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Chemically Reactive Solute Transfer over a Plate in Porous Medium in Presence of Suction

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ABSTRACT

This paper presents the mass transfer characteristics in a boundary layer forced convective flow of a viscous incompressible fluid past a plate embedded in a Darcy-Forchheimer porous medium in presence of suction and first order chemical reaction. With the help of suitable similarity transformations, the governing partial differential equations are reduced to nonlinear ordinary differential equations and are then solved numerically with the help of shooting method. The rate of mass transfer increases with the increasing values of Schmidt number.

***Keywords**:* Forced convective flow, porous medium, similarity solutions, chemical reaction.

Nomenclature:

C	Concentration the fluid
C_w	Concentration at the wall
C_∞	free-stream concentration
f	non-dimensional stream function
$f^{/}$	streamwise velocity
k	Darcy permeability of the porous medium
$k^{/}$	Forchheimer resistance factor
k_1	parameter of the porous medium
k_2	inertial parameter
Sc	Schmidt number
u_∞	free stream velocity

Greek symbols:

ϕ	non-dimensional concentration
η	similarity variable
μ	dynamic viscosity
ν	kinematic viscosity
ψ	stream function
ρ	density of the fluid

1. Introduction

Boundary-layer flow on a flat plate is a classical problem of Fluid Dynamics and it has been considered by many researchers such as Afzal and Hussain [1], Magyari et al. [2], Cortell [3], Ishak et al. [4], Deswita et al. [5]. In recent years, intensive investigations have been carried out for steady flow of viscous incompressible fluids. Blasius [6] first considered the steady laminar boundary layer viscous flow over a flat plate obtained a series solution. Here, the velocity boundary layer developed due to the moving fluid over a stationary flat surface. Considering uniform plate temperature, the heat transfer characteristics for Blasius problem was solved by Pohlhausen [7]. The numerical solution of Blasius flow problem is obtained by Howarth [8]. Due to the many practical applications which can be modeled or approximated as transport phenomena in porous media, convective flows in porous media have been extensively studied during the last several decades considering several different physical effects (Chin et al. [9]).

The heat and mass transfer phenomena in a porous medium find their applications in various engineering disciplines such as soil pollution nuclear waste and disposal geothermal fields. A better understanding of convection through porous medium can benefit several areas like insulation design, grain storage, geothermal systems, heat exchangers, filtering devices, metal processing, catalytic reactors etc. Representative studies on flow through porous media can be found in Cheng and Minkowycz [10], Cheng [11], Wilks [12] etc. Lai and Kulacki [13] discussed the coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium. The Darcy model assumes proportionality between the velocity and the pressure gradient. This model has been extensively used to investigate a number of interesting fluid and heat transfer problems embedded in porous media. However, this model is valid for slow motion of fluid (i.e. for flow with small velocities) through porous media having low permeability. But in nature, certain porous materials, such as foam metals and fibrous exist having high porosities. In such type of media, the boundary and inertia effects which are absent in Darcy's model may alter the flow and heat transfer characteristics. Therefore, it is very important to consider these effects. The study of convective heat transfer in a boundary layer flow and heat transfer of an incompressible fluid past a plate embedded in a Darcy/non-Darcy porous medium has attracted many investigators due to its wide range of applications in geothermal operations, petroleum industries, and many others (Bachok et al. [14]). Darcian and non-Darcian mixed convection about a vertical plate had been reported by Hsu and Cheng [15], Vafai and Tien [16]. Soundalgekar et al. [17] investigated the combined free and forced convection flow past a semi-infinite plate with variable surface temperature. Hong et al. [18] have studied analytically the non-Darcian effects on a vertical plate natural convection in porous media. Kaviany [19], Chen and Ho [20] have studied the effects of flow inertia on vertical, natural convection in saturated, porous media. Kumari et al. [21] have investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium. Damseh et al. [22] has discussed the magnetic field and thermal radiation effects on forced convection flow. Mukhopadhyay and Layek [23] analyzed the effects of porous media on forced convection flow. But in this paper the linear Darcy term describing the distributed body force exerted by the porous medium was retained whereas the non-linear Forchheimer term was neglected. Of late, Mukhopadhyay et al. [24] considered the effects of Darcy-Forchheimer porous media on flow over a plate in presence of thermal radiation.

