Journal of Physical Sciences, Vol. 17, 2013, 87-95
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on 26 December 2013

Some Features of Q–Compact Fuzzy Sets

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Received 13 September 2013; accepted 11 November 2013

ABSTRACT

In this paper, we introduce the concept of Q – compact fuzzy sets and study their several features.

Keywords: Fuzzy topological spaces, Q – compact fuzzy sets.

1. Introduction

Compactness occupies a very important place in fuzzy topological spaces. The purpose of this paper is to introduce and study the concept of Q – compact fuzzy sets in more detail and to obtain several features of the concept. In doing this, we have used the idea of q – coincident of a fuzzy singleton with a fuzzy set or the same between two fuzzy sets. We find that this concept has a different and tangible flavour.

2. Preliminaries

We briefly touch upon the terminological concepts and some definitions, which are needed in the sequel. The following are essential in our study and can be found in the paper referred to.

Definition 2.1. [9] Let X be a non-empty set and I is the closed unit interval [0, 1]. A fuzzy set in X is a function u : X $\rightarrow$ I which assigns to every element x $\in$ X. u(x) denotes a degree or the grade of membership of x. The set of all fuzzy sets in X is denoted by I^X. A member of I^X may also be a called fuzzy subset of X.

Definition 2.2. [7] A fuzzy set is empty iff its grade of membership is identically zero. It is denoted by 0 or ϕ.

Definition 2.3. [7] A fuzzy set is whole iff its grade of membership is identically one in X. It is denoted by 1 or X.

Definition 2.4. [1] Let λ be a fuzzy set in X, then the set { x $\in$ X : λ(x) > 0 } is called the support of λ and is denoted by λ_0 or suppλ,

Definition 2.5. [2] Let u and v be two fuzzy sets in X. Then we define
(i) u = v iff u(x) = v(x) for all x $\in$ X
(ii) u $\subseteq$ v iff u(x) $\leq$ v(x) for all x $\in$ X
(iii) λ = u $\cup$ v iff λ (x) = (u $\cup$ v) (x) = max [u(x), v(x)] for all x $\in$ X
(iv) μ = u $\cap$ v iff μ (x) = (u $\cap$ v) (x) = min [u(x), v(x)] for all x $\in$ X
(v) $\gamma = u^c$ iff γ (x) = 1 – u(x) for all x $\in$ X.

Definition 2.6. [2] Let f : X $\rightarrow$ Y be a mapping and u be a fuzzy set in X. Then the image of u, written $f(u)$, is a fuzzy set in Y whose membership function is given by

$$f(u)(y) = \begin{cases} \sup\{u(x) : x \in f^{-1}(y)\} & if \quad f^{-1}(y) \neq \phi \\ 0 & if \quad f^{-1}(y) = \phi \end{cases}$$

Definition 2.7. [5] Let f be a mapping from a set X into Y and v be a fuzzy set of Y. Then the inverse of v written as f^{-1}(v) is a fuzzy set of X and is defined by f^{-1}(v)(x) = v (f(x)), for x $\in$ X.

De-Morgan's laws 2.8. [9] De-Morgan's Laws valid for fuzzy sets in X i.e. if u and v are any fuzzy sets in X, then
(i) 1– (u $\cup$ v) = (1 – u) $\cap$ (1 – v)
(ii) 1 – (u $\cap$ v) = (1 – u) $\cup$ (1 – v)
For any fuzzy set in u in X, u $\cap$ (1 – u) need not be zero and u $\cup$ (1 – u)need not be one.

Distributive laws 2.9. [9] Distributive laws remain valid for fuzzy sets in X i.e. if u, v and w are fuzzy sets in X, then
(i) u $\cup$ (v $\cap$ w) = (u $\cup$ v) $\cap$ (u $\cup$ w)
(ii) u $\cap$ (v $\cup$ w) = (u $\cap$ v) $\cup$ (u $\cap$ w).

Definition 2.10. [7] A fuzzy set λ in X is called quasi – coincident (in short q – coincident) with a fuzzy set μ in X, denoted by $\lambda q \mu$ iff λ (x) + μ (x) > 1 for some x $\in$ X.

Definition 2.11. [2] Let X be a non-empty set and t $\subseteq I^X$ i.e. t is a collection of fuzzy set in X. Then t is called a fuzzy topology on X if
(i) 0, 1 $\in$ t
(ii) $u_i \in$ t for each i $\in$ J, then $\bigcup_i u_i \in$ t
(iii) u, v $\in$ t, then u $\cap$ v $\in$ t.

