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Estimation of Generalized Rayleigh Components Reliability in a Parallel System Using Dependent Masked Data

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ABSTRACT

When the lifetimes data from the parallel system are masked and the masking probability is dependent with the failure component, we consider the reliability estimations of generalized Rayleigh components in the parallel system. Based on the given masking level and masking probability ratio, Maximum Likelihood Estimations (MLE) and Bayes estimations of parameters are obtained respectively. At last, numerical simulation demonstrates the correctness of theoretical results. We study the influence of masking level on the accuracy of the estimations, and then compare the effect of MLEs and Bayes estimations under non-informative priors and conjugate priors.

Keywords: reliability analysis; dependent masked data; MLE; Bayes estimation; generalized Rayleigh distribution.

1. Introduction

In the field of system reliability analysis, estimations of components reliabilities in a system are often obtained through system life test data. These estimations are very useful since they reflect the actual operational capacity of individual components in system environment. Then we can predict the further reliabilities of components in some new systems, see [1]. System life test data generally consists of system failure time and information on the exact component causing the system failure. In practice, however, the true component responsible for the failure of system is sometimes unknown due to various reasons such as the constraints of cost and time. Therefore, the cause of system failure is masked.

Recently, estimating component reliability based on the masked data has been considered by several authors. Usher and Hodgson[1] firstly proposed the concept of masked data and obtained likelihood function of the life test sample. Literature [2-4] derived the maximum likelihood and Bayes estimates of components' parameters and reliabilities in the series system when the lifetime distributions of components are Weibull, Pareto and Geometric separately. Sarhan[5] extended the reliability analysis of masked data in the parallel case, while the reliability estimating of verse Weibull component was discussed in a parallel system by Zhang[6].

In previously developed models, most papers made an equiprobable assumption for the masking probabilities where the conditional masking probabilities are equal in each cause of failure from the masked set. Under certain circumstances, however, the equiprobable assumption may not meet practical requirement. To avoid the limitation, Lin & Guess[7] presented a proportional probability model for a dependent masked probability in the series system. Based on this model, reliability estimations of Geometric and Pareto components from series system are studied by Sarhan[8] and Xu[9][10], respectively. But there are not any papers associated with dependent masked data in parallel system.

Except for the above distribution, generalized Rayleigh(GR) distribution initially introduced by Surles and Padgett[11], also has wide applications in reliability analysis. GR distribution which can be defined as two-parameter Burr Type X distribution is a particular member of the exponentiated Weibull distribution. Debasis[12] provided the different methods of estimations for GR distribution. However, the discussion of GR components in reliability analysis with masked data is original. In this paper, we mainly consider the estimations of parameters and reliabilities in GR components connected in parallel system using dependent masked data. Both MLE and Bayes approaches are utilized and their results are compared in the simulation.

2. Basic assumptions and maximum likelihood estimation

In order to obtain the results, we set the following assumptions throughout the paper.

1. The system is made up of m independent but non-identical components linked in parallel. This system will fail if and only if all of its components fail.
2. The situation that any two components fail simultaneously is not considered.
3. n identical systems join in the life test. The test is terminated when all systems have failed.
4. The random variables $T_{ij}(i=1,2,\ldots,n, j=1,2,\ldots,m)$ denote the lifetime of the j-th component in the i-th system. So the lifetime of the i-th system is $T_i = \max(T_{i1}, T_{i2}, \cdots, \quad T_{iJ})$, $(i=1,2,\cdots,n)$. T_{ij} are independent with $T_{1j}, T_{2j},, T_{nj}$ being identically distributed and obeying GR distributions with parameters θ_j, α_j.
5. After finishing the life test, we can obtain the observed data $(t_1,S_1),(t_2,S_2),\cdots,(t_n,S_n)$, where $S_i(i=1,\cdots,n)$ express the set of possible failure

cause in i -th system. K_i denote the true cause in i -th system.

6. The masking probability is dependent on the component that leads system to fail and independent with the failure time.

