Journal of Physical Sciences, Vol. 16, 2012, 85-92
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on 31 December 2012

Solving Unbalanced Transportation Problems with Budgetary Constraints

D. Anuradha, P. Pandian and G. Natarajan

Department of Mathematics, School of Advanced Sciences
VIT University, Vellore-14, Tamilnadu, India

anuradhadhanapal1981@gmail.com, pandian61@rediffmail.com,
gnatarajan @vit.ac.in

Received October 19, 2012; accepted December 18, 2012

ABSTRACT

A new method namely, interval-point method without using fuzzy set theory is proposed for finding an optimal solution for unbalanced transportation problems with budgetary constraints where demand and budget are imprecise. With the help of numerical example, the proposed method is illustrated. The proposed method enables the decision makers to choose the optimal distribution according to their budget.

Keywords: Unbalanced transportation problem, Optimal solution, Budgetary constraints, Interval-point method

AMS Mathematics Subject Classification (2010):

1. Introduction

The transportation problem (TP) is a special class of linear programming problem, which deals with shipping commodities from sources to destinations. The main objective of the transportation problem is to determine the shipping schedule that minimize that total shipping cost while satisfying supply and demand limits. In literature, a good amount of research has available to obtain an optimal solution for balanced transportation problems. But in real life situations, the decision maker faces an unbalanced transportation problem in which total supply is less than the total demand. This type of problem is faced by the government agencies like Food Corporation of India, which supplies food grains from different warehouses to different distribution centers. Khanna et al. [7] introduced an algorithm for solving transportation flow under budgetary constraints. Tiwari et al. [8] investigated how the preemptive priority structure can be used in fuzzy goal programming problems. Weighted goal programming for unbalanced single objective transportation problem with budgetary constraint has been discussed by Kishore and Jayswal [1]. Kishore and Jayswal [2] introduced a method, called fuzzy approach, to solve unbalanced transportation problem with budgetary constraints. Peerayuth Charnsethikul and Saeree Sverasreni [5] discussed a method for solving the constrained bottleneck transportation problem under budgetary condition. Pandian and Natarajan [3] introduced the zero point method for finding an optimal solution to a classical transportation problem. Lin and Cheng [9] gave a genetic algorithm for solving a transportation network under a budget Constraint. Senapati and Tapan Kumar [6] investigated fuzzy multi-index transportation problem with budgetary restriction.

In this paper, we propose a new method namely, interval-point method for finding an optimal solution for unbalanced transportation problems with budgetary constraints where demand and budget are imprecise. Here we give more emphasis on meeting the budget rather than on the fulfillment of the demand. For better understanding, the solution procedure is illustrated with a numerical example. The proposed method enables the decision makers to evaluate the economical activities and make self satisfied managerial decisions when they are handling a variety of unbalanced transportation problems with budgetary constraints.

2. Preliminaries

We need the following definitions of the basic arithmetic operators and partial ordering on closed bounded intervals which can be found in [4].

Let $D = \{[a,b], a \le b$ and a and b are in R $\}$ denote the set of all closed bounded intervals on the real line R.

Definition 2.1: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \oplus B = [a+c, b+d]$ and

(ii) $A \otimes B = [p,q]$ where $p = \min\{ac, ad, bc, bd\}$ and $q = \max.\{ac, ad, bc, bd\}$.

Definition 2.2: Let $A = [a,b]$ and $B = [c,d]$ be in D. Then,

(i) $A \le B$ if $a \le c$ and $b \le d$; (ii) $A \ge B$ if $a \ge c$ and $b \ge d$ and (iii) $A = B$ if $a = c$ and $b = d$.

Consider the following unbalanced transportation problem with budgetary constraints (UTPBC):

(P) Find the values of x_{ij} , i = 1,2,…,m ; j = 1,2,…,n such that the following conditions are satisfied:

$$\sum_{j=1}^{n} x_{ij} \le a_i\,,\ i = 1,2,\ldots,m \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} \in [b_j^1, b_j^2],\ j = 1,2,\ldots,n \tag{2}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \in [z_1, z_2] \tag{3}$$

$$x_{ij} \ge 0,\ \ i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n. \tag{4}$$

where c_{ij} is the cost of shipping one unit from supply point i to the demand point j ;

a_i is the supply at supply point i ; $[b_j^1, b_j^2]$ is the imprecise demand at demand point j ;

x_{ij} is the number of units shipped from supply point i to demand point j and

$[z_1, z_2]$ is the imprecise budget.

