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Intuitionistic Q-Fuzzy Normal HX Group

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ABSTRACT

In this paper, we define the concept of an intuitionistic Q-fuzzy normal HX group and some related properties are investigated. We also define the level subsets of an intuitionistic Q-fuzzy normal HX group and discussed some of its properties. A few equivalent propositions of fuzzy HX group constituting an intuitionistic Q-fuzzy normal HX group are given.

Keywords: Intuitionistic fuzzy set, intuitionistic Q-fuzzy set, intuitionistic fuzzy subgroup , intuitionistic Q-fuzzy subgroup, intuitionistic Q-fuzzy HX subgroup and level sub HX groups, intuitionistic normal Q-fuzzy HX subgroup.

1. Introduction

K.H.Kim [2] introduced the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and serife yilmaz [11] introduced the concept of intuitionistic Q-fuzzy R-subgroups of near rings. A.Solairaju and R.Nagarajan [14][15] introduced and defined a new algebraic structure of Q-fuzzy groups. Li Hongxing [6] introduced the concept of HX group and the authors Luo Chengzhong , Mi Honghai , Li Hongxing [8] introduced the concept of fuzzy HX group. V. Lakshmana Gomathi Nayagam, R. Muthuraj, K.H. Manikandan [5] introduced the concept of intuitionistic Q-fuzzy HX groups. In this paper we define a new concept of intuitionistic Q-fuzzy normal HX group and its level subsets and study some of their related properties.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, G = (G , *) is a group, e is the identity element of G, and xy, we mean x * y.

Definition 2.1 [1]

An intuitionistic fuzzy subset (IFS) λ in a set X is defined as an object of the form $\lambda = \{ \langle x , \mu_\lambda(x) , \nu_\lambda(x) \rangle / x \in X \}$, where $\mu_\lambda : X \to [0,1]$ and $\nu_\lambda : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively to be in λ and for every $x \in X$, $0 \leq \mu_\lambda(x) + \nu_\lambda(x) \leq 1$.

Definition 2.2 [7]

Let λ and η be two intuitionistic fuzzy subsets of a set X. We define the following relations and operations:

(i) $\lambda \subset \eta$ iff $\mu_\lambda(x) \leq \mu_\eta(x)$ and $\nu_\lambda(x) \geq \nu_\eta(x)$, for all $x \in X$;

(ii) $\lambda = \eta$ iff $\mu_\lambda(x) = \mu_\eta(x)$ and $\nu_\lambda(x) = \nu_\eta(x)$, for all $x \in X$;

(iii) $\lambda^c = \{ \langle x, \nu_\lambda(x), \mu_\lambda(x) \rangle / x \in X \}$;

(iv) $\lambda \cap \eta = \{ \langle x, \min(\mu_\lambda(x), \mu_\eta(x)), \max(\nu_\lambda(x), \nu_\eta(x)) \rangle / x \in X \}$;

(v) $\lambda \cup \eta = \{ \langle x, \max(\mu_\lambda(x), \mu_\eta(x)), \min(\nu_\lambda(x), \nu_\eta(x)) \rangle / x \in X \}$;

(vi) $\Box \lambda = \{ \langle x, \mu_\lambda(x), 1-\mu_\lambda(x) \rangle / x \in X \}$;

(vii) $\Diamond\lambda = \{ \langle x, 1-\nu_\lambda(x), \nu_\lambda(x) \rangle / x \in X \}$.

Definition 2.3 [2]

An intuitionistic Q-fuzzy subset λ in a set X is defined as an object of the form $\lambda = \{ \langle (x,q) , \mu_\lambda(x,q) , \nu_\lambda(x,q) \rangle \; x \in X \text{ and } q \in Q \}$, where $\mu_\lambda : X \times Q \to [0,1]$ and $\nu_\lambda : X \times Q \to [0,1]$ define the degree of membership and the degree of non-membership of the element $(x,q) \in X \times Q$ respectively and for every $(x,q) \in X \times Q$, $0 \leq \mu_\lambda(x,q) + \nu_\lambda(x,q) \leq 1$.