According to assumption 4, we get the probability density, distribution function and reliability of the components as following:

$$f_j(t) = 2\theta_j \lambda_j^2 t e^{-(\lambda_j t)^2} (1 - e^{-(\lambda_j t)^2})^{\theta_j - 1}, \ \theta_j > 0, \lambda_j > 0, \tag{1}$$

$$F_j(t) = (1 - e^{-(\lambda_j t)^2})^{\theta_j}, \ \theta_j > 0, \lambda_j > 0, \tag{2}$$

$$R_j(t) = 1 - (1 - e^{-(\lambda_j t)^2})^{\theta_j}, \ \theta_j > 0, \lambda_j > 0. \tag{3}$$

Here θ_j and λ_j are the shape and scale parameters, respectively. The two-parameter GR distribution will be denoted by $GR(\theta, \lambda)$.

Based on the assumptions 1 to 5, when masking is independent of failure cause, Sarhan[5] derived the full likelihood

$$L' = \prod_{i=1}^{n} [\sum_{j \in s_i} f_j(t_i) \prod_{l=1, l \neq j}^{m} F_l(t_i)].$$

To relax the independent condition, we consider a conditional probability called masking probability $P(S_i = s_i | T_i = t_i, K_i = j)$. The observed subset is s_i given that system i fails at time t_i, where the true cause is component j. Then the full likelihood is[7]

$$L = \prod_{i=1}^{n} \{\sum_{j \in s_i} [f_j(t_i) \prod_{l=1, l \neq j}^{m} F_l(t_i)] P(S_i = s_i | T_i = t_i, K_i = j)\}. \tag{4}$$

The model built in this paper is under the assumption that the masking probabilities are independent of the failure time, but dependent with the component. Therefore, for any $j \neq j' \in s_i$, we have

$$P(S_i = s_i | T_i = t_i, K_i = j) \neq P(S_i = s_i | T_i = t_i, K_i = j').$$

To illustrate the process of having the estimations in this paper and simplify the computation, we study the problem on two-component parallel systems $(m = 2)$, and assume the parameter $\lambda_1 = \lambda_2 = \lambda$ be known. Let

$$P(S_i = \{1\} | T_i = t_i, K_i = 1) = p_1,$$

$$P(S_i = \{2\} | T_i = t_i, K_i = 2) = p_2,$$

$$P(S_i = \{1,2\} | T_i = t_i, K_i = 1) = 1 - p_1 = p_3,$$

$$P(S_i = \{1,2\} | T_i = t_i, K_i = 2) = 1 - p_2 = p_4.$$

Let $c = p_4 / p_3$ which is called the masking probability ratio. Note that for $c = 1$ the model turn to special case of independent masking, while for $c \neq 1$ that is dependent masking case. Apparently, $c > (<) 1$ when $p_4 > (<) p_3$. Without loss of generality, $0 < c \leq 1$ is assumed.

Simultaneously, let n_1 and n_2 be the numbers of system failures for which the failure cause is known to be component 1 and 2 respectively, while n_{12} denotes the number of failed systems that the cause is not directly observed. That is, $n_j (j=1,2)$ is the number of observations when $S_i = \{j\}$ and n_{12} is the number of observations when $S_i = \{1,2\}$. Note that $n = n_1 + n_2 + n_{12}$. According to Eq. (1) (2) (4), the full likelihood becomes

$$\begin{aligned} L &= 2^n \lambda^{2n} \prod_{i=1}^{n} t_i \exp\{-\sum_{i=1}^{n} (\lambda t_i)^2\} \prod_{i=1}^{n} (1-\exp\{-(\lambda t_i)^2\})^{\theta_1+\theta_2-1} (p_1\theta_1)^{n_1} (p_2\theta_2)^{n_2} (p_3\theta_1 + p_4\theta_2)^{n_{12}} \\ &= 2^n \lambda^{2n} p_1^{n_1} p_2^{n_2} \prod_{i=1}^{n} t_i \exp\{-\sum_{i=1}^{n} (\lambda t_i)^2\} \prod_{i=1}^{n} (1-\exp\{-(\lambda t_i)^2\})^{\theta_1+\theta_2-1} \theta_1^{n_1} \theta_2^{n_2} (\theta_1 + c\theta_2)^{n_{12}} \\ &= A\theta_1^{n_1} \theta_2^{n_2} (\theta_1 + c\theta_2)^{n_{12}} \exp\{(\theta_1 + \theta_2) \sum_{i=1}^{n} \mathrm{In}(1-\exp\{-(\lambda t_i)^2\})\} \\ &= A\theta_1^{n_1} \theta_2^{n_2} (\theta_1 + c\theta_2)^{n_{12}} \exp(\theta_1 T + \theta_2 T), \end{aligned} \tag{5}$$