Darcy's law is an empirical formula relating the pressure gradient, the bulk viscous fluid resistance and the gravitational force for a forced convective flow in a porous medium. For flow through a porous medium with high permeability, Brinkman [25] as well as Chen et al. [26] argue that the momentum equation must reduce to the viscous flow limit and advocate that classical frictional terms be added in Darcy's law. The aim of this paper is to study the steady forced convection flow and heat transfer past a porous plate placed in a fluid-saturated porous medium using the Darcy-Forchheimer model.

Suction of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction. The process of suction/blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery. Suction is applied to chemical processes to remove reactants.

The distribution of solute under going chemical reaction corresponding to boundary layer flow are relevant to many practical applications. The chemical reaction effects were studied by many researchers on several physical aspects. The diffusion of a chemically reactive species in a laminar boundary layer flow over a flat plate was demonstrated by Chambre and Young [27].

First-order reaction is the simplest chemical reactions in which the rate of reaction is directly proportional to the species concentration. It is well-known that even zero-th and second-order reactions can be approximated by first order reactions up to 60% conversion concerning only the changes of compositions with the reaction time. Therefore, the first-order chemical reaction plays an important role in chemical engineering (Midya [28]). The mass transfer analysis in boundary layer flow is of great importance in extending the theory of separation processes and chemical kinetics.

Motivated by the above investigations and possible applications, an attempt is made in the present work to study the problem of mass transfer of an incompressible fluid past a plate embedded in a Darcy-Forchheimer porous medium in presence of first order chemical reaction and suction. The results are analyzed through their graphical representations.

2. Formulation of the problem

Let us consider the problem of a forced convective, steady laminar boundary-layer flow and mass transfer of an incompressible, viscous fluid over a flat plate of very small thickness and much larger breadth, embedded in a porous medium in presence of first order chemical reaction (Fig.1). We also consider that the differential equation governing the fluid motion is based on Darcy-Forchheimer model, which accounts for the drag (represented by the Darcy term) exerted by the porous media as well as the inertia effect (represented by the non-linear Forchheimer term). Under the above assumptions, the boundary layer equations governing the flow and mass transfer are written as

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k}(u-u_\infty)-\frac{k^{/}}{\sqrt{k}}(u^2-u_\infty^2) \tag{2}$$

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-r(C-C_\infty) \tag{3}$$

where the last two terms on the right-hand side of equation (2) describe the non-linear Darcy-Forchheimer resistance of the surrounding porous medium. The x co-ordinate is measured from the leading edge of the plate, y co-ordinate is measured along the normal to the plate. Here u and υ are the components of velocity respectively in the x and y directions, μ is the coefficient of fluid viscosity, ρ is the fluid density, $\nu=\mu/\rho$ is the kinematic viscosity, $k=k_0 x$ is the Darcy permeability of the porous medium, k_0 is the initial permeability, $k^{/}=\frac{k_0^{/}}{\sqrt{x}}$ is the Forchheimer resistance factor, $k_0^{/}$ is the Forchheimer constant, which has been experimentally measured for different porous media, D is the solute diffusion coefficient, r is the variable rate of chemical conversion given by $r=\frac{L}{x}r_0$, L is the reference length and r_0 is a constant; C is the concentration, u_∞ is the free stream velocity.

The appropriate boundary conditions for the problem are given by

$$u=0, \upsilon=-\upsilon_w, C=C_w \quad \text{at } y=0, \tag{4}$$

$$u=u_\infty, C=C_\infty \text{ as } y\to\infty. \tag{5}$$

Here C_w is the concentration at the wall, C_∞ is the free stream concentration.

2.1. Similarity analysis and solution procedure:

To find out the solution we first introduce the stream function $\psi(x,y)$ as

$$u=\frac{\partial\psi}{\partial y}, \upsilon=-\frac{\partial\psi}{\partial x} \tag{6}$$

We also introduce the following dimensionless variables

$$\phi=\frac{C-C_\infty}{C_w-C_\infty}, \tag{7}$$

and $$\eta=y\sqrt{\frac{u_\infty}{\nu x}}, \psi=\sqrt{u_\infty \nu x}\,f(\eta). \tag{8}$$

Using the relations (6) and (7) in the boundary layer equation (2) and in the energy equation (3) we get the following equations