Definition 2.4

Let λ and η be two intuitionistic Q-fuzzy subsets of a set X. We define the following relations and operations: For all $x \in X$ and $q \in Q$,

(i) $\lambda \subset \eta$ iff $\mu_\lambda(x,q) \leq \mu_\eta(x,q)$ and $\nu_\lambda(x,q) \geq \nu_\eta(x,q)$,

(ii) $\lambda = \eta$ iff $\mu_\lambda(x,q) = \mu_\eta(x,q)$ and $\nu_\lambda(x,q) = \nu_\eta(x,q)$,

(iii) $\lambda^c = \{ \langle (x,q), \nu_\lambda(x,q), \mu_\lambda(x,q) \rangle / x \in X \text{ and } q \in Q \}$,

(iv) $\lambda \cap \eta = \{ \langle (x,q), \min \{\mu_\lambda(x,q), \mu_\eta(x,q)\}, \max \{\nu_\lambda(x,q), \nu_\eta (x,q)\}\rangle /$ $x \in X$ and $q \in Q\}$,

(v) $\lambda \cup \eta = \{ \langle (x,q), \max \{\mu_\lambda(x,q), \mu_\eta(x,q)\}, m \{\nu_\lambda(x,q), \nu_\eta (x,q)\}\rangle / x \in X$ and $q \in Q\}$,

(vi) $\Box \lambda = \{ \langle (x,q), \mu_\lambda(x,q), 1-\mu_\lambda(x,q) \rangle / x \in X$ and $q \in Q \}$,

(vii) $\Diamond\lambda = \{ \langle (x,q), 1-\nu_\lambda(x,q), \nu_\lambda(x,q) \rangle / x \in X$ and $q \in Q \}$.

Definition 2.5 [6]

In 2^G-$\{\phi\}$, a nonempty set $\vartheta \subset 2^G$-$\{\phi\}$ is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = $\{ab / a \in A$ and $b \in B\}$, and its unit element is denoted by E.

3.Properties of an Intuitionistic fuzzy HX group

In this section, we discuss some properties of an Intuitionistic fuzzy HX group.

Definition 3.1 [5]

Let G be a group and a nonempty set $\vartheta \subset 2^G$-$\{\phi\}$ is a HX group on G. An intuitionistic fuzzy subset $\lambda = \langle A , \mu_\lambda(A) , \nu_\lambda(A) \rangle$ of a HX group ϑ is said to be an intuitionistic fuzzy HX subgroup of a HX group ϑ if the following conditions are satisfied. For all A and B $\in \vartheta$,

(i) $\mu_\lambda(AB) \geq \min\{ \mu_\lambda(A), \mu_\lambda(B) \}$,

(ii) $\nu_\lambda(AB) \leq \max\{ \nu_\lambda(A), \nu_\lambda(B)) \}$.

(iii) $\mu_\lambda(A^{-1}) = \mu_\lambda(A)$,

(iv) $\nu_\lambda(A^{-1}) = \nu_\lambda(A)$.

Example 3.1 [5]

Let $G = \{ 1 , -1 \}$,then $(G , \bullet)$ is a group and $\vartheta = \{ \{1\} ,\{-1\} \}$, then $(\vartheta , \bullet)$ is a Hx group. Define $\mu_\lambda : \vartheta \to [0,1]$ by $\mu_\lambda (\{1\}) = 0.8$ and $\mu_\lambda (\{-1\}) = 0.5$.

$\nu_\lambda : \vartheta \to [0,1]$ by $\nu_\lambda (\{1\}) = 0.1$ and $\nu_\lambda (\{-1\}) = 0.3$.

Clearly λ is an intuitionistic fuzzy HX group of a HX group ϑ .

Definition 3.2 [5]

An intuitionistic Q-fuzzy subset $\lambda=\{\langle (A,q) , \mu_\lambda(A,q) , \nu_\lambda(A,q) \rangle / A \in \vartheta$ and $q \in Q \}$ of a HX group ϑ is said to be an intuitionistic Q-fuzzy HX subgroup of a HX group ϑ if the following conditions are satisfied. For all A and B $\in \vartheta$ and $q \in Q$,

(i) $\mu_\lambda(AB,q) \geq \min\{ \mu_\lambda(A,q), \mu_\lambda(B,q) \}$,

(ii) $\nu_\lambda(AB,q) \leq \max\{ \nu_\lambda(A,q), \nu_\lambda(B,q)) \}$,
(iii) $\mu_\lambda(A^{-1},q) = \mu_\lambda(A,q)$,
(iv) $\nu_\lambda(A^{-1},q) = \nu_\lambda(A,q)$.