Where

$$A = 2^n \lambda^{2n} p_1^{n_1} p_2^{n_2} \exp\{-\sum_{i=1}^{n} (\lambda t_i)^2\} \prod_{i=1}^{n} t_i \prod_{i=1}^{n} (1-\exp\{-(\lambda t_i)^2\})^{-1}, T = \sum_{i=1}^{n} \mathrm{In}(1-\exp\{-(\lambda t_i)^2\}).$$

The MLEs of θ_1, θ_2 can be obtained by maximizing the likelihood function given by formula (5), with respect to θ_1, θ_2.

One can derive the log-likelihood function as following

$$\mathrm{In}L = \mathrm{In}A + n_1 \mathrm{In}\theta_1 + n_2 \mathrm{In}\theta_2 + n_{12} \mathrm{In}(\theta_1 + c\theta_2) + (\theta_1 T + \theta_2 T).$$

Thus, by setting the derivative zero, the likelihood equations can be obtained as

$$\frac{\partial \mathrm{In}L}{\partial \theta_1} = \frac{n_1}{\theta_1} + \frac{n_{12}}{\theta_1 + c\theta_2} + T = 0, \tag{6}$$

$$\frac{\partial \mathrm{In}L}{\partial \theta_2} = \frac{n_2}{\theta_2} + \frac{cn_{12}}{\theta_1 + c\theta_2} + T = 0. \tag{7}$$

Solving the equations above with respect to θ_1, θ_2, we can directly get

$$\hat{\theta}_2 = \frac{n_2 \hat{\theta}_1}{cn_1 + cT\hat{\theta}_1 - T\hat{\theta}_{11}}. \tag{8}$$

Substituting (8) into (6), we find that θ_1 satisfies the following equation

$$k_1 \hat{\theta}_1^2 + k_2 \hat{\theta}_1 + k_3 = 0, \tag{9}$$

where $k_1 = (c-1)T^2, k_2 = (cn + cn_1 - n_1 - n_{12})T, k_3 = cn_1 n$.

It is easy to get the MLE of θ_1 by solving the Eq. (9), say $\hat{\theta}_1$. Then we can derive the MLE of θ_2 using $\hat{\theta}_1$ and Eq. (8).

Once the MLEs of the unknown parameters θ_1 and θ_2 are obtained, we can get the MLEs of the reliability based on Eq. (3) by replacing the parameters θ_1, θ_2 with their MLEs $\hat{\theta}_1, \hat{\theta}_2$, thanks to invariance property of maximum likelihood estimation.

3. Bayes estimation

One of the limitations of the MLE method is that, the MLEs of $\theta_j (j=1,2)$ in the section 2 are not obtained for the cases in which we have a completely masked level. Therefore, Bayes approach is considered in this section, as Bayes procedure enables us to derive the estimations of unknown parameters and reliability functions. Further, as we shall see, it provides better estimators in the sense of smaller mean squared error.

To present the Bayes analysis, the prior distributions of the parameters θ_1, θ_2 should be considered initially. In this paper, we choose the conjugate prior distributions of θ_1 , θ_2 for their generality and universality. Therefore, the prior probability density function (pdf) takes the following form

$$\pi(\theta_j) = \frac{b_j^{a_j}}{\Gamma(a_j)} \theta_j^{a_j - 1} \mathrm{e}^{-b_j \theta_j}, (\theta_j > 0, a_j > 0, b_j > 0),$$

where $a_j, b_j (j=1,2)$ called hyperparameters is given by historical data or experience of experts.When $a_j = 0, b_j = 0$, we get the standard non-informative priors $\pi(\theta_j) = 1/\theta_j$.

