(ii) Using Cauchy's general principle of convergence, prove that $\{x_n\}$, where $x_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^{n-1} \cdot \frac{1}{n}$, is a convergent sequence. (1+5)+4

(b) State and prove Heine-Borel theorem. Give an illustration which justify Heine-Borel theorem. 1+5+4

(c) Examine if the following series converges :

(i) $\sum_{n=1}^{\infty} \frac{n+1}{10^{10}(n+2)}$

(ii) $\sum_{n=1}^{\infty} \frac{n+1}{n+2}$

(iii) $\sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right)$ 3+3+4

(d) If a sequence $\{x_n\}$ of real numbers is monotone increasing and bounded above, then prove that it converges to its exact upper bound. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is monotone increasing and bounded above. 5+5

B.Sc./2nd Sem (H)/MTM/24(CBCS)

2024

2nd Semester Examination

MATHEMATICS (Honours)

Paper : C 3-T

[Real Analysis]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any ***ten*** questions : 2×10=20

(a) Define least upper bound of a bounded set and obtain it for the set

$$A = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots\right\}$$

(b) Define point of accumulation of a set and find all the points of accumulation of the set

$$E = \left\{\frac{1}{m} + \frac{1}{n} \,\middle|\, m, n = 1, 2, 3, \ldots\right\}$$

(c) If A and B are two closed sets then prove that $A \bigcup B$ and $A \bigcap B$ are both closed sets.

(d) Show that $\left\{\frac{3n+1}{n+2}\right\}$ is a bounded sequence.

(e) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges.

(f) If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then prove that $\lim_{n\to\infty} a_n = 0$.

(g) Define compact set. Give an example of it.

(h) Prove that $\lim_{n\to\infty} \frac{1}{n} = 0 \ (n \in N)$

(i) State Cauchy's principle for the convergence of an infinite series.

(j) Use root test to examine the convergence of the series

$$\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots$$

(k) Show that the series

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$$

is convergent and find its sum.

(l) What do you mean by Conditionally Convergent of a series? Give example.

(m) Examine the convergence of $\sum_{n=1}^{\infty} \frac{n^n}{n!}$.

(n) If a set S is open, then prove that its complement is closed.

(o) What is Countable set? Give an example.

2. Answer any ***four*** questions : 5×4=20

(a) Prove that the set of real numbers is not countable.

(b) State and prove Archimedean property of real numbers.

(c) Define Cauchy Sequence. Prove that the sequence $\{n^2\}$ is not a Cauchy Sequence.

(d) Prove that every bounded sequence has a convergent subsequence.

(e) State and prove density property of real numbers.

(f) Prove that $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots$ converges.

3. Answer any ***two*** questions : 10×2=20

(a) (i) State and prove Bolzano-Weierstrass theorem for sequences.

P.T.O.