3.3 Definition

An intuitionistic Q-fuzzy HX subgroup λ of a HX group ϑ is said to be an intuitionistic Q-fuzzy normal HX subgroup of ϑ if ,

(i) $\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ for all A and B $\in \vartheta$ and $q \in Q$,
(ii) $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A and B $\in \vartheta$ and $q \in Q$.

Theorem 3.1 If λ and η be the intuitionistic Q-fuzzy normal HX subgroups of a HX group ϑ, then $\lambda \cap \eta$ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ .

Proof. Let $\lambda = \{\langle (A,q) , \mu_\lambda(A,q) , \nu_\lambda(A,q) \rangle / A \in \vartheta$ and $q \in Q \}$
$\eta = \{\langle (A,q) , \mu_\eta (A,q) , \nu_\eta (A,q) \rangle / A \in \vartheta$ and $q \in Q \}$,then
$\lambda \cap \eta = \{ \langle (A,q) , \min \{\mu_\lambda(A,q), \mu_\eta(A,q)\}, \max \{\nu_\lambda(A,q), \nu_\eta (A,q)\}\rangle / A \in \vartheta$ and $q \in Q \}$,
Let $\omega_{\lambda \cap \eta} = \min \{\mu_\lambda(A,q), \mu_\eta(A,q)\}$ and $\delta_{\lambda \cap \eta} = \max \{\nu_\lambda(A,q), \nu_\eta (A,q)\}$.
For any A , B $\in \vartheta$ and $q \in Q$,
$\omega_{\lambda \cap \eta} (ABA^{-1},q) = \min \{\mu_\lambda(ABA^{-1},q), \mu_\eta(ABA^{-1},q)\}$,
$\omega_{\lambda \cap \eta} (ABA^{-1},q) \geq \min \{\mu_\lambda(B,q), \mu_\eta(B,q)\} = \omega_{\lambda \cap \eta} (B,q)$ and
$\delta_{\lambda \cap \eta} (ABA^{-1},q) = \max \{\nu_\lambda(ABA^{-1},q), \nu_\eta (ABA^{-1},q)\}$,
$\delta_{\lambda \cap \eta} (ABA^{-1},q) \leq \max \{\nu_\lambda(B,q), \nu_\eta (B,q)\} = \delta_{\lambda \cap \eta} (B,q)$.
Hence, $\lambda \cap \eta$ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. □

Remark

Arbitrary intersection of intuitionistic Q-fuzzy normal HX subgroups of a HX group is an intuitionistic Q-fuzzy normal HX group.

Theorem 3.2 If λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX groupϑ, then □ λ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ **.**

Proof. Let λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ and let □ $\lambda = \{ \langle (A,q) , \mu_\lambda(A,q), 1 - \mu_\lambda(A,q) \rangle / A \in \vartheta$ and $q \in Q \}$. Let $\delta_\lambda(A,q) = 1 - \mu_\lambda(A,q)$.
λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ then
$\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A and B $\in \vartheta$ and $q \in Q$.
Now, $\delta_\lambda(ABA^{-1} , q) = 1 - \mu_\lambda(ABA^{-1} , q) \leq 1 - \mu_\lambda(B , q) = \delta_\lambda(B , q)$.
Thus , □ λ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. □

Theorem 3.3 If λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ, then $\Diamond\lambda$ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ **.**

Proof. Let λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ and let $\Diamond\lambda = \{ \langle (A,q), 1-\nu_\lambda(A,q), \nu_\lambda(A,q) \rangle / A\in\vartheta$ and $q\in Q\}$. Let $\varphi_\lambda(A,q) = 1-\nu_\lambda(A,q)$.

λ be the intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ then $\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A and B $\in\vartheta$ and $q\in Q$.

Now, $\varphi_\lambda(ABA^{-1},q) = 1-\nu_\lambda(ABA^{-1},q) \geq 1-\nu_\lambda(B,q) = \varphi_\lambda(B,q)$.

Thus, $\Diamond\lambda$ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. □

Theorem 3.4 Let G be a classical group and λ be an Q-fuzzy HX subgroup of a HX group ϑ.Then the following conditions are equivalent.

i. λ is an intuitionistic Q-fuzzy normal HX group on ϑ.
ii. $\lambda(ABA^{-1},q) = \lambda(B,q)$, for all $A,B\in \vartheta$ and $q\in Q$.
iii. $\lambda(AB,q) = \lambda(BA,q)$, for all $A,B\in \vartheta$ and $q\in Q$.