Hence the joint prior pdf of θ_1, θ_2 is

$$\pi(\theta_1, \theta_2) = \frac{b_1^{a_1} b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \theta_1^{a_1 - 1} \mathrm{e}^{-b_1\theta_1} \theta_2^{a_2 - 1} \mathrm{e}^{-b_2\theta_2} . \tag{10}$$

Then carry out binomial expansion for formula (5), we have

$$L = A\sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k} \theta_1^{n_1+k} \theta_2^{n_2+n_{12}-k} \exp(\theta_1 T + \theta_2 T), \tag{11}$$

Once the likelihood function, see (11), and the joint prior pdf, see (10), are constructed, we can derive the joint posterior pdf of θ_1, θ_2 as following

$$\pi(\theta_1, \theta_2 \,|data) = I^{-1} \sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k} \theta^{A_1 - 1} \mathrm{e}^{(T-b_1)\theta_1} \theta^{A_2 - 1} \mathrm{e}^{(T-b_2)\theta_2}, \tag{12}$$

where $A_1 = n_1 + a_1 + k, A_2 = n_2 + a_2 + n_{12} - k$, $I = \sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k} \dfrac{\Gamma(A_1)\Gamma(A_2)}{(b_1 - T)^{A_1} (b_2 - T)^{A_2}}$.

Thus the marginal posterior pdf of θ_1, θ_2 can be formulated from (12) as in the following respective relations

$$\pi(\theta_1 \,|data) = I^{-1} \sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k} \theta^{A_1 - 1} \mathrm{e}^{(T-b_1)\theta_1} \Gamma(A_2) / (b_2 - T)^{A_2} ,$$

$$\pi(\theta_1 \,|data) = I^{-1} \sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k} \theta^{A_2 - 1} \mathrm{e}^{(T-b_2)\theta_2} \Gamma(A_1) / (b_1 - T)^{A_1} .$$

Under the square error loss function $L(\theta,d)=(d-\theta)^2$, the Bayes estimators of θ_1,θ_2 are their posterior means, that is $\hat{\theta}_j = E(\theta_j \left| data\right.)$. Hence the Bayes estimators of θ_1,θ_2 are

$$\hat{\theta}_1 = E(\theta_1 \left| data\right.) = \int_0^{+\infty} \theta_1 \pi(\theta_1 \left| data\right.)\mathrm{d}\theta_1 = I^{-1}\sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k}\Gamma(A_1+1)(b_1-T)^{-A_1-1}\Gamma(A_2)(b_2-T)^{-A_2}$$

$$\hat{\theta}_2 = E(\theta_2 \left| data\right.) = \int_0^{+\infty} \theta_2 \pi(\theta_2 \left| data\right.)\mathrm{d}\theta_2 = I^{-1}\sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k}\Gamma(A_2+1)(b_2-T)^{-A_2-1}\Gamma(A_1)(b_1-T)^{-A_1}$$

Similarly, we can obtain the Bayes estimators for the reliability R_1,R_2 below

$$\begin{aligned}\hat{R}_1 &= E(1-(1-\mathrm{e}^{-(\lambda t_0)^2})^{\theta_1} \left| data\right.) = 1 - E(\exp(T_0\theta_1)\left| data\right.)\\ &= 1 - I^{-1}\sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k}\Gamma(A_1)(b_1-T-T_0)^{-A_1}\Gamma(A_2)(b_2-T)^{-A_2},\\ \hat{R}_2 &= E(1-(1-\mathrm{e}^{-(\lambda t_0)^2})^{\theta_2} \left| data\right.) = 1 - E(\exp(T_0\theta_2)\left| data\right.)\\ &= 1 - I^{-1}\sum_{k=0}^{n_{12}} \binom{n_{12}}{k} c^{n_{12}-k}\Gamma(A_2)(b_2-T-T_0)^{-A_2}\Gamma(A_1)(b_1-T)^{-A_1}.\end{aligned}$$

4. Numerical study and discussion of results

In this section, we assume that there are 50 identical systems in the life test at the same time. Each system is consisted of two components linked in parallel, and the lifetimes of the components are GR distributions with parameters $\lambda_1=\lambda_2=1, \theta_1=1, \theta_2=2$. Let $t_0=1$, thus the true values of R_1,R_2 are 0.3679 and 0.6004 respectively.

