2024

2nd Semester Examination

STATISTICS (Honours)

Paper : GE 2-T

(Introductory Probability)

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***five*** of the following questions : 2×5=10

1. Give the limitations of classical definition of probability.

2. Define a random experiment. Give an example.

3. State Chebyshev's inequality.

4. State the theorem of total probability for mutually exclusive events.

5. X is a random variable with probability of occurrence $P(X = x_i) = ca^i, 0 < a < 1, i = 0, 1, 2, \ldots, \infty$. Find c in terms of a.

P.T.O.

6. Define a discrete and continuous random variable. 1+1

7. Give one real-life application of the Weak Law of Large Numbers.

8. What is the relationship between a MGF and moments?

Group - B

Answer any ***four*** of the following questions : 5×4=20

9. Examine if weak law of large numbers holds for the sequence of mutually independent random variables

$$P\left[X_n = 2^{-n}\right] = P\left[X_n = -2^{-n}\right] = \frac{1}{2}.$$

10. State Lindeberg Levy central limit theorem. Show that the sequence of independent Binomial random variables satisfies the theorem.

11. In a statistics class there are 30 students of which 17 are boys and 13 are girls. In a class test, 4 boys and 5 girls obtained marks above 75%. If a student is chosen at random from the class, what is the probability of choosing a girl or a student with marks above 75%?

12. A random variable X has a p.d.f. given by $f(x) = kx$ for $0 \le x \le 2.$ Find the value of k? Calculate $P(0.5 \le X \le 1.5)$. 2+3

13. State and prove Bayes' theorem. 2+3

14. Derive the mean and variance of a Binomial random variable. $2\frac{1}{2}+2\frac{1}{2}$

Group - C

Answer any ***one*** of the following questions : 10×1=10

15. (a) Show that conditional probability satisfies the axioms of axiomatic definition of probability. 4

(b) Determine if Lindeberg Levy Central limit theorem holds for the sequence of independent random variables $\{X_k\}$ where

$$P\left(X_k = 2^{k+1}\right) = P\left(X_k = -2^{k+1}\right) = 2^{-(k+3)};$$

$$P\left(X_k = 0\right) = 1 - 2^{-(k+2)}.$$ 6

16. Define the moment-generating function (MGF). For a random variable X with probabilities $P(X=1)=\frac{1}{2}$, $P(X=1)=\frac{1}{3}$, $P(X=2)=\frac{1}{6}$, find the MGF and use it to compute $E(X)$ and $E\left(X^2\right)$. 3+3+2+2