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Applications of Scaling Group of Transformation on Boundary Layer Flow and Mass Transfer Over a Stretching Sheet Embedded in a Porous Medium

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ABSTRACT

In the present paper, the boundary layer flow and mass transfer over a stretching sheet embedded in porous medium is investigated using scaling group of transformations. By the scaling group of transformations, the governing partial differential equations are transformed to the ordinary differential equations. Then those equations are solved. The analysis shows that without any adhoc assumption on the similarity solutions it can be arrived to a set of similarity solution by scaling group of transformations. Also it is found that for the increase of the permeability parameter the momentum boundary layer thickness decreases. The concentration boundary layer thickness reduces with the increasing values of the Schmidt number.

Keywords: Scaling group of transformations, boundary layer flow, mass transfer, stretching sheet, porous medium.

1. Introduction

The boundary layer flows due to stretching sheet in porous medium are relevant to many engineering problems such as paper production, preparing plastic and metal sheets etc.

Sakiadis (1961a,b) studied the laminar boundary layer flow caused by a rigid surface moving in its own plane. Crane (1970) extended the works of Sakiadis (1961a,b) and considered the flow in a sheet stretched in its own plane. The heat transfer analysis in Newtonian boundary layer flow past a stretching sheet was studied by Gupta and Gupta (1977). Chakrabarti and Gupta (1979) analyzed the magnetohydrodynamic (MHD) flow of Newtonian fluid initially at rest, over a stretching sheet at a different values of parameter related with uniform temperature. Anjali Devi and Ganga (2010) demonstrated dissipation effects on MHD nonlinear flow and heat transfer past a stretching porous surface embedded in a porous medium under a transverse magnetic field.

The mass transfer in boundary layer flow due to a stretching sheet in porous medium also has important applications in many industrial problems. The effect of mass transfer in laminar flow over a stretching sheet was investigated by Andersson et al. (1994). Takhar et al. (2000) analyzed the flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species with n-th order reaction. Afify (2004) explicated the MHD free convective flow of viscous incompressible fluid and mass transfer over a stretching sheet. Liu (2005) studied the momentum, heat and mass transfer of a hydromagnetic flow past a stretching sheet in the presence of a uniform transverse magnetic field. Akyildiz et al. (2006) reported a solution for diffusion of chemically reactive species in a flow of a non-Newtonian fluid over a stretching sheet immersed in a porous medium. Cortell (2007) studied the motion and mass transfer for two classes of viscoelastic fluid over a porous stretching sheet with chemically reactive species. Recently, El-Aziz (2010) explained unsteady flow due to a stretching sheet with mass and heat transfer.

Lie-group analysis, also called symmetry analysis was developed by Sophius Lie to find point-transformations that map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations (Pakdemirli and Yurusoy, 1990). The non-linear character of the partial differential equations governing the motion of the fluid produces difficulties in solving the equations. In fluid mechanics, researchers try to obtain the similarity solutions in such cases. In case of scaling group of transformations, the group-invariant solutions are nothing but the well known similarity solutions (see Mukhopadhyay et al., 2005; Uddin et al., 2011).

In the present paper, the boundary layer flow and mass transfer past a stretching surface embedded in porous medium. A special form of Lie-group of transformations, the scaling group transformations is used to find out the full set of symmetries of the flow problem without adopting any adhoc assumption. So, the governing equations are transformed into a set of self-similar ordinary differential equations. Then the transformed self-similar equations are solved. Exact analytical solution of the momentum equation is obtained and then solution of concentration is obtained numerically. The results are plotted in some figures and discussed the flow and mass transfer characteristics.

2. Formation of the problem

Consider the steady of flow of a viscous incompressible Newtonian fluid and mass transfer over a stretching sheet in porous medium. The governing continuity, momentum and concentration equation in the boundary layer approximation may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{k}u, \quad (2)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}, \quad (3)$$

where u and v are the components of the fluid velocity along the x and y directions respectively, ν is the kinetic viscosity, k is the permeability of the porous medium, C is the concentration and D is the diffusion coefficient.