Considering the MLE approach, we derive the observed data $(t_1,s_1),(t_2,s_2),\cdots(t_r,s_r)$, including the lifetime and cause of failure system. These data are simulated by using Monte-Carlo methed according to the given masking level and masking probability ratio. Then we can get the value of n_1,n_2,n_{12}. Substituting them into the results of estimation in section 2, MLEs of $\theta_j,R_j\,(j=1,2)$ are caculated.

For Bayes analysis, we use the same method to conduct the simulation and obtain the Bayes estimations of $\theta_j,R_j\,(j=1,2)$. Due to the different prior distributions of the parameters θ_1, θ_2, non-informative prior and conjugate prior distribution, one can compute different Bayes estimations under square error loss function. The given hyperparameters in conjugate prior distribution are $a_1=1, a_2=2, b_1=1.5, b_2=1.5$.

Repeat the steps above 1000 times, and then the mean squared errors (MSE) of these estimations are computed as following

$$\mathrm{MSE}_{\hat{\theta}_j} = \frac{\sum_{i=1}^{1000}(\theta_j-\hat{\theta}_j^{(i)})^2}{1000},\ \mathrm{MSE}_{\hat{R}_j} = \frac{\sum_{i=1}^{1000}(R_j-\hat{R}_j^{(i)})^2}{1000}.$$

The results are presented in the table and graphs below.

Some acronyms in the table are illustrated as following
MLE: Maximum Likelihood Estimation;
BN: Bayes estimations with non-information priors;
BC: Bayes estimations with conjugate priors.

Table 1. The mean squared errors of estimations

c	$p=0.1$						$p=0.3$					
	$\hat{\theta}_1$			$\hat{\theta}_2$			$\hat{\theta}_1$			$\hat{\theta}_2$		
	MLE	BN	BC	MLE	BN	BC	MLE	BN	BC	MLE	BN	BC
1	0.0594	0.0608	0.0543	0.1044	0.1007	0.0961	0.0607	0.0604	0.0581	0.1101	0.1040	0.1009
0.9	0.0583	0.0566	0.0490	0.1131	0.1021	0.0951	0.0611	0.0589	0.0557	0.1093	0.1055	0.1029
0.8	0.0601	0.0545	0.0498	0.0963	0.0977	0.0944	0.0634	0.0613	0.0577	0.1163	0.1097	0.1014
0.7	0.0577	0.0511	0.0480	0.0995	0.0943	0.0876	0.0622	0.0590	0.0606	0.1154	0.0963	0.0932
0.6	0.0614	0.0534	0.0387	0.0999	0.0976	0.0898	0.0635	0.0594	0.0546	0.1179	0.1014	0.0954
0.5	0.0598	0.0513	0.0472	0.1010	0.0896	0.0812	0.0646	0.0618	0.0569	0.1202	0.1080	0.0985
0.4	0.0583	0.0494	0.0456	0.1054	0.0967	0.0899	0.0644	0.0581	0.0544	0.1258	0.1047	0.0943
0.3	0.0615	0.0522	0.0487	0.1083	0.0891	0.0778	0.0679	0.0610	0.0568	0.1290	0.1091	0.0955
0.2	0.0640	0.0538	0.0468	0.1121	0.0988	0.0895	0.0744	0.0633	0.0542	0.1366	0.1122	0.1032
0.1	0.0703	0.0555	0.0507	0.1232	0.1004	0.0918	0.0867	0.0677	0.0610	0.1444	0.1089	0.1018
	$p=0.5$						$p=0.7$					
1	0.0711	0.0645	0.0630	0.1213	0.1033	0.0955	0.0887	0.0721	0.0654	0.1436	0.1209	0.1115
0.9	0.0734	0.0674	0.0601	0.1200	0.1075	0.0997	0.0834	0.0704	0.0651	0.1420	0.1177	0.1027
0.8	0.0733	0.0646	0.0617	0.1207	0.1040	0.0941	0.0856	0.0693	0.0628	0.1434	0.1182	0.1055
0.7	0.0764	0.0662	0.0655	0.1264	0.1097	0.0981	0.0894	0.0730	0.0693	0.1453	0.1214	0.1021
0.6	0.0816	0.0681	0.0644	0.1261	0.1053	0.0933	0.0907	0.0804	0.0788	0.1488	0.1253	0.1143
0.5	0.0882	0.0712	0.0684	0.1239	0.1066	0.0992	0.0983	0.0832	0.0715	0.1505	0.1279	0.1151
0.4	0.0980	0.0743	0.0747	0.1411	0.1161	0.1178	0.2933	0.1961	0.1873	0.2971	0.1891	0.1847
0.3	0.1278	0.0951	0.0906	0.1587	0.1297	0.1254	0.4794	0.2808	0.2679	0.4546	0.3630	0.3577
0.2	0.1943	0.1267	0.1211	0.2679	0.1918	0.1746	0.8201	0.3299	0.3189	0.7962	0.4786	0.4531
0.1	0.2891	0.2010	0.1988	0.4848	0.2899	0.2543	1.3662	0.5011	0.4875	1.4410	0.5960	0.5584