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\nu\frac{\partial^3\psi}{\partial y^3}-\frac{\nu}{k}\left(\frac{\partial\psi}{\partial y}-u_\infty\right)-\frac{k^{/}}{\sqrt{k}}\left\{\left(\frac{\partial\psi}{\partial y}\right)^2-u_\infty^2\right\}, \tag{9}$$

and

$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}=D\frac{\partial^2\phi}{\partial y^2}-r\phi \qquad (10)$$

The boundary conditions (4) and (5) then become

$$\frac{\partial\psi}{\partial \mathrm{y}}=0,\frac{\partial\psi}{\partial \mathrm{x}}=0,\phi=1 \quad \text{at } y=0, \qquad (11)$$

$$\frac{\partial\psi}{\partial y}=u_\infty,\phi=0 \quad \text{as } y\to\infty \;, \qquad (12)$$

Using equation (8), the equations (9) and (10) finally can be put in the following form

$$f^{///}+\frac{1}{2}ff^{//}-\mathrm{k}_1(f^{/}-1)-\mathrm{k}_2(f^{/2}-1)=0, \qquad (13)$$

$$\frac{1}{\mathrm{Sc}}\phi^{//}+\frac{1}{2}f\phi^{/}\text{-}\mathrm{R}\phi=0, \qquad (14)$$

where $\mathrm{k}_1=\frac{1}{Da_x\,\mathrm{Re}_x}=\frac{\nu}{\mathrm{k}_0u_\infty}$ is the parameter of the porous medium, $Da_x=\frac{\mathrm{k}}{x^2}=\frac{\mathrm{k}_0}{x}$ is the local Darcy number, $\mathrm{Re}_x=\frac{u_\infty x}{\nu}$ is the local Reynolds number and $\mathrm{k}_2=\frac{\mathrm{k}_0^{/}}{\sqrt{\mathrm{k}_0}}$ is the inertial parameter, $\mathrm{Sc}=\frac{\nu}{D}$ is the Schmidt number, $\mathrm{R}=\frac{Lr_0}{u_\infty}$ is the reaction rate parameter. Here R>0 represents the destructive reaction, R= 0 corresponds to no reaction, and R <0 stands for the generative reaction. k_1^{-1} and k_2 will reflect the effect of Darcian and Forchheimer flows on the present problem. In view of these, the boundary conditions finally become

$$f^{/}=0, f=0,\ \phi=1 \text{ at } \eta=0, \qquad (15)$$

$$f^{/}=1,\ \phi=0 \text{ at } \eta\to\infty. \qquad (16)$$

When $\mathrm{k}_1=0=\mathrm{k}_2$, the equation (15) reduces to the equation of boundary layer flow on a flat plate at zero incidence.

Equations (13) and (14) along with boundary conditions (15)-(16) are solved numerically.

3. Results and discussions

In the paper the boundary layer flow and mass transfer of an incompressible fluid past a plate in presence of first order chemical reaction is investigated. Similarity solution is used to obtain the velocity and solute distribution, which are governed by non-linear differential equations. Computation through employed numerical scheme has been carried out for various values of the parameters such as the parameter of the porous medium (k_1), inertial parameter (k_2), Schmidt number (Sc) and reaction rate parameter (R).

For the verification of the accuracy of the applied numerical method our results are compared corresponding to the velocity and shear stress profiles for k_1=0 and k_2=0 (i.e. in absence of porous medium) with the available published results of Howarth [8] in Fig.2 and are found in excellent agreement.

Variations of the velocity $f^{/}(\eta)$ as function of η for several values of permeability parameter k_1 are shown in Fig.3(a). Fluid velocity $f^{/}(\eta)$ increases with the increasing values of the permeability of the medium. The regime becomes more porous when the permeability of the medium increases and Darcian body force decreases in magnitude (as it is inversely proportional to the permeability). As permeability of the medium increases, this resistance which acts to decelerate the fluid particles in continua diminishes. So progressively less drag is experienced by the flow and flow retardation is thereby decreased that is why the velocity of the fluid increases as the parameter (k_1) increases. Fig.3(b) displays the nature of velocity profiles for different values of inertial parameter k_2. It is found that the velocity decreases with the increase of k_2 within the boundary layer. Figure 3(c) depicts the concentration profiles $\phi(\eta)$ as a function of η for different values of permeability parameter k_1 in presence suction. It can be observed that $\phi(\eta)$ decreases with η for increasing values of k_1. As with the increasing k_1, thickness of the velocity boundary layer decreases so the solute boundary layer thickness becomes thinner. Concentration is found to decrease with the increase of k_2 [Fig.3(d)]. Fig.3(d) also shows that the rate of mass transfer is much faster for higher values of the inertial parameter k_2.

