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Multiplication Operation on Trapezoidal Fuzzy Numbers

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ABSTRACT

In this paper, a methods for solving multiplication operation of two trapezoidal fuzzy numbers are given. The analytical method is most precise, but it is hard to α-cuts interval when the membership function is complicated. This method is illuminated by examples. Fuzzy numbers are used in statistics, engineering, and experimental science. The arithmetic operators on fuzzy numbers are basic content in fuzzy mathematics. Operation of fuzzy number can be generalized from that of crisp interval.

Keywords: membership function, α-cuts, Trapezoidal Fuzzy number.

1. Introduction

In1965, Zadeh [1] introduced the concept of fuzzy set theory to meet those problems. In 1978, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [2]. A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with ”ordinary” (single-valued) numbers [3-6]. Any fuzzy number can be thought of as a function whose domain is a specified set. In many respects, fuzzy numbers depict the physical world more realistically than single-valued numbers. Fuzzy numbers are used in statistics, computer programming, engineering (especially communications), and experimental science. S. M. Wang and J. Watada [6] discuss the laws of large numbers for T-independent L-R fuzzy variables based on continuous archimedean t-norm and expected value of fuzzy variable. H. M. Lee and L. Lin [7] weighted triangular fuzzy numbers to tackle the rate of aggregative risk in fuzzy circumstances during any phase of the software development life cycle. D. Sanchez, M. Delgado and M. A. Vila [8] define imprecise quantities on the basis of a new representation of imprecision introduced by the authors called RL-representation and show that the imprecision of the quantities being operated can be increased, preserved or diminished. Shan-Huo Chen and Guo-Chin Li [9] define representation, ranking, and distance of fuzzy number, and then obtain operations of scalar product, addition and multiplication, and obtain some results.

2. CONCEPT OF FUZZY NUMBER

If a fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, it is called as fuzzy number. So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy.

Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number R. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Generally a fuzzy interval is represented by two end points $\overline{r}$ and $\overline{\overline{\overline{\overline{r}}}}$ and peak points $\overline{\overline{r}}$ and $\overline{\overline{\overline{r}}}$ (Figure 1).

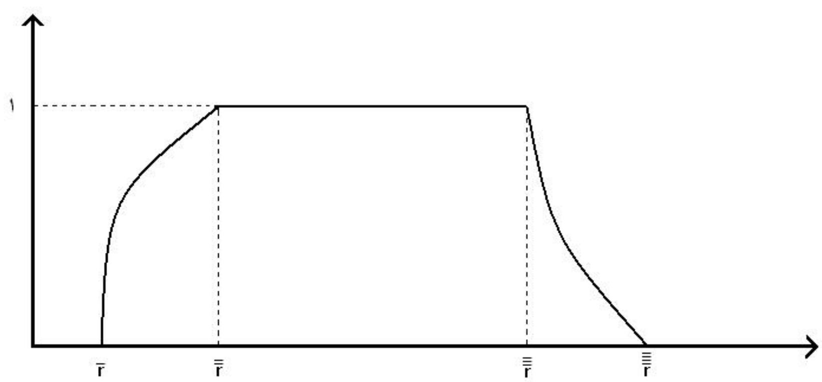

Figure 1: Trapezoidal Fuzzy number T=[$\overline{r}$, $\overline{\overline{r}}$, $\overline{\overline{\overline{r}}}$, $\overline{\overline{\overline{\overline{r}}}}$]

The α-cuts operation can be also applied to the fuzzy number. If we denote α-cuts interval for fuzzy number T as T_α , the obtained interval T_α is defined as

$$T_\alpha = \left[\overline{r}^{(\alpha)}, \overline{\overline{\overline{\overline{r}}}}^{(\alpha)} \right]$$

We can also know that it is an ordinary crisp interval (Figure 2).

