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Entropy Optimization Model on Bimatrix Game

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ABSTRACT
This paper presents an entropy optimization model on bimatrix game (two-person non-zero-sum game). According to the Maximum Entropy Principle the entropy function of the players of bimatrix game has been considered as an objective and using it we formulate the model. This model is a non-linear programming model due to entropy function. The solution concept is based on Kuhn-Tucker optimality conditions. A numerical example is illustrated the results in this paper.
Keywords: Bimatrix Game, Entropy, Nash equilibrium, Kuhn-Tucker conditions.

1. Introduction

Game theory has a remarkable importance in both Operations Research and Systems Engineering due to its great applicability. Many real conflict problems can be modelled as games. However, the encountered conflict problems in economical, military and political fields become more and more complex and uncertain due to the existence of diversified factors. This situation will bring some difficulties in application of classical game theory. To remove this difficulties, we have been employed the entropy on bimatrix game.The elegant theory of matrix (zero-sum) two-person games is hardly applicable in many realistic game theoretic problems in Operations Research, Economics and in other social sciences. To remove this difficulties, we introduce the concept and methodologies of bimatrix (non-zero sum) games in this paper.

Entropy optimization models have been successfully applied to practical problems in many scientific and engineering disciplines. Those disciplines include Information Theory ([10], [11]), Statistical Mechanics, Thermodynamics,

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Statistical Parameter Estimation and Inference, Economics, Business and Finance, Non-linear Spectral Analysis, Patter Recognition [13], Urban and Regional planning, Queueing Theory and Linear Programming. Das and Roy [6], have proposed a new solution concept by considering the entropy function to the objectives of the player. To solve this bimatrix goal game model, we apply fuzzy based Genetic Algorithm Technique. Here we discuss about Entropy Optimization model in the area of two-person non-zero-sum game.

Many algorithms ([1],[3],[5],[8],[9]) have been proposed for solving the constraint maximum entropy or minimum cross-entropy problem. Recently, we derive the model where K.K.T. conditions have played the important role in developing solution to this model and provide numerical results.

2. Mathematical Model

A two-person bimatrix game with single objective can be illustrated as a pair of $m \times n$ matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}, \qquad B = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

If player PI adopts the strategy "row i" and player PII adopts the strategy "column j" then a_{ij} denotes the expected payoff for player PI and b_{ij} denotes the expected payoff for player PII.

Let the mixed strategy of bimatrix game for player PI and PII are defined as follows:

$$Y = \{ y \in R^m ; \sum_{i=1}^{m} y_i = 1; y_i \geq 0, \ i = 1,2,\dots,m \}$$

$$Z = \{ z \in R^n ; \sum_{j=1}^{n} z_j = 1; z_j \geq 0, \ \ j = 1,2,\dots,n \}$$

Definition 1:(Expected payoff)

The expected payoff $v(y,z)$ of the bimatrix game, $A = (a_{ij})$ and $B = (b_{ij}),$ for each strategy pair $y \in Y$ and $z \in Z,$ is defined as follows:

$$v(y,z) = [v_1(y,z), v_2(y,z)]$$

where $$v_1(y,z) = y^t Az = \sum_{j=1}^{n}\sum_{i=1}^{m} a_{ij} y_i z_j, \qquad v_2(y,z) = y^t Bz = \sum_{j=1}^{n}\sum_{i=1}^{m} b_{ij} y_i z_j$$

Definition 2: (Nash equilibrium solution)

For bimatrix game (A, B) of two players, the Nash equilibrium solution (y^*, z^*) is found if,

$$y^{*t} Az^* \geq y^t Az^*, \quad y^{*t} Bz^* \geq y^{*t} Bz$$

where $y \in Y$ and $z \in Z$.

As $v(y,z)$ depends on the strategy of PII, if we assume that PII will choose a strategy $z \in Z$ that gives the minimum value of $v_l(y,z), l=1,2.$ Then, for each $y \in Y$, PI will get

$$v_1(y) = \min_{z\in Z} v_1(y,z) = \min_{z\in Z} y^t Az = \min_{1\le j\le n} \sum_{i=1}^{m} y_i a_{ij}$$

and
$$v_2(y) = \min_{z\in Z} v_2(y,z) = \min_{z\in Z} y^t Bz = \min_{1\le j\le n} \sum_{i=1}^{m} y_i b_{ij}$$

Thus optimal strategy of PI can be obtained by considering the following **Model 1**, irrespective of the action of PII.

