

2024

2nd Semester Examination

MATHEMATICS (Honours)

Paper : C 4-T

[Differential Equations and Vector Calculus]

[CBCS]

Full Marks : 60

Time : Three Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

1. Answer any **ten** questions : 2×10=20

- (a) Define Wronskian.
- (b) Show that x^2 and $x|x|$ are linearly independent on $-\infty < x < \infty$.
- (c) Find the values of a and b for which all real solutions of the equation $y'' + 2ay' + by = \cos x$ (where a and b are real constants) will be periodic.
- (d) If $y_p(x) = x \cos 2x$ be the particular solution of the differential equation $y'' + \alpha y = -4 \sin 2x$ then find the value of α .

P.T.O.

(2)

- (e) Obtain a linear differential equation with real coefficients that is satisfied by the function

$$y = 3e^{-x} \sin 3x.$$

- (f) Reduce the differential equation

$$(x+a)^2 \frac{d^2 y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x$$

to a differential equation with constant coefficients.

- (g) Show that the vectors $4\hat{i} + 2\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $8\hat{i} + 7\hat{k}$ are co-planar.

- (h) If $\vec{r} \cdot d\vec{r} = 0$ then find $|\vec{r}|$.

- (i) If $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when $t = 2$
 $= 4\hat{i} - 2\hat{j} + 3\hat{k}$ when $t = 3$

Find the value of $\int_2^3 \vec{r} \cdot \frac{d\vec{r}}{dt} dt$.

- (j) Absolute value of vector triple product is the volume of a parallelepiped. Justify.
- (k) Differentiate the singular point and regular singular point.
- (l) Discuss the singularity of the differential equation

$$x^2(x-2)^2 \frac{d^2 y}{dx^2} + 2(x-2) \frac{dy}{dx} + (x+1)y = 0$$

- (m) Explain saddle point of a plane autonomous system.

(3)

- (n) Why linear combination of two independent solutions are also a solution of a differential equation?

- (o) Factorise $[xD^2 + (x-1)D - 1]y = x^2$ and reduce it to a first order differential equation.

2. Answer any **four** questions :

5×4=20

- (a) Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x$.

- (b) Solve by the method of variation of parameters

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$

- (c) Show that $[\vec{\beta} \times \vec{\gamma}, \vec{\gamma} \times \vec{\alpha}, \vec{\alpha} \times \vec{\beta}] = [\vec{\alpha}, \vec{\beta}, \vec{\gamma}]^2$.

- (d) Evaluate $\frac{d^2}{dt^2} \left\{ \left(\vec{r} \times \frac{d\vec{r}}{dt} \right) \times \frac{d^2 \vec{r}}{dt^2} \right\}$ where $\vec{r}$ is a vector function of t .

- (e) If $\vec{F} = xy\hat{i} - z\hat{j} + x^2\hat{k}$. Evaluate $\int_C \vec{F} \times d\vec{r}$,
 where $C : x = t, y = 2t, z = t^3; t : 0 \rightarrow 1$.

- (f) Given that $y = x$ is a solution of $(x^2 - 1) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$, find the linearly independent solutions by reducing the order. Write the general solution.

P.T.O.

(4)

3. Answer any **two** questions :

10×2=20

(a) Consider the autonomous equation

$$\frac{dx}{dt} = 10x + x^2$$

$$\frac{dy}{dt} = 20y + x^3$$

- (i) Determine the type of the critical point (0, 0).
- (ii) Obtain the differential equation of the paths and find its general solution.
- (iii) Obtain the differential equation of the paths of the corresponding reduced linear system and find its general solution. Determine the stability of the critical point. Make a sketch of the paths in the phase plane.

(b) (i) If $\vec{r}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, prove that

$$\int_1^2 \vec{r} \times \frac{d^2\vec{r}}{dt^2} dt = -14\hat{i} + 75\hat{j} - 15\hat{k}. \quad 5$$

(ii) Solve by the method of undetermined coefficients.

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2 + \sin x. \quad 5$$

(5)

(c) Use the method of Frobenius to find solution near $x = 0$ of the differential equation

$$x \frac{d^2y}{dx^2} - (x^2 + 2) \frac{dy}{dx} + xy = 0. \quad 10$$

(d) (i) Find the power series solution in power of the following initial value problem

$$(x^2 - 1) \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + xy = 0, \quad y(0) = 4,$$

$$y'(0) = 6. \quad 6$$

(ii) The acceleration of a particle at any time $t \geq 0$ is given by

$$\vec{a} = \frac{d\vec{r}}{dt} = (25 \cos 2t)\hat{i} + (16 \sin 2t)\hat{j} + (9t)\hat{k}.$$

Find $\vec{r}$. 4