Proof. i $\Rightarrow$ ii

Let λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ.Then, $\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for arbitrary A and B $\in\vartheta$ and $q\in Q$. Thus, taking advantage of the arbitrary property of A, we get,

$$\mu_\lambda(A^{-1}BA,q) = \mu_\lambda(A^{-1}B(A^{-1})^{-1},q) \geq \mu_\lambda(B,q),$$

Therefore, $\mu_\lambda(B,q) = \mu_\lambda(A^{-1}(ABA^{-1})A ,q) \geq \mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$.

Hence, $\mu_\lambda(ABA^{-1},q) = \mu_\lambda(B,q)$.

Similarly, we have

$$\nu_\lambda(A^{-1}BA,q) = \nu_\lambda(A^{-1}B(A^{-1})^{-1},q) \leq \nu_\lambda(B,q),$$

Therefore, $\nu_\lambda(B,q) = \nu_\lambda(A^{-1}(ABA^{-1})A ,q) \leq \nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$.

Hence, $\nu_\lambda(ABA^{-1},q) = \nu_\lambda(B,q)$.

Hence, $\lambda(ABA^{-1},q) = \lambda(B,q)$, for all $A,B\in \vartheta$ and $q\in Q$.

ii $\Rightarrow$iii, Substituting B for BA in (ii), we can easily get (iii).

iii $\Rightarrow$ i, According to $\lambda(AB,q) = \lambda(BA,q)$, for all $A,B\in \vartheta$ and $q\in Q$, we obtain $\lambda(ABA^{-1},q) = \lambda(BAA^{-1},q) = \lambda(B,q) \geq \lambda(B,q)$.

That is, $\lambda(ABA^{-1},q) \geq \lambda(B, q)$ implies that $\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for arbitrary A and B $\in\vartheta$ and $q\in Q$.

Hence, λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. □

Theorem 3.5 Let G be an classical abelian group and λ be an intuitionistic Q-fuzzy HX group of an abelian HX group ϑ. Then λ is an intuitionistic Q-fuzzy normal HX group on ϑ.

Proof. Let λ be an intuitionistic Q-fuzzy HX group of an abelian HX group ϑ.
Then AB = BA for every A and B $\in \vartheta$.
Hence $\lambda(AB,q) = \lambda(BA,q)$, for all $A,B \in \vartheta$ and $q \in Q$.
Therefore, λ is an intuitionistic Q-fuzzy normal HX group on ϑ. □

4.Properties of level subsets of an intuitionistic Q-fuzzy HX subgroup

In this section, we introduce the concept of level subset of an intuitionistic Q-fuzzy normal HX subgroup and discuss some of its properties.

4.1 Definition

Let λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. For any α , $\beta \in[0,1]$, we define the set, $\lambda_{<\alpha,\beta>} = \{ A \in \vartheta / \mu_\lambda(A , q) \geq \alpha$ and $\nu_\lambda(A , q) \leq \beta$, for some $q \in Q \}$, is called the level subset of λ.

Theorem 4.1 Let G be a classical group. Then the intuitionistic Q-fuzzy HX subgroup λ of a HX group ϑ is an intuitionistic Q-fuzzy normal HX subgroup on ϑ iff for any α , $\beta \in[0,1]$, $\lambda_{<\alpha,\beta>}$ is a classical normal sub HX group.

Proof. Let λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ.Then,
$\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A and B $\in \vartheta$ and $q \in Q$.
For all A, B $\in \lambda_{<\alpha,\beta>}$ and $q \in Q$, we have,
$\mu_\lambda (A, q) \geq \alpha$ and $\nu_\lambda(A, q) \leq \beta$ and $\mu_\lambda (B, q) \geq \alpha$ and $\nu_\lambda(B, q) \leq \beta$.

Now,
$$\mu_\lambda (AB^{-1}, q) \geq \min \{\mu_\lambda (A, q), \mu_\lambda (B, q)\}.$$
$$\mu_\lambda (AB^{-1}, q) \geq \min \{\alpha , \alpha\} = \alpha.$$
$$\nu_\lambda (AB^{-1}, q) \leq \max \{\nu_\lambda (A, q), \nu_\lambda (B, q)\}.$$
$$\nu_\lambda (AB^{-1}, q) \leq \max \{\beta , \beta\} = \beta.$$
$$AB^{-1} \in \lambda_{<\alpha,\beta>} .$$

For all B $\in \lambda_{<\alpha,\beta>}$, $A \in \vartheta$ and $q \in Q$, we have, $\mu_\lambda (B, q) \geq \alpha$ and $\nu_\lambda(B, q) \leq \beta$.
Since, λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ,
$\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q) \geq \alpha$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q) \leq \beta$ for all A , B $\in \vartheta$ and $q \in Q$.