Model 1 $\max: v_1$, $\max: v_2$

$$s.t.\ \ y^t A \ge (v_1,\ldots,v_1),\quad y^t B \ge (v_2,\ldots,v_2)$$

$$\sum_{i=1}^{m} y_i = 1;\ y_i \ge 0,\ i=1,2,\ldots,m$$

The above model is equivalent to the following **Model 2.**

Model 2 $\min: \frac{1}{v_1} = \sum_{i=1}^{m} Y_i$, $\min: \frac{1}{v_2} = \sum_{i=1}^{m} Y_i'$

$$s.t.\ \ \sum_{i=1}^{m} a_{ij} Y_i \ge 1,\ j=1,2,\ldots,n\quad,\quad \sum_{i=1}^{m} b_{ij} Y_i' \ge 1,\ j=1,2,\ldots,n$$

$$Y_i = \frac{y_i}{v_1} \ge 0,\ i=1,2,\ldots,m \tag{1}$$

$$Y_i' = \frac{y_i}{v_2} \ge 0,\ i=1,2,\ldots,m \tag{2}$$

Similarly, as $v(y,z)$ depends on the strategy of PI , if we assume that PI will choose a strategy $y \in Y$ that gives the maximum value of $v_l(y,z), l=1,2.$ Then, for each $z \in Z$, PII will get

$$w_1(y) = \max_{y\in Y} v_1(y,z) = \max_{y\in Y} y^t Az = \max_{1\le i\le m} \sum_{j=1}^{n} a_{ij} z_j$$

and
$$w_2(y) = \max_{y\in Y} v_2(y,z) = \max_{y\in Y} y^t Bz = \max_{1\le i\le m} \sum_{j=1}^{n} b_{ij} z_j$$

Thus optimal strategy of PII can be obtained by considering the following **Model 3**, irrespective of the action of PI.

Model 3 $\min: w_1$, $\min: w_2$

$$s.t.\ \ Az \le (w_1,\ldots,w_1)\quad,\quad Bz \le (w_2,\ldots,w_2)$$

$$\sum_{j=1}^{n} z_j = 1;\ z_j \geq 0,\ \ j = 1,2,\ldots,n$$

The above model is equivalent to the following **Model 4.**

Model 4 $\quad \max : \dfrac{1}{w_1} = \sum_{j=1}^{n} Z_j\ ,\qquad \max : \dfrac{1}{w_2} = \sum_{j=1}^{n} Z_j^{'}$

$$s.t.\ \ \sum_{j=1}^{n} a_{ij} Z_j \leq 1,\ i = 1,2,\ldots,m$$

$$\sum_{j=1}^{n} b_{ij} Z_j^{'} \leq 1,\ i = 1,2,\ldots,m$$

$$Z_j = \frac{z_j}{w_1} \geq 0,\ \ j = 1,2,\ldots,n \tag{3}$$

$$Z_j^{'} = \frac{z_j}{w_2} \geq 0,\ \ j = 1,2,\ldots,n \tag{4}$$

Again each player is interested in making moves which will be as surprising and as uncertain to the other player as possible. For this reason, the players are involved in maximizing their entropies or involved in minimizing their cross-entropies. The mathematical form of entropies are as follows:

$$H_1 = -\sum_{i=1}^{m} Y_i\, ln(Y_i)\ ,\ \ H_2 = -\sum_{j=1}^{n} Z_j\, ln(Z_j)$$

$$H_1^{'} = -\sum_{i=1}^{m} Y_i^{'}\, ln(Y_i^{'})\ \ ,\ H_2^{'} = -\sum_{j=1}^{n} Z_j^{'}\, ln(Z_j^{'})$$

