16. Let a random vector (X_1, X_2) follow a bivariate normal distribution with $E(X_1) = E(X_2) = 1$, $V(X_1) = 4V(X_2) = 4$ and $\text{Cov}(X_1, X_2) = 1$. Derive the moment generating function of (X_1, X_2).

Total Pages : 4 B.Sc./2nd Sem (H)/STS/24(CBCS)

2024

2nd Semester Examination

STATISTICS (Honours)

Paper : C 4-T

(Probability and Probability Distributions-II)

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***five*** of the following questions : 2×5=10

1. State the properties of a cumulative distribution function.

2. Show that the function $f(x)$ given by

$$f(x) = \begin{cases} |x| & ; -1 < x < 1 \\ 0 & ; \text{otherwise} \end{cases}$$

is a possible probability density function.

3. State the uniqueness property of moment generating function of a random variable.

4. State the loss of memory property of Exponential distribution.

5. State Markov's inequality.

6. Derive an expression for the mean of a Gamma distribution.

7. If X and Y are independent random variables then show that $E(XY) = E(X)E(Y)$.

8. Show that if a random variable X follows a lognormal distribution, then log X follows a normal distribution.

Group - B

Answer any ***four*** of the following questions : 5×4=20

9. Show that if $X_1, X_2, \ldots, X_n$ be n independent normal random variables each with mean μ and variance σ^2, then $\sum_{i=1}^{n} X_i \sim N(n\mu, n\sigma^2)$.

10. The probability density function of a random variable X is $f(x) = \dfrac{1}{\pi\left\{1+(x-1)^2\right\}}; -\infty < x < \infty$. Show that the mean of the distribution does not exist.

11. Find the cumulative distribution function of a double exponential distribution with p.d.f. $f(x) = \dfrac{1}{2\sigma} e^{-\frac{|x-\mu|}{\sigma}}$; $-\infty < x, \mu < \infty, \sigma > 0$.

12. Determine the value of the constant such that the function defined by $f(x, y) = \begin{cases} \dfrac{k(1+x+y)}{(1+x)^4(1+y)^4}, & 0 \le x, y < \infty \\ = 0, & \text{otherwise} \end{cases}$

Is $f(x, y)$ a probability density function of a bivariate distribution? 2+3

13. Consider a bivariate normal distribution of the variables (X_1, X_2) with parameters $\mu_1 = 0, \mu_2 = 2, \sigma_{11} = 2,$ $\sigma_{22} = 1$ and $\rho = 0.5$. Write the bivariate normal density function, derive the regression of X_2 on $X_1 = x$ and hence show that it is linear in x. 2+2+1

14. Define probability generating function of a random variable. Discuss its important properties. 1+4

Group - C

Answer any ***one*** of the following questions : 10×1=10

15. (i) If a random vector $(X, Y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then show that the random vector $\left(U = \dfrac{X-\mu_1}{\sigma_1}, V = \dfrac{Y-\mu_2}{\sigma_2}\right) \sim BN(0, 0, 1, 1, \rho)$. 6

(ii) State and prove Chebyshev's inequality. 4

P.T.O.