The unbalanced transportation problem related to the problem (P) is given below:

(IP) Minimize $\quad [z_1, z_2] = \left[\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \right]$

subject to

$$\left[\sum_{j=1}^{n} x_{ij}, \sum_{j=1}^{n} x_{ij} \right] = [a_i, a_i],\ i = 1,2,\ldots,m\,;$$

$$\left[\sum_{i=1}^{m} x_{ij}, \sum_{i=1}^{m} x_{ij} \right] = [b_j^1, b_j^2],\ j = 1,2,\ldots,n\,;$$

$$x_{ij} \ge 0,\ \ i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n \text{ and integers.}$$

Now, we need the following theorem which finds a relation between optimal solutions of an interval integer transportation problem and a pair of induced transportation problems and also, is used in the proposed method which can be found in Pandian and Natarajan [4].

Theorem 2.1. If the set { y°_{ij}, for all i and j } is an optimal solution of the upper bound transportation problem (UP) of the problem (IP) where

(UP) Minimize $z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,\ldots,m \;;$$

$$\sum_{i=1}^{m} x_{ij} = b_j^2, \; j = 1,2,\ldots,n \;;$$

$$x_{ij} \geq 0, \; i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n \text{ and integers}$$

and the set $\{ x_{ij}^{\circ}$, for all i and j } is an optimal solution of the lower bound transportation problem (LP) of the problem (IP) where

(LP) Minimize $z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,\ldots,m \;;$$

$$\sum_{i=1}^{m} x_{ij} = b_j^1, \; j = 1,2,\ldots,n \;;$$

$$x_{ij} \geq 0, \; i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n \text{ and integers},$$

then the set of intervals $\{[x_{ij}^{\circ}, y_{ij}^{\circ}]$, for all i and j } is an optimal solution of the problem (IP) provided $x_{ij}^{\circ} \leq y_{ij}^{\circ}$, for all i and j.

3. Interval-Point Method

We, now propose a new method namely, interval-point method for finding an optimal solution to the problem (P).

The interval-point method proceeds as follows:

Step 1: Construct an unbalanced interval transportation problem (IP) related to the given UTPBC.

Step 2: Construct the upper bound unbalanced transportation problem (UP) of the problem (IP) and solve the problem (UP) by the zero point method [3]. Let { y_{ij}°, for all i and j} be an optimal solution to the problem (UP).

Step 3: Construct the lower bound unbalanced transportation problem (LP) of the given the problem (IP) and solve the problem (LP) with the upper bound constraints $x_{ij} \leq y_{ij}^{\circ}$, for all i and j by the zero point method [3]. Let $\{x_{ij}^{\circ}$, for all i and j$\}$ be the optimal solution to the problem (LP) with $x_{ij}^{\circ} \leq y_{ij}^{\circ}$, for all i and j.

Step 4: The optimal solution to the problem (IP) is $\{[x_{ij}^{\circ}, y_{ij}^{\circ}]$, for all i and j$\}$ by the Theorem 2.1.. The optimal objective value of the problem (IP) is $[Z_L, Z_U]$

Step 5: Let $Z \in [Z_L, Z_U]$ be the given budget cost for transportation cost of the given UTPBC. Now, we write Z as in the form $Z = Z_L + (Z_U - Z_L)\mu$, for some μ, $0 \leq \mu \leq 1$. This implies $\mu = \dfrac{Z - Z_L}{(Z_U - Z_L)}$.

Step 6: Compute the values of decision variables

$x_{ij} = [x_{ij}^{\circ}, y_{ij}^{\circ}] = x_{ij}^{\circ} + (y_{ij}^{\circ} - x_{ij}^{\circ})\mu$ where μ is as in Step 5..

Step 7: The optimal solution to the given UTPBC is

$x_{ij} = x_{ij}^{\circ} + (y_{ij}^{\circ} - x_{ij}^{\circ})\left(\dfrac{Z - Z_L}{(Z_U - Z_L)}\right)$ for the given budget is Z.

4. Numerical Example

The interval-point method for solving unbalanced transportation problem with budgetary constraints is illustrated by the following example.

Example 1. There are four godowns (source points) from where the food grains are supplied to three different destinations (demand points). c_{ij} 's are the cost coefficients expressed in rupee per metric ton and a_i 's , b_j^1 's and b_j^2 's are expressed in lakhs of metric ton. The transportation matrix is given in the following table:

	D_1	D_2	D_3	Supply
O_1	5	8	3	$\leq$ 10
O_2	7	4	5	$\leq$ 4
O_3	2	6	9	$\leq$ 4
O_4	4	6	6	$\leq$ 12
Demand	$\in$ [6,12]	$\in$ [[7,14]	$\in$ [7,14]	

Determine an optimal distribution plan to transport the items from the source points to the destination points for the budget Rs.100.