Fig. 4(a)-4(b) represent the effects of suction on velocity profiles in the absence and presence of porous media respectively. With the increasing S (S >0), the fluid velocity is found to increase [Fig. 4(a), 4(b)] i.e. suction causes to increase the velocity of the fluid in the boundary layer region. In case of blowing (S<0) opposite behaviour is noted. Since the effect of suction is to suck away the fluid near the wall, the momentum boundary layer is reduced due to suction (S >0). Consequently the velocity increases. Hence the velocity gradient and so the skin friction increases with increasing S (S > 0). Fig. 4(c)-4(d) exhibit the nature of concentration profiles due to suction in both the cases of non-porous and porous media. The solute boundary layer thickness decreases with the suction parameter S which causes an increase in the rate of mass transfer. This effect is more pronounced in presence of porous media. As the effect of blowing is opposite of suction so it is not presented here.

Fig. 5 exhibits the effects of Schmidt number (Sc) on the concentration profiles in presence of first order chemical reaction. This clearly explains that the solute distribution increases due to the decrease of solute diffusion coefficient. Actually, the Schmidt number is inversely proportional to the diffusion coefficient D. Thus, the increase of Sc results in the decrease of the diffusion coefficient. An increase in Schmidt number reduces the solute boundary layer thickness.

Fig. 6 displays the effects of reaction rate parameter on solute distribution. The concentration within the fluid decreases for the increasing values of reaction rate parameter R. It is seen that concentration field increases for generative chemical reaction

(R< 0) where as it decreases for destructive chemical reaction (R> 0). The concentration boundary layer decreases in case of destructive chemical reaction.

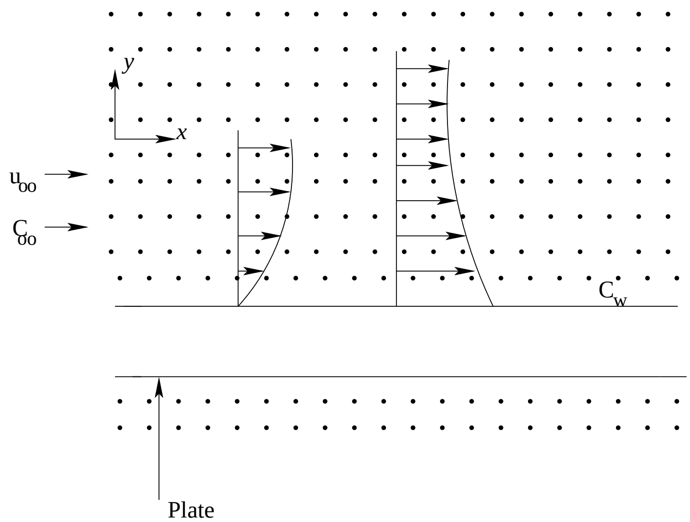

Figure 1: Sketch of the physical problem

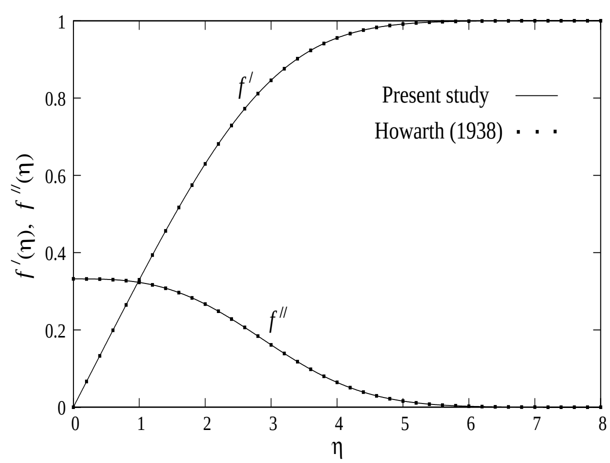

Figure 2: Velocity $f^{/}(\eta)$ and shear stress $f^{//}(\eta)$ profiles for k_1=0 and k_2=0 in the absence of suction.