The mathematical form of cross-entropies are as follows:

$$H_1^{c} = \sum_{i=1}^{m} Y_i\, ln(\frac{Y_i}{p_i^{0}})\ ,\ \ H_2^{c} = \sum_{j=1}^{n} Z_j\, ln(\frac{Z_j}{q_j^{0}})$$

$$H_1^{'c} = \sum_{i=1}^{m} Y_i^{'}\, ln(\frac{Y_i^{'}}{p_i^{'0}})\ \ ,\ H_2^{'c} = \sum_{j=1}^{n} Z_j^{'}\, ln(\frac{Z_j^{'}}{q_j^{'0}})$$

where $\{p_i^{0}, i = 1,2,\ldots,m\}$, $\{p_i^{'0}, i = 1,2,\ldots,m\ \}$, $\{q_j^{0}, j = 1,2,\ldots,n\}$ and $\{q_j^{'0}, j = 1,2,\ldots,n\}$ are given priori distribution and, in the absence of it, they are follow uniform probability distribution.

2.1 Entropy Optimization Model on Bimatrix Game:

Let us first establish the entropy optimization model for maximization type by considering the following principle. “Out of all possible distributions that are consistent with moment constraint, choose one that has the maximum entropy". This principle was proposed by Jaynes [11] and has been known as Principle of Maximum Entropy or Janes' Maximum Entropy principle. From this point of view, we formulate a new mathematical model namely Entropy Optimization Model on two- person bimatrix game in which the entropy function has been considered as an objective function. The entropy optimization model for player II is represented as follows.

Model 5 max : $H_2 + H_2^{'}$

$$s.t.\ \sum_{i=1}^{n} a_{ij} Z_j \le 1,\ i = 1,2,\ldots,m \tag{5}$$

$$\sum_{i=1}^{n} b_{ij} Z_j^{'} \le 1,\ i = 1,2,\ldots,m \tag{6}$$

$$H_2 = -\sum_{j=1}^{n} Z_j\, ln(Z_j)\ ,\quad Z_j = \frac{z_j}{w_1} \ge 0,\ j = 1,2,\ldots,n$$

$$H_2^{'} = -\sum_{j=1}^{n} Z_j^{'}\, ln(Z_j^{'}),\quad Z_j^{'} = \frac{z_j}{w_2} \ge 0,\ j = 1,2,\ldots,n$$

Similarly, we establish the entropy optimization model for minimization type by considering the following principle. "Out of all probability distributions satisfying the given moment constraints, choose the distribution that minimizes the cross-entropy with respect to the given priori distribution and, in the absence of it, choose the distribution that minimizes the cross-entropy with respect to the uniform distribution". This principle was proposed by Kullback-Leibler [13] and has been known as Principle of Minimum Cross Entropy or Kullback-Leibler's Minimum Cross Entropy principle. From this point of view, we formulate a new mathematical model namely Entropy Optimization Model on two-person bimatrix game in which the entropy function has been considered as an objective function. The Entropy Optimization Model for player I is represented as follows:

Model 6 min : $H_1^c + H_1^{'c}$

$$s.t.\ \sum_{i=1}^{m} a_{ij} Y_i \ge 1,\ j = 1,2,\ldots,n$$

$$\sum_{i=1}^{m} b_{ij} Y_i^{'} \ge 1,\ j = 1,2,\ldots,n$$

$$H_1^c = \sum_{i=1}^{m} Y_i\, ln(\frac{Y_i}{p_i^0})\ ,\ H_1^{'c} = \sum_{i=1}^{m} Y_i^{'}\, ln(\frac{Y_i^{'}}{p_i^{'0}})$$

$$Y_i = \frac{y_i}{v_1} \ge 0, Y_i^{'} = \frac{y_i}{v_2} \ge 0, i = 1,2,\ldots,m\,.$$

3 Solution Procedure

Model 5 is non-linear programming model with concave objective function and convex feasible region so the K.K.T. conditions must gives optimum solution, which is maximum. For this purpose we construct the Lagrangian to this model as follows.

