Mathematics, 11 (2008) 412-427.
4. A.Charnes and W.W.Cooper, Deterministic equivalents for optimizing the satisficing under Chance-constraints, Operations Research, 2 (1963) 18-39.
5. L. Cooper and L.J. Leblanc, Stochastic transportation problems and other newtork related convex problems, Naval Research Logistics Quarterly, 24 (1977) 327–337.
6. R. Hassin, and E. Zemel, Probabilistic analysis of the capacitated transportation problem, Mathematics of Operations Research, 13 (1988) 80-89.
7. F.L.Hitchcock, The diStribution of product from several sources to numerous localities, Journal of Mathematical Physics, 20 (1941) 224-30.
8. K.Holmberg, Cross decomposition applied to the stochastic transportation problem, Computational Optimization and Applications, 4 (1995) 293-316.
9. S.Hulsurkar, M.P.Biswal, and S.B.Sinha, Fuzzy Programming Approach to Multi-objective Stochastic Linear Programming Problems, Fuzzy Sets and Systems, 88 (1997) 173-181.
10. H.Isermann, Solving the transportation problem with mixed constraints, ZOR, 26 (1982) 251–257.
11. P. Kall, Stochastic programming, Springer-Verlag, Berlin Heidelbera, New York.
12. T.C.Koopmans, Uptimum utilization of the transportation system, Econometrica, 17 (1949) 3-4.
13. S.Kataoka, A Stochastic programming model, Econometrica, 31 (1963) 181–196.
14. J.P.Leclercq, Stochastic Programming : An interactive multicriteria approach, European Journal of Operational Research, 10 (1982) 33-41.
15. D.R.Mahapatra, S.K.Roy and M.P.Biswal, Stochastic Based on Multi-objective Transportation Problems Involving Normal Randomness, Advanced Modeling and Optimization, 12 (2010) 205-223.
16. C. Mohon, and H. T. Nguyen, A Fuzzifying Approach to Stochastic Programming, Opsearch, 34 (1997) 73–96.
17. L.Qi, The A-forest iteration method for the stochastic generalized transportation problem, Mathematics of Operations Research, 12 (1987) 1-21.
18. E. Roghanian, S.J. Sadjadi and M.B. Aryanezhad, A probabilistic bi-level linear multi-objective programming problem to supply chain planning, Applied Mathematics and Computation, 188 (2007) 786-800.
19. M.Roubens, and J.Jr.Teghem, Comparision of methodologies for fuzzy and stochastic Multiobjective programming, Fuzzy Sets and Systems, 26 (1986) 65-82.
20. N.P.Sahoo and M.P.Biswal, Some stochastic Multiobjective linear programming problems, Proceeding of the APORS, (2003) 168-180.
21. W.Szwarc, The transportation problem with stochastic demand, Management Science, 11 (1964) 33-50.
22. R.Verma, M.P. Biswal and A.Biswas, Fuzzy programming approach to probabilistic multi-objective transportation problems with Pareto optimum solutions, The Journal of fuzzy Mathematics, 4 (1996) 301-314.
23. A.C.Williams, A stochastic transportation problem, Operations Research, 11 (1963) 759-770