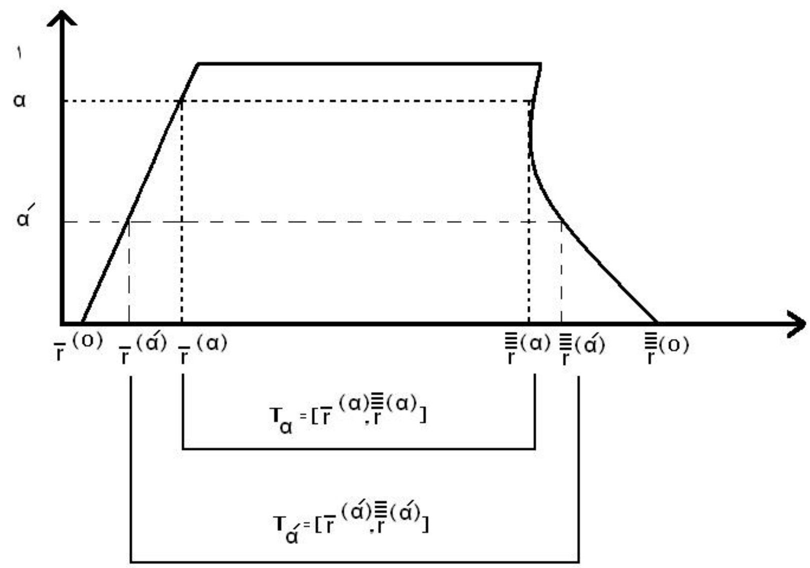

Figure 2: α-cuts of trapezoidal fuzzy number.

$$T_\alpha = \left[\overline{r}^{(\alpha)}, \overline{\overline{\overline{r}}}^{(\alpha)}\right]$$

$$(\alpha' < \alpha) \Rightarrow \left(\overline{r}^{(\alpha')} < \overline{r}^{(\alpha)}, \overline{\overline{\overline{r}}}^{(\alpha')} < \overline{\overline{\overline{r}}}^{(\alpha)}\right)$$

The convex condition may also be written as,

$$(\alpha' < \alpha) \Rightarrow (T_\alpha \subset T_{\alpha'})$$

Operation of fuzzy number can be generalized from that of crisp interval. Lets have a look at the operations of interval.

$$\forall\ \overline{r}_1, \overline{\overline{\overline{r}}}_1, \overline{r}_2, \overline{\overline{\overline{r}}}_2 \in R$$

$$T_1 = \left[\overline{r}_1, \overline{\overline{\overline{r}}}_1\right], T_2 = \left[\overline{r}_2, \overline{\overline{\overline{r}}}_2\right]$$

Assuming T_1 and T_2 as numbers expressed as interval, main operations of interval are

1- Addition

$$\left[\overline{r}_1, \overline{\overline{\overline{r}}}_1\right](+)\left[\overline{r}_2, \overline{\overline{\overline{r}}}_2\right] = \left[\overline{r}_1 + \overline{r}_2, \overline{\overline{\overline{r}}}_1 + \overline{\overline{\overline{r}}}_2\right]$$

2- Subtraction

$$\left[\overline{r}_1, \overline{\overline{\overline{r}}}_1\right](-)\left[\overline{r}_2, \overline{\overline{\overline{r}}}_2\right] = \left[\overline{r}_1 - \overline{\overline{\overline{r}}}_2, \overline{\overline{\overline{r}}}_1 - \overline{r}_2\right]$$

3- Multiplication

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](\bullet)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1\bullet\overline{r}_2\wedge\overline{r}_1\bullet\overline{\overline{\overline{r}}}_2\wedge\overline{\overline{\overline{r}}}_1\bullet\overline{r}_2\wedge\overline{\overline{\overline{r}}}_1\bullet\overline{\overline{\overline{r}}}_2,\overline{r}_1\bullet\overline{r}_2\vee\overline{r}_1\bullet\overline{\overline{\overline{r}}}_2\vee\overline{\overline{\overline{r}}}_1\bullet\overline{r}_2\vee\overline{\overline{\overline{r}}}_1\bullet\overline{\overline{\overline{r}}}_2\right]$$