$$L(Z,Z',R,R',S,S') = -\sum_{j=1}^{n} Z_j \, ln(Z_j) - \sum_{j=1}^{n} Z_j^{'} \, ln(Z_j^{'})$$
$$-\sum_{i=1}^{m} r_i (\sum_{j=1}^{n} a_{ij} Z_j - 1 + s_i^2) - \sum_{i=1}^{m} r_i^{'} (\sum_{j=1}^{n} b_{ij} Z_j^{'} - 1 + s_i^{'2}) \qquad (7)$$

where $R = (r_1, r_2, \ldots, r_m)$, $R' = (r_1', r_2', \ldots, r_m')$ are Lagrange multipliers and $S = (s_1, s_2, \ldots, s_m)$, $S' = (s_1', s_2', \ldots, s_m')$ are associated with inequlity constraints. The optimality condition gives

$$\left.\begin{aligned}
&Z_j = \exp(-1 - \sum_{i=1}^{m} a_{ij} r_i), \; j = 1,2,\ldots,n \\
&Z_j^{'} = \exp(-1 - \sum_{i=1}^{m} b_{ij} r_i^{'}), \; j = 1,2,\ldots,n \\
&\sum_{j=1}^{n} a_{ij} Z_j \le 1, \; i = 1,2,\ldots,m \\
&\sum_{j=1}^{n} b_{ij} Z_j^{'} \le 1, \; i = 1,2,\ldots,m \\
&r_i (\sum_{i=1}^{n} a_{ij} Z_j - 1) = 0, \; i = 1,2,\ldots,m \\
&r_i^{'} (\sum_{i=1}^{n} b_{ij} Z_j^{'} - 1) = 0, \; i = 1,2,\ldots,m \\
&r_i \ge 0, r_i^{'} \ge 0, i = 1,2,\ldots,m \\
&Z_j \ge 0, \; Z_j^{'} \ge 0, j = 1,2,\ldots,n
\end{aligned}\right\} \qquad (8)$$

Hence the Lagrangian $L(Z,R,S)$ is maximum for those values of Z and R which satisfy the relations (8). Hence for these values of Z, the equation (7) is also maximize and the corresponding value of Z, we obtain the value of the game v and the optimal strategy z for player PII. But the relations (8) are implicit form, so it is not easy to determine $L(Z,R,S)$. Using the equation (7) and the equations (8), we formulate the following model which is solve by Lingo package.

Model 7 $\quad L(Z,Z',R,R',S,S') = -\sum_{j=1}^{n} Z_j \, ln(Z_j) - \sum_{j=1}^{n} Z_j^{'} \, ln(Z_j^{'})$

$$-\sum_{i=1}^{m} r_i (\sum_{j=1}^{n} a_{ij} Z_j - 1 + s_i^2) - \sum_{i=1}^{m} r_i^{'} (\sum_{j=1}^{n} b_{ij} Z_j^{'} - 1 + s_i^{'2}) \qquad (9)$$

subject to

$$\left.\begin{aligned}
& Z_j = \exp(-1 - \sum_{i=1}^{m} a_{ij} r_i),\ j = 1,2,\ldots,n \\
& Z_j^{'} = \exp(-1 - \sum_{i=1}^{m} b_{ij} r_i^{'}),\ j = 1,2,\ldots,n \\
& \sum_{i=1}^{n} a_{ij} Z_j \le 1,\ i = 1,2,\ldots,m \\
& \sum_{i=1}^{n} b_{ij} Z_j^{'} \le 1,\ i = 1,2,\ldots,m \\
& r_i (\sum_{i=1}^{n} a_{ij} Z_j - 1) = 0,\ i = 1,2,\ldots,m \\
& r_i^{'} (\sum_{i=1}^{n} b_{ij} Z_j^{'} - 1) = 0,\ i = 1,2,\ldots,m \\
& r_i \ge 0, r_i^{'} \ge 0, i = 1,2,\ldots,m \\
& Z_j \ge 0,\ Z_j^{'} \ge 0, j = 1,2,\ldots,n
\end{aligned}\right\} \qquad (10)$$

After calculating $Z_j, Z_j^{'}$ from **Model 7** say $Z_j^{*}, Z_j^{'*}$ we determine the value of game v^* and optimal strategy z^* for player PII by the following relations.