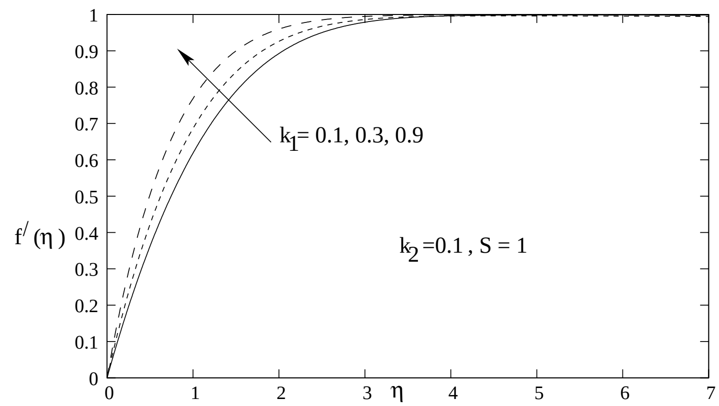

Figure 3a: Variation of velocity $f^{/}(\eta)$ with η for several values of permeability parameter k_1.

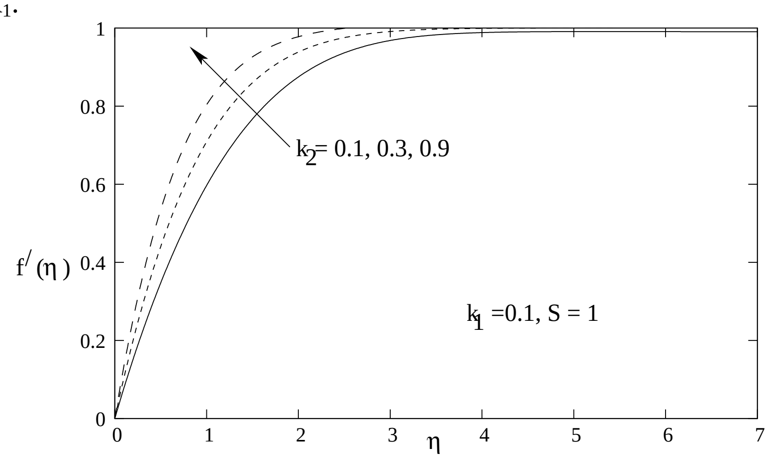

Figure 3b: Variation of velocity $f^{/}(\eta)$ with η for several values of inertial parameter k_2.

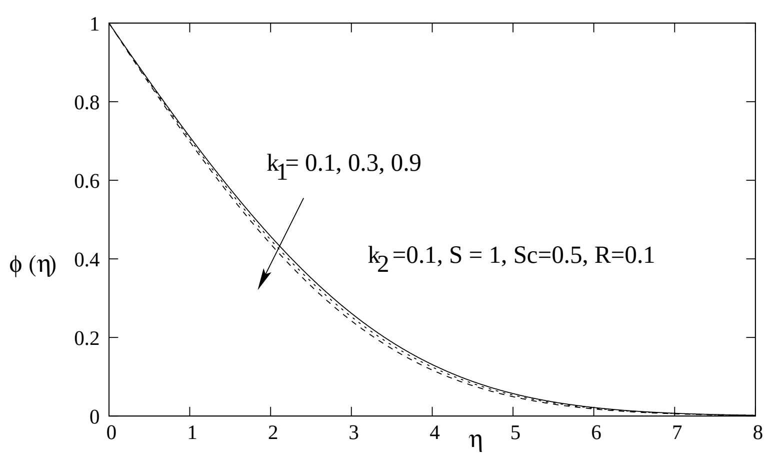

Figure 3c: Variation of concentration $\phi(\eta)$ with η for several values of permeability parameter k

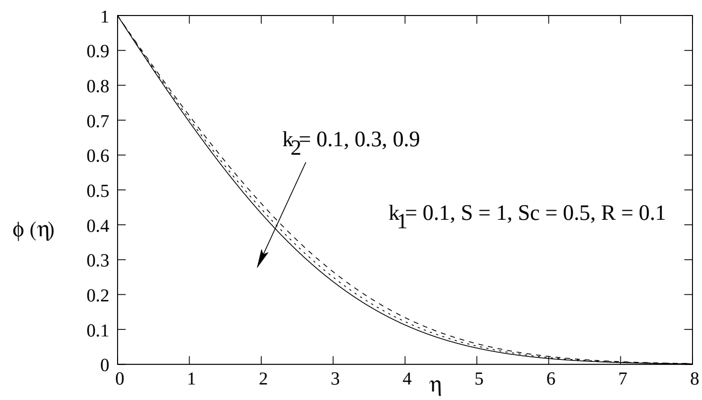

Figure 3d: Variation of concentration $\phi(\eta)$ with η for several values of inertial parameter k_2.