$ABA^{-1} \in \lambda_{<\alpha,\beta>}$. Hence $\lambda_{<\alpha,\beta>}$ is a classical normal sub HX group.

Conversely, for any α , $\beta \in[0,1]$, $\lambda_{<\alpha,\beta>} \neq \phi$ and $\lambda_{<\alpha,\beta>}$ is a classical normal sub HX group. Then ,we have,
$\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A , B $\in \vartheta$ and $q \in Q$.
Otherwise, if there exists A_0 or $B_0 \in \vartheta$ and $q \in Q$ such that,
$\mu_\lambda(A_0B_0A_0^{-1},q) < \mu_\lambda(B_0,q)$ and $\nu_\lambda(A_0B_0A_0^{-1}, q) > \nu_\lambda(B_0,q)$.
Take $\alpha_0 = 0.5 [\mu_\lambda(B_0,q) + \mu_\lambda(A_0B_0A_0^{-1},q)]$and $\beta_0 = 0.5 [\nu_\lambda(B_0,q) + \nu_\lambda(A_0B_0A_0^{-1}, q)]$.

Evidently, α_0 , $\beta_0 \in[0,1]$,we can infer that,
$\mu_\lambda(B_0,q) > \alpha_0$, $\mu_\lambda(A_0B_0A_0^{-1},q) < \alpha_0$ and $\nu_\lambda(B_0,q) < \beta_0$ and $\nu_\lambda(A_0B_0A_0^{-1}, q) > \beta_0$.
Consequently, we have $B_0 \in \lambda_{<\alpha_0,\beta_0>}$ and $A_0B_0A_0^{-1} \notin \lambda_{<\alpha_0,\beta_0>}$.

This contradicts that $\lambda_{<\alpha_0,\beta_0>}$ is a classical normal sub HX group.

Hence, we get,
$\mu_\lambda(ABA^{-1},q) \geq \mu_\lambda(B,q)$ and $\nu_\lambda(ABA^{-1},q) \leq \nu_\lambda(B,q)$ for all A , B $\in \vartheta$ and $q \in Q$.
Hence, λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. □

Definition 4.2

Let λ be an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. The normal sub HX groups $\lambda_{<\alpha,\beta>}$, α , $\beta \in[0,1]$ are called level normal sub HX groups of λ.

Theorem 4.2 An intuitionistic Q-fuzzy subset λ of ϑ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ if and only if the level subsets $\lambda_{<\alpha,\beta>}$, α , $\beta \in$ Image λ, are normal sub HX groups of ϑ.

Proof. It is clear. □

Theorem 4.3 λ is an intuitionistic Q-fuzzy normal HX subgroup of a HX group ϑ. The sequence of level normal sub HX groups $\{ \lambda_{<\alpha n,\beta n>} / n \in N, N = 0, 1, 2, \ldots \}$ is a nested family if α_n is decreasing and β_n is an increasing sequnences.

Proof. The level normal sub HX groups of an intuitionistic Q-fuzzy normal HX subgroup λ of a HX group ϑ form a chain. Since $\mu_\lambda(E, q) \geq \mu_\lambda(A, q)$ and $\nu_\lambda(E, q) \leq \nu_\lambda(A, q)$ for all A in ϑ and $q \in Q$, therefore $\lambda_{<\alpha_0,\beta_0>}$, where $\mu_\lambda(E, q) = \alpha_0$ and $\nu_\lambda(E, q) = \beta_0$ is the smallest and we have the chain :
$\{E\} \subseteq \lambda_{<\alpha_0,\beta_0>} \subseteq \lambda_{<\alpha_1,\beta_1>} \subseteq \lambda_{<\alpha_2,\beta_2>} \subseteq \ldots \subseteq \lambda_{<\alpha_n,\beta_n>} = \vartheta$, where
$\alpha_0 > \alpha_1 > \alpha_2 > \ldots\ldots > \alpha_n$ and $\beta_0 < \beta_1 < \beta_2 < \ldots\ldots < \beta_n$. □

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