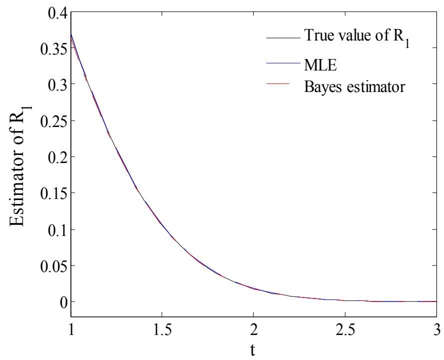

Fig. 1. MLE and Bayes estimations of R_1

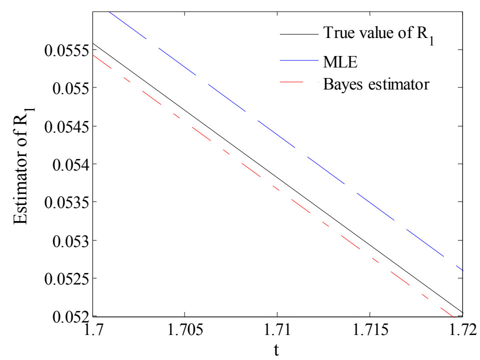

Fig. 2. Magnification of Fig. 1

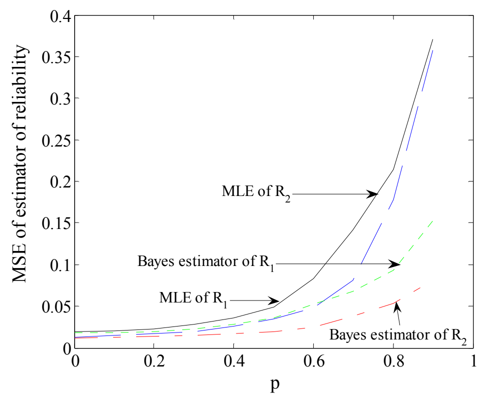

Fig. 3. The MSE associated with the MLE and Bayes estimations of R_1, R_2 against masking level p when $c = 0.1$.

According to the MSEs illustrated in the table and figures for reliability, we can conclude that:

(I) For the given masking probability ratio c, all the MSEs associated with the estimations of $\theta_j, R_j\, (j = 1,2)$ increase with increasing masking level p.

(II) The MSEs associated with MLEs of $\theta_j, R_j\, (j = 1,2)$ are always grater than that associated with Bayes estimations of $\theta_j, R_j\, (j = 1,2)$. When the masking level p goes up, MSEs of MLEs rise more rapidly than Bayes estimations.

(III) When comparing the MSEs of Bayes estimation under different priors from the table, we find that Bayes estimations with non-informative priors compared with conjugate priors conduct slightly higher MSEs.

Therefore, the effect of Bayes estimation is better than MLE, and with Bayes approach, the effect of choosing conjugate priors is better than non-informative priors.

5. Conclusion

In this paper, we discuss estimations of GR components reliability in parallel system using dependent masked data. MLE and Bayes methods are exploited for estimating. Compared with previous literatures, this paper considers a new distribution and model in reliability analysis. Finally, effectiveness and feasibility of our modeling and derivation are verified in the numerical simulation. Furthermore, the comparison of MLE and Bayes approach has been shown by the table and figures.

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