4- Division

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](/)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1/\overline{r}_2\wedge\overline{r}_1/\overline{\overline{\overline{r}}}_2\wedge\overline{\overline{\overline{r}}}_1/\overline{r}_2\wedge\overline{\overline{\overline{r}}}_1/\overline{\overline{\overline{r}}}_2,\overline{r}_1/\overline{r}_2\vee\overline{r}_1/\overline{\overline{\overline{r}}}_2\vee\overline{\overline{\overline{r}}}_1/\overline{r}_2\vee\overline{\overline{\overline{r}}}_1/\overline{\overline{\overline{r}}}_2\right]$$

5- Inverse interval

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right]^{-1}=\left[1/\overline{r}_1\wedge 1/\overline{\overline{\overline{r}}}_1,1/\overline{r}_1\vee 1/\overline{\overline{\overline{r}}}_1\right]$$

excluding the case $\overline{r}_1=0$ or $\overline{\overline{\overline{r}}}_1=0$.

When previous sets A and B is defined in the positive real number R^+, the operations of multiplication, division, and inverse interval are written as,

$3'$- Multiplication

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](\bullet)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1\bullet\overline{r}_2,\overline{\overline{\overline{r}}}_1\bullet\overline{\overline{\overline{r}}}_2\right]$$

$4'$- Division

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](/)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1/\overline{\overline{\overline{r}}}_2,\overline{\overline{\overline{r}}}_1/\overline{r}_2\right]$$

$5'$- Inverse interval

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right]^{-1}=\left[1/\overline{\overline{\overline{r}}}_1,1/\overline{r}_1\right]$$

6- Minimum

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](\wedge)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1\wedge\overline{r}_2,\overline{\overline{\overline{r}}}_1\wedge\overline{\overline{\overline{r}}}_2\right]$$

7- Maximum

$$\left[\overline{r}_1,\overline{\overline{\overline{r}}}_1\right](\vee)\left[\overline{r}_2,\overline{\overline{\overline{r}}}_2\right]=\left[\overline{r}_1\vee\overline{r}_2,\overline{\overline{\overline{r}}}_1\vee\overline{\overline{\overline{r}}}_2\right]$$

Example 1. There are two intervals T_1 and T_2,

$T_1=[4,5],T_2=[-3,8]$, Then following operation might be set.

$T_1(+)T_2=[4-3,5+8]=[1,13]$

$T_1(-)T_2=[4-(-3),5-8]=[7,-3]$

$T_1(\bullet)T_2=[4\bullet(-3)\wedge 4\bullet 8\wedge 5\bullet(-3)\wedge 5\bullet 8,4\bullet(-3)\vee 4\bullet 8\vee 5\bullet(-3)\vee 5\bullet 8_2]=[-15,40]$

$T_1(/)T_2=[4/(-3)\wedge 4/8\wedge 5/(-3)\wedge 5/8,4/(-3)\vee 4/8\vee 5/(-3)\vee 5/8_2]=[-1,66,0.625]$

$T_1^{-1}=[4,5]^{-1}=[1/5,1/4]$

$T_2^{-1} = [-3,8]^{-1} = [1/(-3) \wedge 1/8, 1/(-3) \vee 1/8]$

$T_1 (\wedge) T_2 = [4 \wedge (-3), 5 \wedge 8] = [-3,5]$

$T_1 (\vee) T_2 = [4 \vee (-3), 5 \vee 8] = [4,8]$

3. ANALYTICAL METHOD

Among the various shapes of fuzzy number, trapezoidal fuzzy number (TFN) is the most popular one. It is a fuzzy number represented with four points as follows:

$$T = \left[\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}} \right]$$

This representation is interpreted as membership function (Figure 3).