The corresponding boundary conditions are

$$u = ax \quad and \quad v = 0 \quad at \quad y = 0 \quad and \quad u \to 0 \quad as \quad y \to \infty \quad (4)$$

$$C = C_w \quad at \quad y = 0 \quad as \quad C \to 0 \quad as \quad y \to \infty \quad (5)$$

where $a > 0$ is stretching constant, C_w denotes the concentration at the stretching sheet.

3. Reduction of PDE to ODE

Let us introduce the non-dimensional quantities as

$$\bar{x}=\frac{x}{L} \;,\; \bar{y}=\frac{y}{L}\sqrt{Re} \;,\; \bar{u}=\frac{u}{U}$$

$$\bar{v}=\frac{v}{U}\sqrt{Re} \;,\quad C=C_w\bar{C} \quad or \quad \bar{C}=\frac{C}{C_w}\;. \quad (6)$$

Here L is a characteristic length, U is the free stream velocity and $Re=\frac{uL}{\nu}$ is the Reynolds number.

Substituting the relation's (6) into the equation (1) to (3)

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \quad (7)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}=\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\frac{\nu}{k}\bar{u} \quad (8)$$

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}=D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\,. \quad (9)$$

The boundary condition (4) are deduce to,

$$\bar{u}=a\bar{x} \quad and \quad \bar{v}=0 \quad at \quad \bar{y}=0 \quad and \quad \bar{u}\to 0 \quad as \quad \bar{y}\to\infty\,. \quad (10)$$

We introduce the stream function φ as

$$\bar{u}=\frac{\partial\psi}{\partial\bar{y}} \quad and \quad \bar{v}=\frac{\partial\psi}{\partial\bar{x}} \quad (11)$$

Clearly the relation (11) satisfies the equation (7). In view of the relation (11), the equation (8) and (9) reduce respectively to,

$$\frac{\partial\psi}{\partial\bar{y}}\frac{\partial^2\psi}{\partial\bar{x}\partial\bar{y}}-\frac{\partial\psi}{\partial\bar{x}}\frac{\partial^2\psi}{\partial\bar{y}^2}=\frac{\partial^3\psi}{\partial\bar{y}^3}-\frac{\nu}{k}\frac{\partial\psi}{\partial\bar{y}} \qquad (12)$$

$$\frac{\partial\psi}{\partial\bar{y}}\frac{\partial\bar{C}}{\partial\bar{x}}-\frac{\partial\psi}{\partial\bar{x}}\frac{\partial\bar{C}}{\partial\bar{y}}=D\frac{\partial^2\bar{C}}{\partial\bar{y}^2}\ . \qquad (13)$$

3.1 Scaling group of transformations*:*

We now introduce the simplified from Lie-group transformations, namely, the scaling group of transformations (Mukhopadhyay et al., 2005; Uddin et al., 2011) as:

$$x^*=\bar{x}e^{\epsilon\alpha_1},y^*=\bar{y}e^{\epsilon\alpha_2},\psi^*=\psi e^{\epsilon\alpha_3},u^*=\bar{u}e^{\epsilon\alpha_4},v^*=\bar{v}e^{\epsilon\alpha_5},C^*=\bar{C}e^{\epsilon\alpha_6} \qquad (14)$$

The transformation (3.14) may be considered as a point transformation, which transformed the coordinates $(\bar{x}.\bar{y},\psi,\bar{u},\bar{v},\bar{C})$ to the coordinates$(x^*,y^*,\psi^*,u^*,v^*,C^*)$. Taking the relation (3.14) in to account in equation (12) and (13), we respectively,

$$e^{\epsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{*2}}\right)$$
$$=e^{\epsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial y^{*3}}-e^{\epsilon(\alpha_2-\alpha_3)}\frac{\nu}{k}\frac{\partial\psi^*}{\partial y^*} \qquad (15)$$

$$e^{\epsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial C^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial C^*}{\partial y^*}\right)=e^{\epsilon(2\alpha_2-\alpha_6)}\frac{\partial^2\psi^*}{\partial y^{*2}} \qquad (16)$$