The pair (X, t) is called a fuzzy topological space and in short, fts. Every member of t is called a t-open fuzzy set. A fuzzy set is t-closed iff its complements is t-open. In the sequel, when no confusion is likely to arise, we shall call a t-open (t-closed) fuzzy set simply an open (closed) fuzzy set.

Definition 2.12. [7] Let λ be a fuzzy set in an fts (X, t). Then the interior of λ is denoted by λ^0 or intλ and defined by $\lambda^0 = \cup \{ \mu : \mu \subseteq \lambda$ and $\mu \in$ t }.

Definition 2.13. [2] Let (X, t) and (Y, s) be two fuzzy topological spaces. A mapping f : $(X, t) \to (Y, s)$ is called an fuzzy continuous iff the inverse of each s-open fuzzy set is t-open.

Definition 2.14. [7] Let (X, t) be an fts and A $\subseteq$ X. Then the collection t_A = { u|A : u$\in$t } = { u$\cap$A : u$\in$t } is fuzzy topology on A, called the subspace fuzzy topology on A and the pair (A, t_A) is referred to as a fuzzy subspace of (X, t).

Definition 2.15. [3] Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively and f is a mapping from (X, t) to (Y, s), then we say that f is a mapping from (A, t_A) to (B, s_B) if $f(A) \subseteq B$.

Definition 2.16. [3] Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively. Then a mapping f : $(A, t_A) \to (B, s_B)$ is relatively fuzzy continuous iff for each
$v \in s_B$, the intersection $f^{-1}(v) \cap A \in t_A$.

Definition 2.17. [4] An fts (X, t) is said to be fuzzy Hausdorfff iff for all x, y $\in$ X, x $\neq$ y, there exist u, v $\in$ t such that u(x) = 1, v(y) = 1 and u $\cap$ v = 0.

Definition 2.18. [1] Let (X, T) be a topological space. A function $f : X \to$ **R** (with usual topology) is called lower semi-continuous (l. s. c.) if for each $a \in$ **R**, the set $f^{-1}(a, \infty) \in T$. For a topology T on a set X, let $\omega(T)$ be the set of all l. s. c. functions from (X, T) to I (with usual topology); thus $\omega(T) = \{u \in I^X : u^{-1}(a, 1] \in T, a \in I_1\}$. It can be shown that $\omega(T)$ is a fuzzy topology on X.

Let P be a property of topological spaces and FP be its fuzzy topology analogue. Then FP is called a ' good extension' of P " iff the statement (X, T) has P iff $(X, \omega(T))$ has FP" holds good for every topological space (X, T). Thus characteristic functions are l.s.c.

3. Characterizations of Q–compact fuzzy sets

Now we obtain some tangible features of Q – compact fuzzy sets.

Definition 3.1. Let (X, t) be an fts and λ be a fuzzy set in X. Let M = {u_i : i$\in$ J } $\subseteq I^X$ be a family of fuzzy sets. Then M = {u_i } is called a Q – cover of λ if λ(x) +

u_i(x) $\geq$ 1 for all x $\in$ X and for some u_i $\in$ M. If each u_i is open, then M = $\{u_i\}$ is called an open Q – cover of λ.

Definition 3.2. A fuzzy set λ in X is said to be Q – compact iff every open Q – cover of λ has a finite Q – subcover i.e. there exist u_{i_1}, u_{i_2}, ….., u_{i_n} $\in\{u_i\}$ such that λ(x) + u_{i_k}(x) $\geq$ 1 for all x $\in$ X and for some u_{i_k} $\in\{u_i\}$. If $\lambda \subset \mu$ and $\mu \in I^X$, then μ is also Q – compact.

Theorem 3.3. Let λ be a fuzzy set in an fts (X, t) and A $\subseteq$ X. Then the following are equivalent :

(i) λ is Q – compact with respect to t.

(ii) λ is Q – compact with respect to the subspace fuzzy topology t_A on A.

Proof : (i) $\Rightarrow$ (ii) : Let $\{u_i$: i$\in$ J } be a t_A– open Q – cover of λ. Then by definition of subspace fuzzy topology, there exists v_i $\in$ t such that u_i = A $\cap$ v_i $\subseteq v_i$. Hence λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X and consequently λ(x) + v_i(x) $\geq$ 1 for all x $\in$ X . Therefore { v_i : i$\in$ J} is a t – open Q – cover of λ.