$$\left.\begin{aligned}
& \frac{1}{w_1^*} = \sum_{j=1}^{n} Z_j^* \ , \quad \frac{1}{w_2^*} = \sum_{j=1}^{n} Z_j^{'*} \\
& z_j^* = w_1^* Z_j^*,\ j = 1,2,\ldots,n
\end{aligned}\right\} \qquad (11)$$

Similarly to solve **Model 6** it is clear that, the **Model 6** is non-linear programming model with convex objective function and convex feasible region so the K.K.T. conditions must gives optimum solution, which is minimum. For this purpose we construct the Lagrangian of this model as follows.

$$L1(Y, Y', T, T', U, U') = \sum_{i=1}^{m} Y_i \, ln(\frac{Y_i}{p_i^0}) + \sum_{i=1}^{m} Y_i^{'} \, ln(\frac{Y_i^{'}}{p_i^{'0}})$$

$$-\sum_{j=1}^{n} t_j (\sum_{i=1}^{m} a_{ij} Y_i - 1 + u_j^2) - \sum_{j=1}^{n} t_j^{'} (\sum_{i=1}^{m} b_{ij} Y_i^{'} - 1 + u_j^{'2}) \qquad (12)$$

where $T = (t_1, t_2, \ldots, t_n)$, $T' = (t_1', t_2', \ldots, t_n')$ are Lagrange multipliers and $S = (u_1, u_2, \ldots, u_n)$, $S' = (u_1', u_2', \ldots, u_n')$ associated with inequality constraints in (10). The optimality condition gives

$$\left.\begin{array}{l}
Y_i = p_i^0 \exp(\sum_{j=1}^{n} a_{ij} t_j - 1),\ i = 1,2,\ldots,m \\
Y_i' = p_i'^0 \exp(\sum_{j=1}^{n} b_{ij} t_j' - 1),\ i = 1,2,\ldots,m \\
\sum_{i=1}^{m} a_{ij} Y_i \geq 1,\ j = 1,2,\ldots,n \\
\sum_{i=1}^{m} b_{ij} Y_i' \geq 1,\ j = 1,2,\ldots,n \\
t_j(\sum_{i=1}^{m} a_{ij} Y_i - 1) = 0,\ j = 1,2,\ldots,n \\
t_j'(\sum_{i=1}^{m} b_{ij} Y_i' - 1) = 0,\ j = 1,2,\ldots,n \\
t_j \geq 0, t_j' \geq 0,\ j = 1,2,\ldots,n,\ Y_i \geq 0, Y_i' \geq 0, i = 1,2,\ldots,m
\end{array}\right\} \quad (13)$$

Hence the Lagrangian $L1(Y,Y',T,T',U,U')$ is minimum for those values of Y,Y' and T,T' which satisfy the relations (13). Hence for these values of Y,Y' the equation (12) is minimize and the corresponding value of Y,Y', we obtain the value of the game w_1, w_2 and the optimal strategy y for player PI. But the relations (13) are implicit form so it is not easy to determine $L1(Y,Y',T,T',U,U')$. Using the equation (12) and the equations (13), we formulate the following model which is solve by Lingo package.

Model 8 $$L1(Y,Y',T,T',U,U') = \sum_{i=1}^{m} Y_i\, ln(\frac{Y_i}{p_i^0}) + \sum_{i=1}^{m} Y_i'\, ln(\frac{Y_i'}{p_i'^0})$$

$$- \sum_{j=1}^{n} t_j (\sum_{i=1}^{m} a_{ij} Y_i - 1 + u_j^2) - \sum_{j=1}^{n} t_j' (\sum_{i=1}^{m} b_{ij} Y_i' - 1 + u_j'^2) \quad (14)$$

