2024

2nd Semester Examination

STATISTICS (Honours)

Paper : C 3-T

(Mathematical Analysis)

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***ten*** of the following questions : 2×10=20

1. Define neighbourhood of a point.
2. State the completeness property of **R**.
3. Define supremum of an ordered set. Show that it is unique.
4. Prove that $\Gamma(n+1) = n\Gamma n;\, n > 0$
5. Find the limit of the sequence $\left\{\left(\frac{(3n)!}{(n!)^3}\right)^{1/n}\right\}$.

P.T.O.

6. Define local maximum and local minimum of a function in two variables.

7. If $y = a\cos(\log x) + b\sin(\log x)$ then show that $x^2 y_2 + xy_1 + y = 0$.

8. State Lagrange's mean value theorem.

9. What do you mean by conditional convergence of a series?

10. For real constants a and b, let

$$f(x) = \frac{a\sin x - 2x}{x}, \quad \text{when } x < 0;$$
$$f(x) = bx, \quad \text{when } x \geq 0.$$

If f is a differentiable function, then find the value of $(a+b)$.

11. State two fundamental properties of integration.

12. Evaluate $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

13. Show that the function $f(x) = \psi(x) + \phi(x)$ is a discontinuous function where $\psi(x)$ is a continuous function and $\phi(x)$ is a discontinuous function.

14. Define Jacobian.

15. Evaluate $\iint e^{2x+3y} dxdy$ over the triangle bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$.

Group - B

Answer any ***four*** of the following questions : 5×4=20

16. Show that the sequence $< a_n >$ defined by $a_1 = 1$ and $a_n = \sqrt{2 + a_{n-1}}$ converges to the positive root of the equation $x^2 - x - 2 = 0$.

17. State Leibnitz theorem. Use the theorem to find y_n if $y = e^{ax} x^3$.

18. Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$, $p > 0$.

19. Evaluate $\iint xy(x+y)\, dxdy$ over the area between $y = x^2$ and $y = x$.

20. When does the function $\sin 3x - 3\sin x$ attain maximum or minimum value in $(0, 2\pi)$?

21. Establish the relation between Gamma and Beta function that is $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

P.T.O.

Group - C

Answer any ***two*** of the following questions : 10×2=20

22. (i) Test the convergence of the series $\left(\frac{1}{2}\right)^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2+\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2+\cdots\cdots$ 4

(ii) If $y=\frac{\sin^{-1}x}{\sqrt{1-x^2}}, |x|<1,$ then show that $\left(1-x^2\right)y_{n+2}-\left(2n+3\right)xy_{n+1}-\left(n+1\right)^2 y_n=0.$ 6

23. (i) Evaluate $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}\,dy\,dx$ by changing the order of the integration. 5

(ii) Use Taylor's infinite series expansion to show that $\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\cdots$ for all real x. 5

24. (i) Show that the rectangle inscribed in a circle has maximum area when it is a square. 5

(ii) Prove that $\beta\left(m+1, n\right)=\frac{m}{m+n}\beta\left(m, n\right)$. 5

25. (i) If a function $f\left(x\right)=\frac{\sin\left(a^2x^2\right)}{x}$, $x\neq 0$ and $f\left(0\right)=k$, find the value of k for which $f\left(x\right)$ is continuous at $x=0$. 5

(ii) Prove by induction that the Leibnitz rule for successive n derivative of two functions u and v is given by $\left(uv\right)^n=\sum_{i=0}^{n}\binom{n}{i}u^{n-i}v^i$. 5