As λ is Q – compact in (X, t), then λ has finite Q – subcover i.e. there exist v_{i_k} $\in$ $\{v_i\}$ (k $\in$ J_n) such that λ(x) + v_{i_k}(x) $\geq$ 1 for all x $\in$ X. But then λ(x) + (A $\cap$ v_{i_k}) (x) $\geq$ 1 for all x $\in$ X and therefore λ(x) + u_{i_k}(x) $\geq$ 1 for all x $\in$ X. Thus $\{u_i\}$ contains a finite Q – subcover of { u_1, u_2, …. , u_n } and hence (λ, t_A) is Q – compact.

(ii) $\Rightarrow$ (i) : Let { v_i : i$\in$ J } be a t - open Q – cover of λ. Set u_i = A $\cap$ v_i, then λ(x) + v_i(x) $\geq$ 1 for all x $\in$ X. Hence λ(x) + (A $\cap$ v_i) (x) $\geq$ 1 for all x $\in$ X implies that λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X. But $u_i \in t_A$, so $\{u_i : i \in J\}$ is a t_A - open Q – cover of λ. As λ is Q – compact in (A, t_A), then there exist u_{i_k} $\in$ { u_i } (k $\in$ J_n) such that λ(x) + u_{i_k}(x) $\geq$ 1 for all x $\in$ X . This implies that λ(x) + (A $\cap$ v_{i_k}) (x) $\geq$ 1 for all x $\in$ X and consequently λ(x) + v_{i_k}(x) $\geq$ 1 for all x $\in$ X. Thus { v_i } contains a finite Q – subcover $\{v_{i_k}\}$ and therefore λ is Q – compact with respect to t.

Theorem 3.4. Let λ be a Q – compact fuzzy set in an fts (X, t). If $\mu \subset \lambda$ and $\mu \in t^c$, then μ is also Q – compact.

Proof: Let $\{u_i : i \in J\}$ be an open Q – cover of μ. Then $\{u_i\} \cup \{\mu^c\}$ is an open Q – cover of λ. As μ(x) + u_i(x) $\geq$ 1 for all x $\in$ X , then λ(x) + max (u_i(x), μ^c(x)) $\geq$ 1 for all x $\in$ X. Hence μ(x) + u_i(x) $\leq$ λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X. Since λ is Q –

compact in (X, t), then each open Q – cover of λ has a finite Q – subcover i.e. there exist a finite subset $J_n \subset$ J, such that { u_{i_k} : k $\in$ J_n } $\cup$ { μ^c } is an open Q – cover of λ. Then { u_{i_k} : k $\in$ J_n } is a finite subfamily of $\{u_i : i \in J\}$ and is an open Q – cover of μ i.e. { u_{i_k} : k $\in$ J_n } is a finite Q – subcover of μ. Hence μ is Q – compact.

Theorem 3.5. Let λ and μ be Q – compact fuzzy sets in an fts (X, t). Then $\lambda \cup \mu$ is also Q – compact in (X, t).

Proof : Let M = $\{u_i$: i$\in$ J } and N = { v_i : i$\in$ J } be any open Q – cover of λ and μ respectively. As λ and μ are Q – compact fuzzy sets in (X, t), then M and N have finite open Q – subcovers, say M_{i_k} = { u_{i_k} : k $\in$ J_n } and N_{i_k} = { v_{i_k} : k $\in$ J_n }. It is clear that M_{i_k} or N_{i_k} is an open Q – subcover of $\lambda \cup \mu$ and this implies that $\lambda \cup \mu$ is Q – compact.

Theorem 3.6. Let (X, t) and (Y, s) be two fts's and λ be a Q – compact fuzzy set in (X, t). Let f : $(X, t) \to (Y, s)$ be fuzzy continuous, one – one and onto. Then f(λ) is also Q – compact in (Y, s).

Proof : Let $\{u_i : i \in J\}$ be an open Q – cover of f(λ) in (Y, s) i.e. (f(λ)) (x) + u_i (x) $\geq$ 1 for all x $\in$Y. Since f is fuzzy continuous, then $f^{-1}(u_i) \in$ t and hence { $f^{-1}(u_i)$: i $\in$ J } is an open Q – cover of λ. As λ is Q – compact in (X, t), then λ has a finite Q – subcover i.e. there exist $u_{i_k} \in \{u_i\}$ (k $\in$ J_n) such that λ (x) + (f^{-1}(u_{i_k})) (x) $\geq$ 1 for all x $\in$ X. Again, let u be any fuzzy set in Y. Since f is onto, then for any y $\in$Y, we have

f(f^{-1}(u)) (y) = sup { f^{-1}(u) (z) : z $\in$ f^{-1}(y), f^{-1}(y) $\neq$ ϕ } = sup {u(f(z)) : f(z) = y } = sup {u(y) } = u(y) i.e. f(f^{-1}(u)) = u. This is true for any fuzzy set in Y. As f is one – one and onto, so f(1) = 1. Hence f(λ (x) + (f^{-1}(u_{i_k})) (x)) $\geq$ f(1) (since f is one –one and onto) implies that (f (λ)) (x) + (u_{i_k})) (x) $\geq$ 1 for all x $\in$ X. Therefore f(λ) is Q – compact in (Y, s).