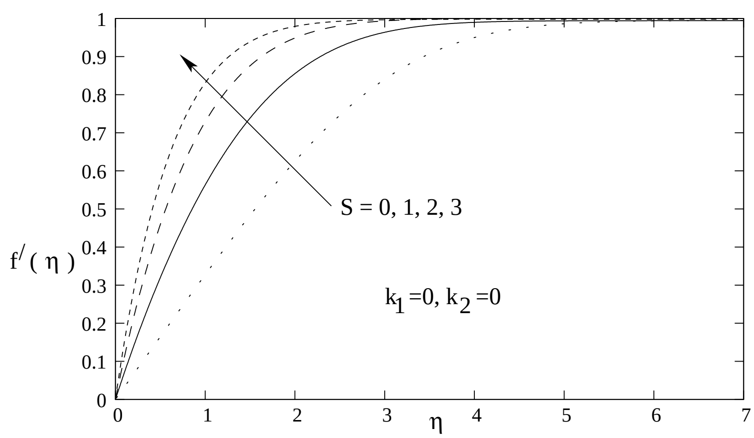

Figure 4a: Variation of velocity $f^{/}(\eta)$ with η for several values of suction parameter S in case of non-porous medium.

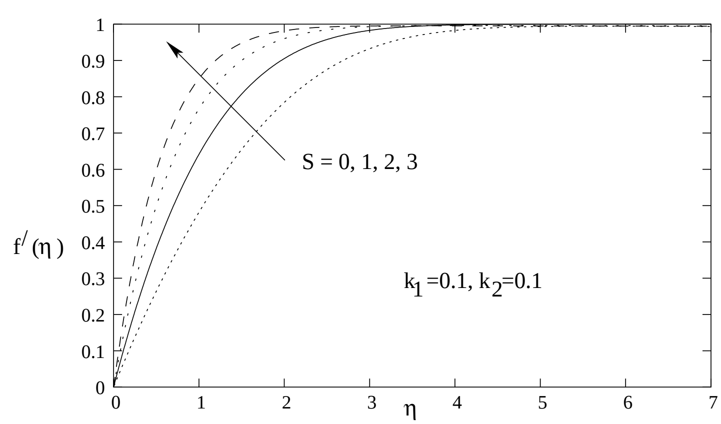

Figure 4b: Variation of velocity $f^{/}(\eta)$ with η for several values of suction parameter S in case of porous medium.

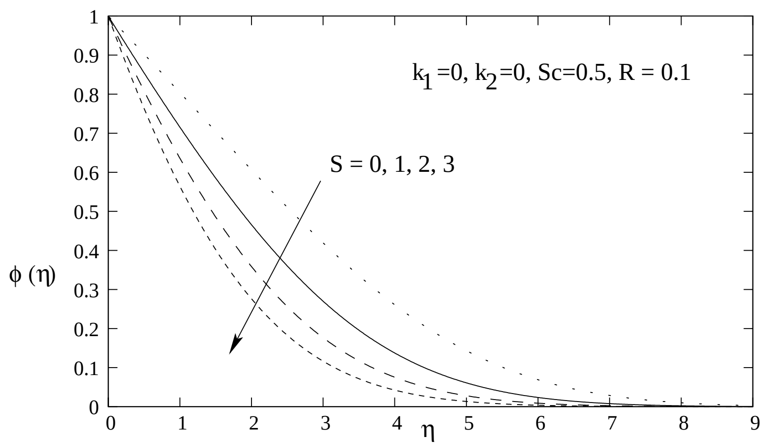

Figure 4c: Variation of concentration $\phi(\eta)$ with η for several values of suction parameter S in case of non-porous medium.