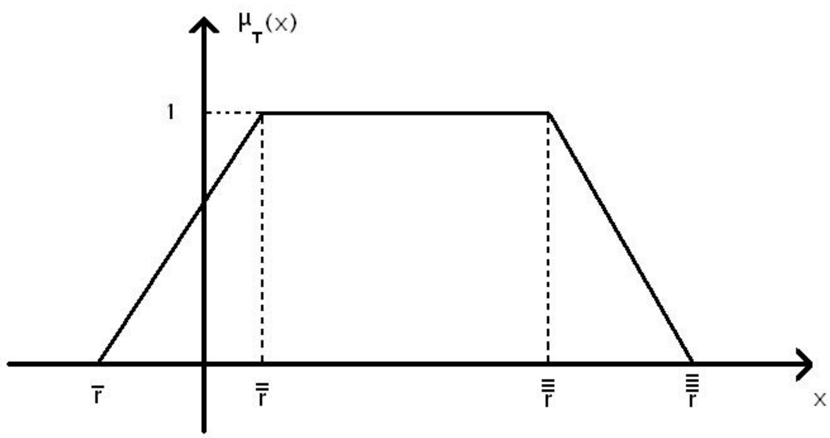

Figure 3: Trapezoidal fuzzy number $T = \left[\bar{r}, \bar{\bar{r}}, \bar{\bar{\bar{r}}}, \bar{\bar{\bar{\bar{r}}}} \right]$.

$$\mu_T(x) = \begin{cases} 0 & if \quad x < \bar{r} \\ x - \bar{r} / \bar{\bar{r}} - \bar{r} & if \quad \bar{r} \le x < \bar{\bar{r}} \\ 1 & if \quad \bar{\bar{r}} \le x < \bar{\bar{\bar{r}}} \\ \bar{\bar{\bar{\bar{r}}}} - x / \bar{\bar{\bar{\bar{r}}}} - \bar{\bar{\bar{r}}} & if \quad \bar{\bar{\bar{r}}} \le x < \bar{\bar{\bar{\bar{r}}}} \\ 0 & if \quad \bar{\bar{\bar{\bar{r}}}} < x \end{cases}$$

Suppose trapezoidal fuzzy numbers $\tilde{T}_1$ and $\tilde{T}_2$ are defined as,

$$\tilde{T}_1 = \left[\bar{r}_1, \bar{\bar{r}}_1, \bar{\bar{\bar{r}}}_1, \bar{\bar{\bar{\bar{r}}}}_1\right]$$

$$\tilde{T}_2 = \left[\bar{r}_2, \bar{\bar{r}}_2, \bar{\bar{\bar{r}}}_2, \bar{\bar{\bar{\bar{r}}}}_2\right]$$

We have following conditions.

A) The results from addition or subtraction between trapezoidal fuzzy numbers result also triangular fuzzy numbers.
B) The results from multiplication or division are not trapezoidal fuzzy numbers.
C) Max or min operation does not give trapezoidal fuzzy number.

$$\tilde{T}_1(+)\tilde{T}_2 = \left(\bar{r}_1 + \bar{r}_2, \bar{\bar{r}}_1 + \bar{\bar{r}}_2, \bar{\bar{\bar{r}}}_1 + \bar{\bar{\bar{r}}}_2, \bar{\bar{\bar{\bar{r}}}}_1 + \bar{\bar{\bar{\bar{r}}}}_2\right)$$

$$\tilde{T}_1(-)\tilde{T}_2 = \left(\bar{r}_1 - \bar{\bar{r}}_2, \bar{\bar{r}}_1 - \bar{r}_2, \bar{\bar{\bar{r}}}_1 + \bar{\bar{\bar{r}}}_2, \bar{\bar{\bar{\bar{r}}}}_1 + \bar{\bar{\bar{\bar{r}}}}_2\right)$$

$$-\left(\tilde{T}_2\right) = \left(-\bar{\bar{r}}_2, -\bar{r}_2, \bar{\bar{\bar{\bar{r}}}}_2, \bar{\bar{\bar{r}}}_2\right)$$

$$T_{1\ \alpha} = \left[t_{1\alpha}^{L}, t_{1\alpha}^{R}\right] = \left[\bar{r}_1 + \left(\bar{\bar{r}}_1 - \bar{r}_1\right)\alpha, \bar{\bar{\bar{\bar{r}}}}_1 + \left(\bar{\bar{\bar{r}}}_1 - \bar{\bar{\bar{\bar{r}}}}_1\right)\alpha\right]$$

$$T_{2\ \alpha} = \left[t_{2\alpha}^{L}, t_{2\alpha}^{R}\right] = \left[\bar{r}_2 + \left(\bar{\bar{r}}_2 - \bar{r}_2\right)\alpha, \bar{\bar{\bar{\bar{r}}}}_2 + \left(\bar{\bar{\bar{r}}}_2 - \bar{\bar{\bar{\bar{r}}}}_2\right)\alpha\right]$$