subject to
$$\left.\begin{array}{l}
Y_i = p_i^0 \exp(\sum_{j=1}^{n} a_{ij} t_j - 1),\ i = 1,2,\ldots,m \\
Y_i' = p_i'^0 \exp(\sum_{j=1}^{n} b_{ij} t_j' - 1),\ i = 1,2,\ldots,m \\
\sum_{i=1}^{m} a_{ij} Y_i \geq 1,\ j = 1,2,\ldots,n\ ,\quad \sum_{i=1}^{m} b_{ij} Y_i' \geq 1,\ j = 1,2,\ldots,n \\
t_j(\sum_{i=1}^{m} a_{ij} Y_i - 1) = 0,\ j = 1,2,\ldots,n,\ t_j'(\sum_{i=1}^{m} b_{ij} Y_i' - 1) = 0,\ j = 1,2,\ldots,n, \\
t_j \geq 0, t_j' \geq 0,\ j = 1,2,\ldots,n,\ Y_i \geq 0, Y_i' \geq 0, i = 1,2,\ldots,m
\end{array}\right\} \quad (15)$$

After calculating Y_i from **Model 8** say Y_i^*, we determine the value of game v^* and optimal strategy y^* for player I by the following relations.

$$\left.\begin{aligned} &\frac{1}{v_1^*}=\sum_{i=1}^{m}Y_i^* \quad , \frac{1}{v_2^*}=\sum_{i=1}^{m}Y_i^{'*} \\ &y_i^*=v_1^*Y_i^*,\ i=1,2,\ldots,m \end{aligned}\right\} \qquad (16)$$

4 Numerical Example

Let us consider a bimatrix game as follows:

$$A=\begin{bmatrix} 7 & 9 & 1 & 3 \\ 5 & 2 & 4 & 8 \\ 4 & 6 & 3 & 9 \\ 8 & 2 & 5 & 6 \end{bmatrix}, \qquad B=\begin{bmatrix} 6 & 3 & 5 & 9 \\ 8 & 1 & 4 & 7 \\ 2 & 5 & 6 & 3 \\ 9 & 4 & 1 & 5 \end{bmatrix}$$

Using the **Model 7,** we obtain the following non-linear model.

Model 9 $\max:\ L(Z,Z',R,R',S,S')=-\sum_{j=1}^{4}Z_j\,ln(Z_j)-\sum_{j=1}^{4}Z'_j\,ln(Z'_j)$

$$-r_1(9Z_1+1Z_2+4Z_3+s_1^2-1)$$
$$-r_2(0Z_1+6Z_2+3Z_3+s_2^2-1)$$
$$-r_3(5Z_1+2Z_2+8Z_3+s_3^2-1)$$

subject to

$$\left.\begin{aligned} &Z_j=\exp(-1-\sum_{i=1}^{m}a_{ij}r_i),\ j=1,2,\ldots,n \\ &Z_j^{'}=\exp(-1-\sum_{i=1}^{m}b_{ij}r_i^{'}),\ j=1,2,\ldots,n \\ &\sum_{i=1}^{n}a_{ij}Z_j\le 1,\ i=1,2,\ldots,m, \quad \sum_{i=1}^{n}b_{ij}Z_j^{'}\le 1,\ i=1,2,\ldots,m \\ &r_i(\sum_{i=1}^{n}a_{ij}Z_j-1)=0,\ i=1,2,\ldots,m\ ,\ r_i^{'}(\sum_{i=1}^{n}b_{ij}Z_j^{'}-1)=0,\ i=1,2,\ldots,m \\ &r_i\ge 0, r_i^{'}\ge 0, i=1,2,\ldots,m,\ Z_j\ge 0,\ Z_j^{'}\ge 0, j=1,2,\ldots,n \end{aligned}\right\} \qquad (17)$$

Using equations (11) we obtain the following relations.

$$\frac{1}{w_1^*} = Z_1 + Z_2 + Z_3 + Z_4 \text{ , } \frac{1}{w_2^*} = Z_1' + Z_2' + Z_3' + Z_4'$$

$$z_1^* = w^* Z_1^* \text{ , } z_2^* = w^* Z_2^* \text{ , } z_3^* = w^* Z_3^* \text{ , } z_4^* = w^* Z_4^*$$

Again using the **Model 8,** we obtain the following non-linear model.