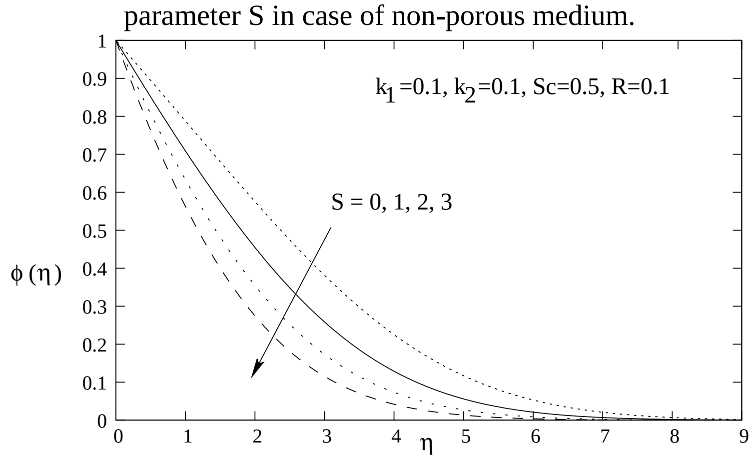

Figure 4d: Variation of concentration $\phi(\eta)$ with η for several values of suction parameter S in case of porous medium.

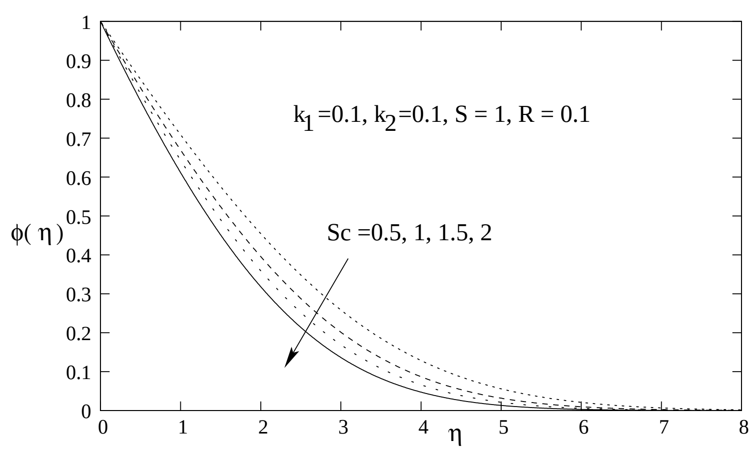

Figure 5: Variation of concentration $\phi(\eta)$ with η for several values of Schmidt Number Sc.

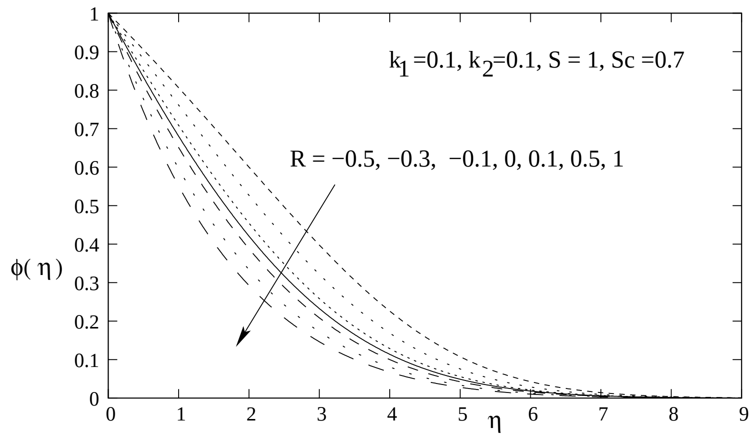

Figure 6: Variation of concentration $\phi(\eta)$ with η for several values of reaction rate parameter R.

4. Conclusion

This paper presents the steady forced convective boundary layer flow and mass transfer over a plate embedded in a Darcy-Forchheimer porous medium in presence of first order chemical reaction. The following observations are made from this study:

(i) The velocity and concentration decrease with increasing permeability parameter (k_1) and inertial parameter (k_2).

(ii) Concentration at a point increases with increasing values of reaction rate parameter.

(iii) Concentration boundary layer decreases in case of destructive chemical reaction.

(iv) The rate of mass transfer increases with the increasing values of Schemidt number Sc.

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