Theorem 3.7. Let (A, t_A) and (B, s_B) be fuzzy subspaces of fts's (X, t) and (Y, s) respectively.

Let λ be a Q–compact fuzzy set in (A, t_A) and f : $(A, t_A) \to (B, s_B)$ be relatively fuzzy continuous, one – one and onto. Then f(λ) is also Q – compact in (B, s_B).

Proof : Suppose λ is Q – compact in (A, t_A). Let M = { v_i : i$\in$ J } be an open Q – cover of f(λ) in (B, s_B) i.e. (f(λ)) (x) + v_i (x) $\geq$ 1 for all x $\in$ X. Since $v_i \in s_B$, then there exist $u_i \in$ s such that $v_i = u_i \cap$ B. Hence (f(λ)) (x) + ($u_i \cap$ B) (x) $\geq$ 1 for all x $\in$ X. As f is fuzzy relatively continuous, then $f^{-1}(v_i) \cap$ A $\in t_A$ and hence { $f^{-1}(v_i) \cap$ A : i$\in$ J } is an open Q – cover of λ. Since λ is Q – compact in $\left(A, t_A\right)$, then there exist $v_{i_k} \in \{v_i\}$ (k $\in J_n$) such that λ(x) + ($f^{-1}(v_{i_k}) \cap$ A) (x) $\geq$ 1 for all x $\in$ X and k $\in J_n$. Again, let v be any fuzzy set in B. Since f is onto, then for any y $\in$B, we have f(f^{-1}(v)) (y) = sup { f^{-1}(v) (z) : z $\in f^{-1}$(y), f^{-1}(y) $\neq \phi$ } = sup {v(f(z)) : f(z) = y } = sup {v (y) } = v(y) i.e. f(f^{-1}(v)) = v. This is true for any fuzzy set in B. As f is one – one and onto, so f(1) = 1. Hence f (λ(x) + ($f^{-1}(v_{i_k}) \cap$ A) (x)) $\geq$ f(1) (since f is one –one and onto) implies that (f (λ)) (x) + ($v_{i_k} \cap$ f(A)) (x) $\geq$ 1 for all x $\in$ X. Therefore f(λ) is Q – compact in $\left(B, s_B\right)$.

The following example will show that the open subset of a Q – compact fuzzy set need not be Q – compact.

Example 3.8. Let X = { a, b } and I = [0, 1]. Let $u_1, u_2, u_3, u_4 \in I^X$ defined by u_1(a) = 0.3, u_1(b) = 0.5 ; u_2 (a) = 0.2, u_2 (b) = 0.6 ; u_3(a) = 0.3, u_3(b) = 0.6 ; u_4 (a) = 0.2, u_4 (b) = 0.5. Now take t = { 0, 1, u_1, u_2, u_3, u_4 }, then we see that $\left(X, t\right)$ is an fts. Again, let $\lambda, \mu \in I^X$ defined by λ(a) = 0.9, λ(b) = 0.6 and μ(a) = 0.3, μ(b) = 0.5. Then we see that $\lambda \subset \mu$ and $\mu \in$ t. Now, λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X and for some u_i. Hence by definition of Q – compact, λ is Q – compact. But μ(a) + u_i(a) $\geq$ 1. Hence μ is not Q – compact.

The following example will show that Q – compact fuzzy set in an fts $\left(X, t\right)$ need not be closed.

Example 3.9. Let X = { a, b } and I = [0, 1]. Let $u_1, u_2 \in I^X$ defined by u_1(a) = 0.2, u_1(b) = 0.4 and u_2 (a) = 0.5, u_2 (b) = 0.6. Now, put t = { 0, 1, u_1, u_2 }, then we see that $\left(X, t\right)$ is an fts. Let $\lambda \in I^X$ defined by λ(a) = 0.9, λ(b) = 0.7. Then λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X and for some u_i. Hence by definition of Q – compact, λ is Q – compact. But λ is not closed, as its complements λ^c is not open in $\left(X, t\right)$.