Model 10
$$L1(Y,Y',T,T',U,U') = \sum_{i=1}^{m} Y_i \, ln(\frac{Y_i}{p_i^0}) + \sum_{i=1}^{m} Y_i^{'} \, ln(\frac{Y_i^{'}}{p_i^{'0}})$$
$$-t_1(7Y_1 + 5Y_2 + 4Y_3 + 8Y_4 + u_1^2 - 1)$$
$$-t_2(9Y_1 + 2Y_2 + 6Y_3 + 2Y_4 + u_2^2 - 1)$$
$$-t_3(Y_1 + 4Y_2 + 3Y_3 + 5Y_4 + u_3^2 - 1)$$
$$-t_4(3Y_1 + 8Y_2 + 9Y_3 + 6Y_4 + u_4^2 - 1)$$
$$-t_1'(6Y_1' + 8Y_2' + 2Y_3' + 9Y_4' + u_1^{'2} - 1)$$
$$-t_2'(3Y_1' + 1Y_2' + 5Y_3' + 4Y_4' + u_2^{'2} - 1)$$
$$-t_3'(5Y_1' + 4Y_2' + 6Y_3' + 1Y_4' + u_3^{'2} - 1)$$
$$-t_4'(9Y_1' + 7Y_2' + 3Y_3' + 8Y_4' + u_4^{'2} - 1)$$

subject to
$$\left.\begin{array}{l} Y_i = p_i^0 \exp(\sum_{j=1}^{n} a_{ij} t_j - 1), \; i = 1,2,\ldots,m \\ Y_i^{'} = p_i^{'0} \exp(\sum_{j=1}^{n} b_{ij} t_j^{'} - 1), \; i = 1,2,\ldots,m \\ \sum_{i=1}^{m} a_{ij} Y_i \geq 1, \; j = 1,2,\ldots,n \quad , \quad \sum_{i=1}^{m} b_{ij} Y_i^{'} \geq 1, \; j = 1,2,\ldots,n \\ t_j(\sum_{i=1}^{m} a_{ij} Y_i - 1) = 0, \; j = 1,2,\ldots,n \\ t_j^{'}(\sum_{i=1}^{m} b_{ij} Y_i^{'} - 1) = 0, \; j = 1,2,\ldots,n \\ t_j \geq 0, t_j^{'} \geq 0, \; j = 1,2,\ldots,n \;, Y_i \geq 0, Y_i^{'} \geq 0, i = 1,2,\ldots,m \end{array}\right\} \quad (18)$$

Using equations (16), we obtain the following relations.

$$\frac{1}{w_1^*} = Y_1 + Y_2 + Y_3 + Y_4, \quad \frac{1}{w_2^*} = Y_1' + Y_2' + Y_3' + Y_4'$$

$$y_1^* = w_1^* Y_1^*, \; y_2^* = w_1^* Y_2^*, \; y_3^* = w_1^* Y_3^*$$

4.1 Result

Using Lingo 9.0 package, the solution of **Model 9** and **Model 10** for player PI and player PII can be represented in the following Table-1.

Players	*expected pay-offs*	*POSS*
PI	$v^* = (3.575, 3.3958)$	$y^* = (0.028, 0.053, 0.630, 0.289)$
PII	$w^* = (4.7184, 4.9814)$	$z^* = (0.177, .3058, .4132, 0.1040)$

Table - 1

Using Lingo 9.0 package the solution of **Model 2** and **Model 4** for player PI and player PII can be represented in the following Table-2.

Players	*expected pay-offs*	*POSS*
PI	$v^* = (3.378, 3.399)$	$y^* = (0.00, 0.07, 0.65, 0.28)$
PII	$w^* = (4.1, 5.45)$	$z^* = (0.05, 0.35, 0.60, 0.00)$

Table - 2

5 Conclusion

The application of entropy optimization for two-person non-zero-sum game problem has a vibrant research area. The entropy optimization model is non-linear model whose objective function is also an entropy function. In this model the objective function is concave (or convex) function but feasible region is constructed by convex function, so the K.K.T. optimality conditions assures us that both the models must have optimal solution. Finally, we conclude that the solution of the entropy optimization model on bimatrix game gives an alternative solution with classical bimatrix game model (shown in Table-1 and Table-2), which is highly significant for real life game problem.

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