Now, the interval transportation problem (IP) to the given problem is given below:

	D_1	D_2	D_3	Supply
O_1	[5,5]	[8,8]	[3,3]	[10,10]
O_2	[7,7]	[4,4]	[5,5]	[4,4]
O_3	[2,2]	[6,6]	[9,9]	[4,4]
O_4	[4,4]	[6,6]	[6,6]	[12,12]
Demand	[6,12]	[[7,14]	[7,14]	

Now, the upper bound problem of (IP), (UP) of the problem (IP) is given below:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
Demand	12	14	14	

Now, using the zero point method, the optimal solution to the problem(UB) is $y^{\circ}_{13} = 10$, $y^{\circ}_{22} = 4$, $y^{\circ}_{31} = 4$, $y^{\circ}_{41} = 8$ and $y^{\circ}_{42} = 4$.

Now, the lower bound problem of (IP), (LB) with the upper bounded constraints is given below:

	D_1	D_2	D_3	Supply
O_1	5	8	3	10
O_2	7	4	5	4
O_3	2	6	9	4
O_4	4	6	6	12
Demand	6	7	7	

with $x_{ij} \leq y^{\circ}_{ij}$, i =1,2,…,m and j = 1,2,…,n.

Now, using the zero point method, the optimal solution to the problem (LP) is $x^{\circ}_{13} = 7$, $x^{\circ}_{22} = 4$, $x^{\circ}_{31} = 4$, $x^{\circ}_{41} = 2$ and $x^{\circ}_{42} = 3$.

Now, as in the Step 3., we consider the optimal solution to the problem (IP) is $[x_{13}^{\circ}, y_{13}^{\circ}] = [7,10]$, $[x_{22}^{\circ}, y_{22}^{\circ}] = [4,4]$, $[x_{31}^{\circ}, y_{31}^{\circ}] = [4,4]$, $[x_{41}^{\circ}, y_{41}^{\circ}] = [2,8]$ and $[x_{42}^{\circ}, y_{42}^{\circ}] = [3,4]$ and also, the minimum interval transportation cost is [71, 110].

Now, as in the Step 4., we have the total transportation cost as $Z = 71 + 39\mu$ where Z is the given budget. This implies that $\mu = \dfrac{Z - 71}{39}$.

Now, as in the Step 5. and the Step 6., we have

$$x_{13} = 7 + 3\left(\frac{Z - 71}{39}\right);\ x_{22} = 4;\ x_{31} = 4;\ x_{41} = 2 + 6\left(\frac{Z - 71}{39}\right) \text{ and}$$

$$x_{42} = 3 + \left(\frac{Z - 71}{39}\right).$$

Given Z =100. Then, the optimal solution to the given UTPBC for Z =100, is $x_{13} = 9.23$; $x_{22} = 4$; $x_{31} = 4$; $x_{41} = 6.46$ and $x_{42} = 3.74$. The total number of units transported = 27.43 tons.

Remark 1: In Kishore and Anurag Jayswal [2], the total number of units transported in the unbalanced transportation problem with budgetary constraints (Example 1.) is 26.66 tons, but we obtain that the total number of units transported is 27.43 tons by the interval-point method.

5. Conclusion

In this paper, we consider the unbalanced transportation problem with budgetary constraints. A new method is proposed for finding an optimal solution. The proposed method is easy to understand and compute. This method enables the decision makers to optimize the economical activities and make the correct managerial decisions depending on their financial position.

REFERENCES

1. N.Kishore and Anurag Jayswal, Prioritized goal programming formulation of an unbalanced transportation problem with budgetary constraints, Industrial Engineering Journal, 9 (2001) 16-19.
2. N.Kishore and Anurag Jayswal, Prioritized goal programming formulation of an unbalanced transportation problem with budgetary constraints: A fuzzy approach, Opsearch, 39 (2002) 151-160.
3. P.Pandian and G. Natarajan, A new method for finding an optimal solution for transportation problems, International J. of Math. Sci. and Engg. Appls., 4 (2010) 59-65.
4. P.Pandian and G.Natarajan, A new method for finding an optimal solution of fully interval integer transportation problems, Applied Mathematical Sciences, 4 (2010) 1819-1830.

5. Peerayuth Charnsethikul and Saeree Sverasreni, The constrained bottleneck transportation problem, Journal of Mathematics and Statistics, 3 (2007) 24-27.
6. Samiran Senapati and Tapan Kumar Samanta, 2012, Optimal distribution of commodities under Budgetary restriction: a fuzzy approach, International Journal of Advanced Engineering Research and Studies, 1 (2012) 208-211.
7. Saroj Khanna, H.C.Bakhshi and M.C.Puri, Maximizing transportation flow under budgetary constraints, NZOR, 11 (1983) 41-50.
8. R.N.Tiwari, S.Dharmar and J.R.Rao, Priority structure in fuzzy Goal Programming, Fuzzy Sets and Systems, 19 (1986) 251-259.
9. Yi-Kuei Lin and Cheng-Ta Yeh, Maximizing network reliability for stochastic transportation networks under a budget constraint by using a genetic algorithm, International Journal of Innovative Computing, Information and Control, 7 (2011) 7033- 7050.