α-cuts of $\tilde{W} = \tilde{T}_1(\times)\tilde{T}_2$ is

$$\left[t_{1\alpha}^{L}, t_{1\alpha}^{R}\right] \times \left[t_{2\alpha}^{L}, t_{2\alpha}^{R}\right] = \left[w_{\alpha}^{L}, w_{\alpha}^{R}\right]$$

Where

$$w_{\alpha}^{L} = \min\left\{t_{1\alpha}^{L} t_{1\alpha}^{L}, t_{1\alpha}^{L} t_{2\alpha}^{R}, t_{1\alpha}^{R} t_{2\alpha}^{L}, t_{1\alpha}^{R} t_{2\alpha}^{R}\right\}$$

And

$$w_{\alpha}^{R} = \max\left\{t_{1\alpha}^{L} t_{1\alpha}^{L}, t_{1\alpha}^{L} t_{2\alpha}^{R}, t_{1\alpha}^{R} t_{2\alpha}^{L}, t_{1\alpha}^{R} t_{2\alpha}^{R}\right\}.$$

We suppose $\bar{\bar{\bar{\bar{r}}}}_1 \geq \bar{\bar{\bar{r}}}_1 \geq \bar{\bar{r}}_1 \geq \bar{r}_1$ and $\bar{\bar{\bar{\bar{r}}}}_2 \geq \bar{\bar{\bar{r}}}_2 \geq \bar{\bar{r}}_2 \geq \bar{r}_2$ firstly.

$$t_{1\alpha}^{L} = \bar{r}_1 + \left(\bar{\bar{r}}_1 - \bar{r}_1\right)\alpha,$$

$$t_{1\alpha}^{R} = \bar{\bar{\bar{\bar{r}}}}_1 + \left(\bar{\bar{\bar{r}}}_1 - \bar{\bar{\bar{\bar{r}}}}_1\right)\alpha,$$

$$t_{2\alpha}^{L} = \bar{r}_2 + \left(\bar{\bar{r}}_2 - \bar{r}_2\right)\alpha,$$

$$t_{2\alpha}^{R} = \bar{\bar{\bar{\bar{r}}}}_2 + \left(\bar{\bar{\bar{r}}}_2 - \bar{\bar{\bar{\bar{r}}}}_2\right)\alpha$$

$$w_{\alpha}^{L}=\min\left\{t_{1\alpha}^{L}t_{1\alpha}^{L},t_{1\alpha}^{L}t_{2\alpha}^{R},t_{1\alpha}^{R}t_{2\alpha}^{L},t_{1\alpha}^{R}t_{2\alpha}^{R}\right\}=t_{1\alpha}^{L}t_{2\alpha}^{L}$$

$$=\left(\overline{\overline{r}}_1-\overline{r}_1\right)\left(\overline{\overline{r}}_2-\overline{r}_2\right)\alpha^2+\left(\overline{r}_1\overline{\overline{r}}_2+\overline{\overline{r}}_1\overline{r}_2-2\overline{r}_1\overline{r}_2\right)\alpha+\overline{r}_1\overline{r}_2$$

Substituting $w_{\alpha}^{L}=g$, we get

$$\alpha=\frac{-\left(\overline{r}_1\overline{\overline{r}}_2+\overline{\overline{r}}_1\overline{r}_2-2\overline{r}_1\overline{r}_2\right)+\sqrt{\left(\overline{r}_1\overline{\overline{r}}_2-\overline{\overline{r}}_1\overline{r}_2\right)^2+4\left(\overline{\overline{r}}_1-\overline{r}_1\right)\left(\overline{\overline{r}}_2-\overline{r}_2\right)g}}{2\left(\overline{\overline{r}}_1-\overline{r}_1\right)\left(\overline{\overline{r}}_2-\overline{r}_2\right)}\qquad\left(omit\ \ \alpha<0\right)$$