Hence we get from (3.15) and (3.16)

$\alpha_1+2\alpha_2-2\alpha_3=3\alpha_2-\alpha_3=\alpha_2-\alpha_3$

$\alpha_1+\alpha_2-\alpha_3-\alpha_6=2\alpha_2-\alpha_6$

Solving $\alpha_2=0\ ,\ \alpha_1=\alpha_3\ ,\ \alpha_4=\alpha_1\ and\ \alpha_5=0$ then the transformation becomes,

$$x^*=xe^{\epsilon\alpha_1},y^*=y,\psi^*=\psi e^{\epsilon\alpha_1},u^*=ue^{\epsilon\alpha_1},v^*=v,C^*=\bar{C}e^{\epsilon\alpha_6} \qquad (17)$$

The boundary conditions become,

$$\frac{\partial\psi^*}{\partial y^*}=ax^*,\quad \frac{\partial\psi^*}{\partial x^*}=0\quad at\quad y^*=0\quad and\quad \frac{\partial\psi^*}{\partial y^*}\to 0\quad at\quad \psi^*\to\infty$$

$$C^*=1\quad at\quad y^*=0\quad and\quad C^*\to 0\quad as\quad y^*\to\infty$$

where $C^*=1$ gives $\alpha_6=0$.

3.2 Absolute invariant*:*

Firstly, we consider the absolute invariant, η which is a fuction of the independent variables and taken as

$$\eta=\bar{y}\bar{x}^s.$$

Since the quantity η is absolute invariant, we get

$\bar{y}\bar{x}^s=yx^s$

Now, $\bar{y}\bar{x}^{s} = yx^{s}e^{s\alpha_1 p} = yx^{s}$ if $p = 0$ (since α_1 cannot be 0). Hence the first invariant as $\eta = y^{*}$.

For this purpose we can write,

$x^{*} = A\bar{x}$, $A = e^{\epsilon}$, $y^{*} = \bar{y}$, $\psi^{*} = A\psi$

To establish $y^{*} = \bar{y}$ we have

$y^{*}x^{*s} = \bar{y}A^{s}\bar{x}^{s} = A^{s}(\bar{y}\bar{x}^{s})$

Putting $s = 0$ we obtain $y^{*} = \bar{y}$ thus $\eta = y^{*}$ is an invariant.

We now find second absolute invariant $G = f(\eta)$ which involves the independent variable φ.let us consider that $G = x^{-r}\psi$.

We will find r such that

$$x^{-r}\psi = x^{*r}\psi^{*}.$$

Now

$$x^{*r}\psi^{*} = (A\bar{x})^{r}A\psi = A^{r+1}\bar{x}^{r}.$$

Putting $r + 1 = 0 \implies r = -1$.

Then the second absolute invariant G is given by $G = \bar{x}^{-1}\psi^{*}$

Now putting $g = f(x)$ we can write

$\psi^{*} = x^{*}f(\eta)$

and

$$H = x^{p}\bar{C} \ , \ x^{*p}C^{*} = e^{-\epsilon(p\alpha_1)}x^{p}C \ \Rightarrow \ p = 0 \ .$$

Thus

$$H = C^{*} \ \Rightarrow \ C^{*} = \bar{\Phi}(\eta) \ , \ H = \bar{\Phi}(\eta) \ .$$

Thus from three absolute invariants, we get the transformations as given below

$$\eta = y^{*} \ , \ \psi^{*} = x^{*}f(\eta) \ , \ \bar{C} = \bar{\Phi}(\eta) \ .$$