The following example will show that the subset of a Q – compact fuzzy set in an fts $\left(X, t\right)$ need not be Q – compact.

Example 3.10. Let X = { a, b } and I = [0, 1]. Let $u_1, u_2 \in I^X$ defined by u_1(a) = 0.3, u_1(b) = 0.5 and u_2 (a) = 0.6, u_2 (b) = 0.5. Now, put t = { 0, 1, u_1, u_2 }, then we see that $\left(X, t\right)$ is an fts. Let $\lambda, \mu \in I^X$ defined by λ(a) = 0.8, λ(b) = 0.6 and μ(a) = 0.3,

μ (b) = 0.4. Hence we see that $\mu \subset \lambda$ and λ is clearly Q – compact . But μ is not Q – compact, since μ (a) + u_i (a) $\geq$ 1 and μ (b) + u_i (b) $\geq$ 1.

The following example will show that the interior of a Q – compact fuzzy set in an fts (X, t) need not be Q – compact.

Example 3.11. Let X = { a, b } and I = [0, 1]. Let u_1, u_2, u_3, $u_4 \in I^X$ defined by u_1(a) = 0.2, u_1(b) = 0.5 ; u_2 (a) = 0.3, u_2 (b) = 0.4 ; u_3(a) = 0.3, u_3(b) = 0.5 ; u_4 (a) = 0.2, u_4 (b) = 0.4.

Now, take t = { 0, 1, u_1, u_2, u_3, u_4 }, then we see that (X, t) is an fts. Let $\lambda \in I^X$ defined by λ (a) = 0.8, λ (b) = 0.6. Now λ (x) + u_i (x) $\geq$ 1 for all x $\in$ X and for some u_i. Hence λ is Q – compact. We observe that u_1, u_2, u_3, $u_4 \subset \lambda$. Therefore $\lambda^0 = u_1 \cup u_2 \cup u_3 \cup u_4 = u_3$ i.e. λ^0 (a) = 0.3, λ^0 (b) = 0.5. Now λ^0 (a) + u_i (a) $\geq$ 1. Hence λ^0 is not Q – compact.

Theorem 3.12. Let λ be a Q – compact fuzzy set in a fuzzy Hausdorff space (X, t) with $\lambda_0 \subset$ X (proper subset). Suppose x $\notin \lambda_0$ (λ (x) = 0), then there exist u, v $\in$ t such that u(x) = 1, $\lambda_0 \subseteq v^{-1}$(0, 1] and u $\cap$ v = 0.

Proof : Let y $\in \lambda_0$. Then clearly x $\neq$ y. As (X, t) is fuzzy Hausdorff, then there exist u_y, $v_y \in$ t such that u_y (x) = 1, v_y (y) = 1 and $u_y \cap v_y$ = 0. Hence λ (x) + u_y (x) $\geq$ 1, x $\in$ X and λ (y) + v_y (y) $\geq$ 1, y $\in \lambda_0$ i.e. { u_y, v_y : y $\in \lambda_0$ } is an open Q – cover of λ. Since λ is Q – compact in (X, t), then there exist u_{y_1}, u_{y_2}, ……., $u_{y_n} \in \{u_y\}$ and v_{y_1}, v_{y_2}, ……., $v_{y_n} \in \{v_y\}$ such that λ (x) + u_{y_k} (x) $\geq$ 1 for all x $\in$ X when λ (x) = 0 and some $u_{y_k} \in \{u_y\}$ and λ (y) + v_{y_k} (y) $\geq$ 1 for all y $\in$ X when λ (y) > 0 and some $v_{y_k} \in \{v_y\}$. Now, let v = $v_{y_1} \cup v_{y_2} \cup \ldots\ldots \cup v_{y_n}$ and u = $u_{y_1} \cap u_{y_2} \cap \ldots\ldots \cap u_{y_n}$. Then we see that v and u are open fuzzy sets, as they are the union and finite intersection of open fuzzy sets respectively i.e. v, u $\in$ t. Furthermore, $\lambda_0 \subseteq v^{-1}$(0, 1] and u(x) = 1, as u_{y_k} (x) = 1 for each k.

Finally, we have to show that u $\cap$ v = 0. As $u_{y_k} \cap v_{y_k}$ = 0 implies u $\cap v_{y_k}$ = 0, by distributive law, we see that u $\cap$ v = u $\cap$ ($v_{y_1} \cup v_{y_2} \cup \ldots\ldots \cup v_{y_n}$) = 0.