$$w_{\alpha}^{R}=\max\left\{t_{1\alpha}^{L}t_{1\alpha}^{L},t_{1\alpha}^{L}t_{2\alpha}^{R},t_{1\alpha}^{R}t_{2\alpha}^{L},t_{1\alpha}^{R}t_{2\alpha}^{R}\right\}=t_{1\alpha}^{R}t_{2\alpha}^{R}$$

$$=\left(\overline{\overline{\overline{r}}}_1-\overline{\overline{\overline{\overline{r}}}}_1\right)\left(\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{\overline{r}}}}_2\right)\alpha^2+\left(\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{r}}}_2+\overline{\overline{\overline{r}}}_1\overline{\overline{\overline{\overline{r}}}}_2-2\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{\overline{r}}}}_2\right)\alpha+\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{\overline{r}}}}_2$$

Substituting $w_{\alpha}^{R}=g$, we get

$$\alpha=\frac{-\left(\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{r}}}_2+\overline{\overline{\overline{r}}}_1\overline{\overline{\overline{\overline{r}}}}_2-2\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{\overline{r}}}}_2\right)+\sqrt{\left(\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{r}}}_1\overline{\overline{\overline{\overline{r}}}}_2\right)^2+4\left(\overline{\overline{\overline{r}}}_1-\overline{\overline{\overline{\overline{r}}}}_1\right)\left(\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{\overline{r}}}}_2\right)g}}{2\left(\overline{\overline{\overline{r}}}_1-\overline{\overline{\overline{\overline{r}}}}_1\right)\left(\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{\overline{r}}}}_2\right)}\qquad\left(omit\ \ \alpha>1\right)$$

Hence membership function of $\tilde{W}=\tilde{T}_1(\times)\tilde{T}_2$ is

$$\mu_{\tilde{W}}(g)=\begin{cases}\dfrac{-\left(\overline{r}_1\overline{\overline{r}}_2+\overline{\overline{r}}_1\overline{r}_2-2\overline{r}_1\overline{r}_2\right)+\sqrt{\left(\overline{r}_1\overline{\overline{r}}_2-\overline{\overline{r}}_1\overline{r}_2\right)^2+4\left(\overline{\overline{r}}_1-\overline{r}_1\right)\left(\overline{\overline{r}}_2-\overline{r}_2\right)g}}{2\left(\overline{\overline{r}}_1-\overline{r}_1\right)\left(\overline{\overline{r}}_2-\overline{r}_2\right)} & \overline{r}_1\overline{r}_2\le g\le\overline{\overline{r}}_1\overline{\overline{r}}_2\\ \dfrac{-\left(\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{r}}}_2+\overline{\overline{\overline{r}}}_1\overline{\overline{\overline{\overline{r}}}}_2-2\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{\overline{r}}}}_2\right)+\sqrt{\left(\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{r}}}_1\overline{\overline{\overline{\overline{r}}}}_2\right)^2+4\left(\overline{\overline{\overline{r}}}_1-\overline{\overline{\overline{\overline{r}}}}_1\right)\left(\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{\overline{r}}}}_2\right)g}}{2\left(\overline{\overline{\overline{r}}}_1-\overline{\overline{\overline{\overline{r}}}}_1\right)\left(\overline{\overline{\overline{r}}}_2-\overline{\overline{\overline{\overline{r}}}}_2\right)} & \overline{\overline{\overline{r}}}_1\overline{\overline{\overline{r}}}_2\le g\le\overline{\overline{\overline{\overline{r}}}}_1\overline{\overline{\overline{\overline{r}}}}_2\end{cases}\qquad(*)$$

In otherwise=0.

Example 2. Suppose $\tilde{A}=(1,2,4,5),\tilde{B}=(2,3,5,6)$, calculate membership function of $\tilde{W}=\tilde{A}(\times)\tilde{B}$.