Then the equation (3.15) and (3.16) becomes

$f'^{2} - ff'' = \upsilon f''' - \frac{\nu}{k}f'$

$0 - Scf\bar{\Phi}' = \bar{\Phi}''$

$$i,e \quad \upsilon f''' + ff'' - f'^{2} - \frac{\nu}{k}f' = 0 \tag{18}$$

$$\bar{\Phi}'' + Scf\bar{\Phi}' = 0 \tag{19}$$

Then the boundary condition become

$$f'(\eta) = a \ , \ f(\eta) = 0 \quad at \quad \eta = 0 \ , \quad f'(\eta) \to 0 \quad as \quad y \to \infty \tag{20}$$

$$\phi(\eta) = 1 \quad at \quad \eta = 0 \ , \quad \phi(\eta) \to 0 \quad as \quad y \to \infty \tag{21}$$

Again, we introduce the following transformations for η, f and ϕ in equations (18)-(21):

$$\eta = \upsilon^{\alpha'}a^{\beta'}\bar{\eta} \ , \ f = \upsilon^{\alpha''}a^{\beta''}\bar{f} \ and \ \bar{\Phi} = \upsilon^{\alpha'''}a^{\beta'''}\phi^{*} \tag{22}$$

and we obtain $\alpha' = \alpha'' = \frac{1}{2}\beta' = \frac{1}{2}$, $\beta'' = -\frac{1}{2}$ and $\alpha''' = \beta''' = 0$.

Finally, in view of the above transformations and taking

$\bar{\eta} = \eta$, $\bar{f} = f$ and $\phi^* = \phi$

the equations (18) and (19) reduce to the following forms:

$$f''' + ff'' - f'^2 - k^* f' = 0 \tag{23}$$

$$\phi'' + Scf\phi' = 0, \tag{24}$$

where $k^* = \upsilon/ka$ is the permeability parameter of the porous medium, $Sc = \upsilon/D$ is the Schmidt number.

Then the boundary conditions (20) and (21) become

$$f'(\eta) = 1,\ f(\eta) = 0 \ \text{ at } \eta = 0,\ f'(\eta) \to 0 \ \text{ as } \eta \to \infty \tag{25}$$

$$\phi(\eta) = 1 \ \text{ at } \eta = 0,\ \phi(\eta) \to 0 \ \text{ as } \ \eta \to \infty \tag{26}$$

4. Solution of the Problem

The equation (23) along with the boundary condition (25) is solved analytically (Sarpkaya, 1961) and the exact solution is given by

$$f(\eta) = \frac{1 - e^{-\sqrt{1+k^*}\,\eta}}{\sqrt{1+k^*}}, \quad \eta \geq 0. \tag{27}$$

After substitution of the function $f(\eta)$ and using finite-difference technique in the equation (24) along with the boundary conditions (26) is solved numerically. The expression for wall shear stress is given by $f''(0) = \left|\sqrt{1+k^*}\right|$ which increases with the increase of the permeability parameter k^*.

5. Results and Discussion

The analytic solution of velocity has presented for various values of the permeability parameter k^*. The concentration equation is solved numerically and the results are shown graphically for various values of permeability parameter k^* and the Schmidt number *Sc*.

The dimensionless velocity profiles $f'(\eta)$ for various values of the permeability parameter k^* are plotted in Figure1. From the figure it is noted that with increase of k^*, the velocity profile for any fixed value of η decreases. Consequently, the momentum boundary layer thickness reduces with the increase in k^*. The values of $f(\eta)$ are presented in Figure2 for various values of k^* and it is found that the value of $f(\eta)$ at a point decreases. The dimensionless concentration profiles are shown in Figure3 for various values of k^*. The value of concentration at particular value of η increases with the increase of the permeability parameter.

The concentration distribution for several values of *Sc* are plotted in Figure4 taking the permeability parameter $k^* = 1$. It is found that there is a significant effect of Schmidt number on the concentration distribution. Due to increase of Schmidt number *Sc* the concentration at point decreases substantially. Consequently the concentration boundary layer thickness reduces. For the increase of Schmidt number the diffusion coefficient decreases and so the concentration boundary layer thickness decreases.