Theorem 3.13. Let λ and μ are disjoint Q – compact fuzzy sets in a fuzzy Hausdorff space (X, t) with λ_0, $\mu_0 \subset$ X (proper subsets). Then there exist u, v $\in$ t such that $\lambda_0 \subseteq u^{-1}$(0, 1], $\mu_0 \subseteq v^{-1}$(0, 1] and u $\cap$ v = 0.

Proof : Let y $\in \lambda_0$. Then y $\notin \mu_0$, as λ and μ are disjoint. As μ is Q – compact in (X, t), then by previous theorem, there exist u_y, $v_y \in$ t such that u_y(y) = 1, $\mu_0 \subseteq v_y^{-1}(0, 1]$ and $u_y \cap v_y$ = 0. As u_y(y) = 1, then λ(x) + u_y(x) $\geq$ 1, x $\in$ X and λ(y) + v_y(y) $\geq$ 1, y $\in \lambda_0$ i.e. { u_y, v_y : y $\in \lambda_0$ } is an open Q – cover of λ. Since λ is Q – compact in (X, t), then there exist u_{y_1}, u_{y_2}, ……., $u_{y_n} \in \{u_y\}$ and v_{y_1}, v_{y_2}, ……., $v_{y_n} \in \{v_y\}$ such that λ(x) + u_{y_k}(x) $\geq$ 1 for all x $\in$ X when λ(x) = 0 and some $u_{y_k} \in \{u_y\}$ and λ(y) + v_{y_k}(y) $\geq$ 1 for all y $\in$ X when λ(y) > 0 and some $v_{y_k} \in \{v_y\}$. Furthermore, μ(x) + u_{y_k}(x) $\geq$ 1 for all x $\in$ X when μ(x) > 0 and some $u_{y_k} \in \{u_y\}$ and μ (y) + v_{y_k}(y) $\geq$ 1 for all y $\in$ X when μ (y) = 0 and some $v_{y_k} \in \{v_y\}$. Now, let u = $u_{y_1} \cup u_{y_2} \cup \ldots\ldots \cup u_{y_n}$ and v = $v_{y_1} \cap v_{y_2} \cap \ldots\ldots \cap v_{y_n}$. Thus we see that $\lambda_0 \subseteq u^{-1}(0, 1]$ and $\mu_0 \subseteq v^{-1}(0, 1]$, as $\mu \subseteq v_{y_k}$ for each k. Hence u and v are open fuzzy sets, as they are the union and finite intersection of open fuzzy sets respectively i.e. u, v $\in$ t.

Finally, we have to show that u $\cap$ v = 0. As $u_{y_k} \cap v_{y_k}$ = 0 implies $u_{y_k} \cap$ v= 0, by distributive law, we see that u $\cap$ v = ($u_{y_1} \cup u_{y_2} \cup \ldots\ldots \cup u_{y_n}$) $\cap$ v = 0.

The following example will show that the Q – compact fuzzy set of a fuzzy Hausdorff space need not be closed.

Example 3.14. Let X = { a, b } and I = [0, 1]. Let u_1, $u_2 \in I^X$ defined by u_1(a) = 1, u_1(b) = 0 and u_2(a) = 0, u_2(b) = 1. Now, put t = { 0, 1, u_1, u_2 }, then we see that (X, t) is a fuzzy Hausdorff space. Let $\lambda \in I^X$ defined by λ(a) = 0.6, λ(b) = 0.4. Thus we see that λ(x) + u_i(x) $\geq$ 1 for all x $\in$ X and for some u_i. Hence by definition of Q – compact fuzzy set, λ is Q – compact. But λ is not closed, as its complement λ^c is not open in (X, t).

The following example will show that the "good extension" property does not hold for Q – compact fuzzy sets.

Example 3.15. Let X = { a, b, c } and T = { {a}, {b}, {a, b}, ϕ, X }. Let M = {a, b}. Then clearly M is compact in (X, T). 1_M is not Q – compact in $(X, \omega(T))$ as there does not exist u $\in \omega$(T) such that 1_M(c) + u(c) $\geq$ 1. Let $\lambda \in I^X$ with λ(a) = 0.5, λ(b) = 0, λ(c) = 1. Then clearly λ is Q – compact in $(X, \omega(T))$, but $\lambda^{-1}(0, 1]$ = {a, c} is not compact in (X, T). It is , therefore, observe that the "good extension property" does not hold good for Q – compact fuzzy sets.

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