Substituting $\overline{r}_1=1,\overline{\overline{r}}_1=2,\overline{\overline{\overline{r}}}_1=4,\overline{\overline{\overline{\overline{r}}}}_1=5,\overline{r}_2=2,\overline{\overline{r}}_2=3,\overline{\overline{\overline{r}}}_2=5,\overline{\overline{\overline{\overline{r}}}}_2=6$ into equation $(*)$, the result is the following expression

$$\mu_{\tilde{W}}(g)=\begin{cases}\dfrac{1}{2}\left(-3+\sqrt{1+4g}\right) & 2\le g\le 6\\ \dfrac{1}{2}\left(11+\sqrt{1+4g}\right) & 20\le g\le 30\end{cases}$$

In otherwise=0.

The membership functions of $\tilde{A} = (1,2,4,5), \tilde{B} = (2,3,5,6)$ are shown in Figure 4 and Figure 5. The membership function of $\tilde{W} = \tilde{A}(\times)\tilde{B}$ is shown in Figure 6.

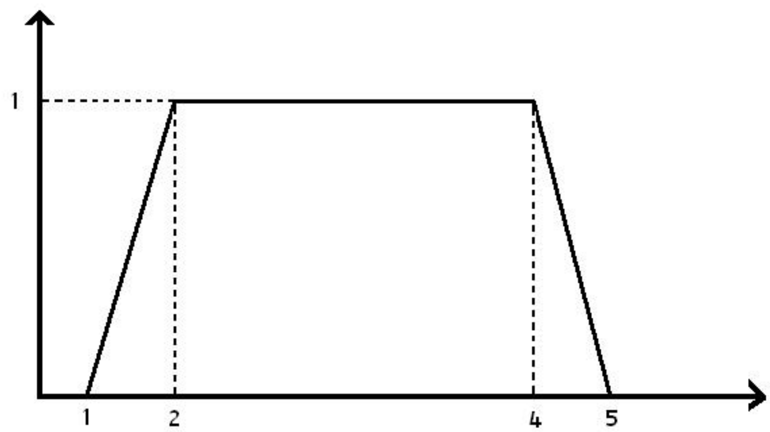

Figure 4: Membership function of $\tilde{A} = (1,2,4,5)$.

The situation is complicated when it does not satisfy $\overline{\overline{\overline{\overline{r_1}}}} \geq \overline{\overline{\overline{r_1}}} \geq \overline{\overline{\overline{r_1}}} \geq \overline{r_1}$, $\overline{\overline{\overline{\overline{r_2}}}} \geq \overline{\overline{\overline{r_2}}} \geq \overline{\overline{\overline{r_2}}} \geq \overline{r_2}$. The exact analytic equations are hard to get.

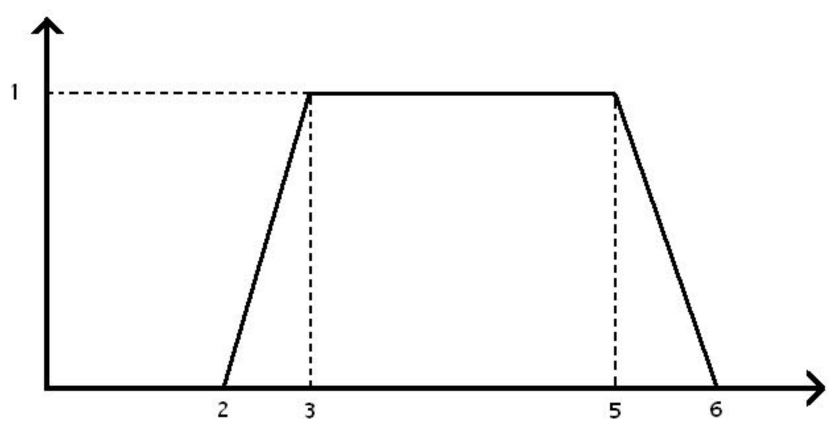

Figure 5: Membership function of $\tilde{B} = (2,3,5,6)$.