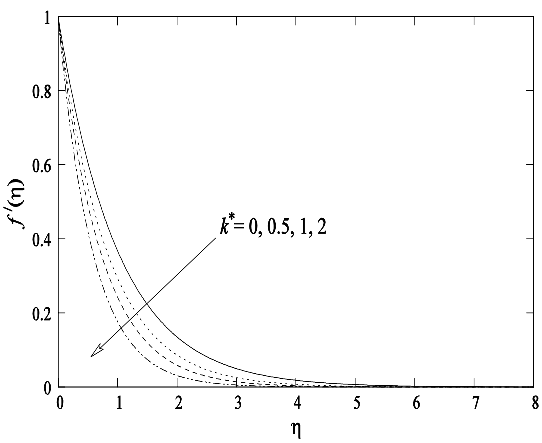

Figure 1. The velocity profile $f'(\eta)$ for increasing η for various value of k^*.

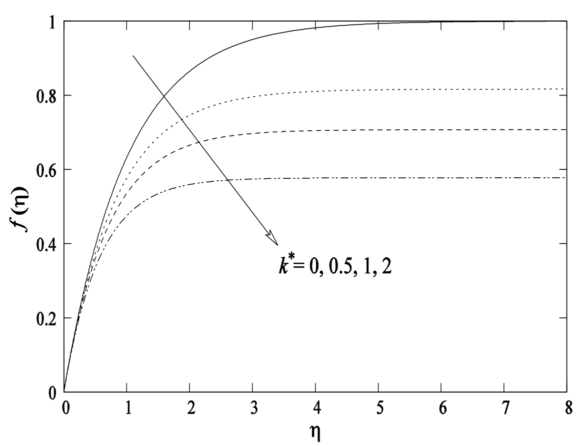

Figure 2. The values of $f(\eta)$ for increasing η for various value of k^*.

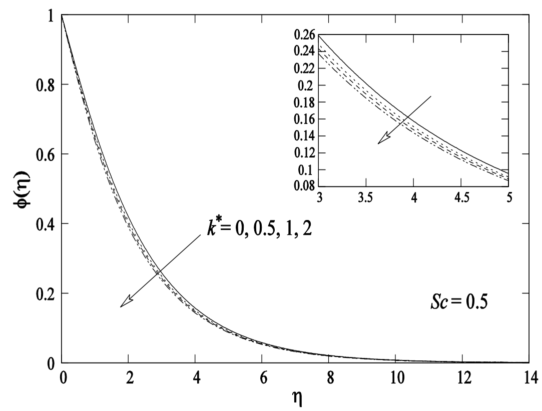

Figure3 The concentration profiles $\phi(\eta)$ for increasing η for various value of k^*.

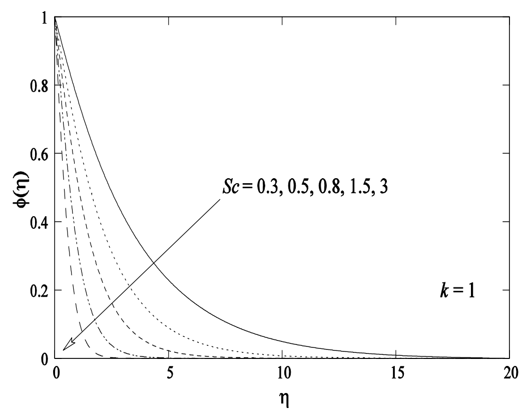

Figure4 The concentration profiles $\phi(\eta)$ for increasing η with various values of *Sc*.

6. Conclusions

The objective of the investigation is to show the applications of scaling group of transformations in boundary layer flow and mass transfer over a stretching sheet in porous medium. Using scaling group of transformations, the governing equations are transformed to the self-similar ODEs. Then those are solved. Due to increase of the permeability parameter the momentum boundary layer thickness reduces and the concentration at a point also decreases. The concentration boundary layer thickness decreases with the Schmidt number.

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