# 2025

## M.Sc. 3rd Semester Examination

## PAPER : M250310 (MOOCS)

## NUMERICAL ANALYSIS

Full Marks : 100 Time : Three Hours

*The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.*

### Group - A

Answer any *nine* questions : 2×9=18

1. Let $x_1 = 43.5$ and $x_2 = 76.9$ be two approximate numbers and 0.02 and 0.008 are the corresponding absolute errors respectively. Then find the relative errors. 1+1

2. Let $f(x)$ be a fifth degree polynomial and the coefficient of leading term is 5. If we take $h = 2$, then what will be the fifth order forward difference of $f(x)$?

3. Set of data is given by :

| | | | | |
|---|---|---|---|---|
| $x$ | : | 1.5 | 20.0 | 2.5 |
| $y$ | : | 1.14471 | 1.25992 | 1.35721 |
| $y'$ | : | 0.25438 | 0.20999 | 0.18096 |

P.T.O.

What is the degree of the Hermite interpolating polynomial?

4. Given that $S(x) = 14T_2(x) + 10T_4(x) - 20T_6(x)$. Find the value of $S(0)$.

5. Which iteration scheme is used to find the square root of a positive number 11? Find the correct one.

   (i) $x_{n+1} = x_n + 22 / x_n$

   (ii) $x_{n+1} = x_n + 11 / x_n$

   (iii) $x_{n+1} = x_n - 11 / x_n$

   (iv) $2x_{n+1} = x_n + 11 / x_n$

6. Choose the correct statement(s)

   (i) Gauss-Seidel iteration method is applicable if the system is diagonally dominated.

   (ii) Gauss-Seidel iteration method is application for any system of linear equations.

   (iii) The rate of convergence of Gauss-Seidel iteration method is 2.

   (iv) The Gauss-Seidel iteration method converge faster than Jacobi's method.

7. Which is the correct one? For a system of linear equations, the least square method

   (i) is applicable only for consistent system of equations

   (ii) gives solution with minimum error

(iii) is useful for the inconsistent system

(iv) is useful for square and rectangular system of equations

8. Suppose the value of $y = f(x)$ is known at three points $x_0$, $x_1$ and $x_2$. If $L_i(x), i = 0, 1, 2$ represents the Lagrange's function, then find the value of $L_0(x) + L_1(x) + L_2(x)$ for all $x$.

9. Identify the correct answer(s)

   (i) Jacobi's method finds all eigenvalues of any square matrix.

   (ii) Rutishauser's method finds all the eigenvalues of a square matrix $A$, if all eigenvalues of $A$ are real.

   (iii) Power method finds largest (in magnitude) eigenvalue of any square matrix.

   (iv) Jacobi's method finds all eigenvalues of a symmetric matrix.

   (v) Power method finds all eigenvalues of any square matrix.

   (vi) Leverrier-Faddeev method finds all eigenvalues of any square matrix.

10. What are the Cote's coefficients for Simpson 1/3 formula?

P.T.O.

11. What is the value of the double integration of

$$I = \int_0^1 \int_0^1 (x+y)\,dx\,dy$$

by using Trapezoidal rule? Take $h = k = 0.5$ in both $x$ and $y$ directions.

12. Find the correct statements. The fourth order Runge-Kutta method can be used to solve,

(i) a pair of first order ODEs

(ii) a second order IVP

(iii) a second order BVP

(iv) a system of first order ODEs

13. The finite difference scheme for the BVP $y''+y'+x=0$ with boundary conditions $y(0)=0,\ y(1)=0$ and $nh = 1$ is $Ay_{i+1}+By_i+Cy_{i-1}+D=0$. Then find the values of $A, B, C, D$.

14. Answer the correct one. Crank-Nicolson method converts a parabolic PDE into

(i) a system of ordinary differential equation

(ii) two first order PDEs

(iii) a system of ODEs

(iv) a system of tri-diagonal linear equations

## Group - B

Answer any ***five*** questions : 4×5=20

15. A computer uses a 4-digit normalized floating-point system with base 10.

    (i) Represent 0.004567 in normalized floating-point form with 4 significant digits.

    (ii) Calculate the absolute and relative round-off errors for both numbers. 2+1+1

16. Given the following data :

| $x$ | $f(x)$ |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 7 |
| 3 | 13 |

    (i) Construct the forward difference table.

    (ii) Determine the first forward differences $\Delta f(x)$ and second forward differences $\Delta^2 f(x)$. 2+1+1

17. The following table is given :

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 2 | 1 |

Use Hermite interpolation to estimate $f(0.5)$.

P.T.O.

18. Consider the system of equations :

$$x^2 + y^2 - 4 = 0$$
$$x - y - 1 = 0$$

(i) Write the Newton-Raphson iteration formula for the system.

(ii) Perform one iteration starting from $(x_0, y_0) = (1,0)$. 2+2

19. Given :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \; b = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

(i) Find $A^{-1}$.

(ii) Solve $Ax = b$ using $A^{-1}$.

20. Solve the following system of linear equations using Gaussian elimination :

$$\begin{aligned} 2x + y - z &= 8 \\ -3x - y + 2z &= -11 \\ -2x + y + 2z &= -3 \end{aligned}$$

21. Use the forward difference method to approximate the derivative of $f(x) = e^x$ at $x = 1$ using step size $h = 0.1$.

22. Explain how the Monte Carlo method can be used to estimate the integral

$$\int_0^1 x^2 dx$$

## Group - C

Answer any ***four*** questions : 8×4=32

23. Find the Lagrange's interpolation formula to find the value of $y$ when $x$ = 10, if the following values are given.

| $x$ | 5 | 6 | 9 | 11 |
|---|---|---|---|---|
| $y$ | 12 | 13 | 14 | 16 |

24. Use the fourth-order Runge-Kutta method to solve the initial value problem : $dy/dx = x + y, y(0) = 1$

Compute $y$ at $x$ = 0.1 using step size $h$ = 0.1.

25. Use Simpson's 1/3 rule to evaluate the integral :

$$\int_0^2 \left(1+x^2\right) dx$$

Take $n = 4$ subintervals.

26. Evaluate approximately by trapezoidal rule, the integral

$$\int_0^1 \left(4x-3x^2\right) dx,$$

by taking $n$ = 10. Also compute the exact integral and find the absolute error and the relative error. 4+2+1+1

P.T.O.

27. Describe the Gauss-Elimination method to solve system of equations.

28. Find the eigenvalues and eigenvectors of the matrix :

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

**Internal Assessment : 30 marks**

---