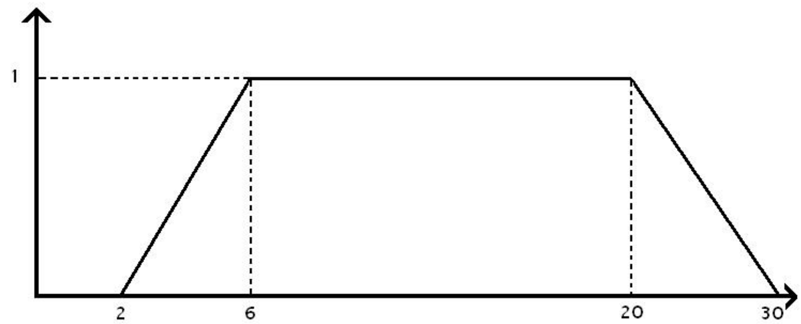

Figure 6: Membership function of $\tilde{W} = \tilde{A}\,(\times)\,\tilde{B}$ by analytical method .

4. Multiplication of General Fuzzy Number

As multiplication of fuzzy number, we can also multiply scalar value to , α-cuts interval of fuzzy number.

Example 3. Suppose that the membership of T_1, T_2 are $\mu_{\tilde{T}_1}(x) = e^{1-(x-1)^2}$ and $\mu_{\tilde{T}_2}(x) = e^{1-(y+1)^2}$. so

α-cuts of $\tilde{T}_1$ is

$$\left[t_{1\alpha}^{L}, t_{1\alpha}^{R}\right] = \left[1-\sqrt{1-\ln\ \alpha}, 1+\sqrt{1-\ln\ \alpha}\right]$$

α-cuts of $\tilde{T}_2$ is

$$\left[t_{2\alpha}^{L}, t_{2\alpha}^{R}\right] = \left[-1-\sqrt{1-\ln\ \alpha}, -1+\sqrt{1-\ln\ \alpha}\right]$$

Where

$$w_{\alpha}^{L} = \min\left\{t_{1\alpha}^{L}t_{1\alpha}^{L}, t_{1\alpha}^{L}t_{2\alpha}^{R}, t_{1\alpha}^{R}t_{2\alpha}^{L}, t_{1\alpha}^{R}t_{2\alpha}^{R}\right\} = -2+\ln\ \alpha - 2\sqrt{1-\ln\ \alpha},$$

$$w_{\alpha}^{R} = \max\left\{t_{1\alpha}^{L}t_{1\alpha}^{L}, t_{1\alpha}^{L}t_{2\alpha}^{R}, t_{1\alpha}^{R}t_{2\alpha}^{L}, t_{1\alpha}^{R}t_{2\alpha}^{R}\right\} = \begin{cases} -\ln\ \alpha & 0 \le \alpha \le e^{-1} \\ -2+\ln\ \alpha + 2\sqrt{1-\ln\ \alpha} & e^{-1} < \alpha \le 1 \end{cases}$$

Substituting $w_{\alpha}^{R} = g$, we get

$$\sqrt{1-\ln\ \alpha} = -1+\sqrt{-g} \qquad \left(omit\ \sqrt{1-\ln\ \alpha} < 0\right)$$

Therefore

$$1-\ln\ \alpha = \left(-1+\sqrt{-g}\right)^2 \Rightarrow \ln\ \alpha = 1-\left(-1+\sqrt{-g}\right)^2 \Rightarrow \alpha = e^{1-\left(-1+\sqrt{-g}\right)^2}$$

When $0 \le \alpha \le e^{-1}$, Substituting $w_{\alpha}^{R} = g$, we get

$$\alpha = e^{-g}$$

When $e^{-1} < \alpha \le 1$, Substituting $w_{\alpha}^{R} = g$, we get

$$\sqrt{1-\ln\ \alpha} = -1-\sqrt{-g} \qquad \left(omit\ \ \alpha < -1\right)$$

So

$$1-\ln\ \alpha = \left(-1-\sqrt{-g}\right)^2 \Rightarrow \ln\ \alpha = 1-\left(-1-\sqrt{-g}\right)^2 \Rightarrow \alpha = e^{1-\left(-1-\sqrt{-g}\right)^2}$$

Therefore

$$\alpha = \begin{cases} e^{1-\left(-1+\sqrt{-g}\right)^2} & g \le 0 \\ e^{-g} & g > 0 \end{cases}.$$

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