2025

M.Tech. 1st Semester Examination

COMPUTER SCIENCE WITH AI

Paper : MCIC401X0

[Mathematical Foundations]

(Automata Part)

Full Marks : 100 Time : Three Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any ***five*** questions : 2×5=10

1. Define bipartite graph with an example. [CO1]

2. Find a closed form for the generating function for the following sequence :
   1, 0, –1, 0, 1, 0, –1, 0, 1, ... [CO1]

3. Show that the maximum number of edges in a simple undirected graph with n vertices is $\frac{n(n-1)}{2}$. [CO1]

P.T.O.

4. Draw the graph G corresponding to adjacency matrix given below :

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}. \qquad [CO1]$$

5. Is it possible to guess the type of language from a given grammar? Is the reverse true? Give reasons in support of your answer. [CO3]

6. Distinguish between Decidable, Recognizable and undecidable problems. [CO4]

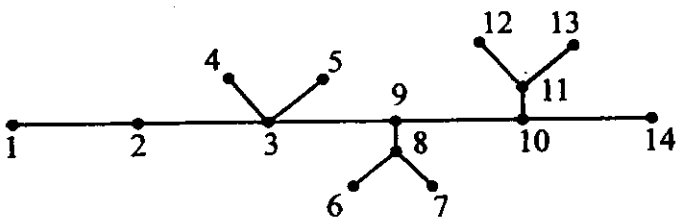
7. List the properties of Reducibility. [CO4]

8. Prove that if L_1 is regular and L_2 is context-free, then $L_1 \, uL_2^C$ is recursive. [CO4]

Group - B

Answer any ***four*** questions : 15×4=60

9. (i) Define the radius, diameter and centre of a graph. Find the radius, diameter and centre for the following graph :



(ii) Write a short note on 'Spanning tree'.

(iii) Prove that every tree has either one or two centre. 6+3+6[CO1]

10. (i) State and prove Euler's theorem for a connected planar graph.

(ii) If G is connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n-6$ and using this, verify that $K_{3,3}$ (Kuratowski's second graph) is planar or not.

(iii) Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 5+5+5[CO1]

11. (i) Define the term "Chromatic number" for graph colouring. Find the chromatic number of Peterson graph.

(ii) Show that the chromatic number of a cycle with n vertices (C_n) is 2 if n is even and 3 if n is odd.

(iii) Prove that the chromatic polynomial of any cycle C_n of length n is $p_n(\lambda) = (\lambda-1)^n + (-1)^n(\lambda-1)$. 5+5+5

12. (i) Determine the generating function of the following sequences : $a_r = 5^r + (-1)^r 3^r + 8^r + {}^3c_r$

P.T.O.

(ii) Use generating function to solve the recurrence relation :

$a_{n+2} - 2a_{n+1} + a_n = 2^n$ taking $a_0 = 2$, $a_1 = 1$.

7+8[CO1]

13. Define grammar and language. State the usefulness of the Chomsky hierarchy. Explain the Chomsky's hierarchy and corresponding type 0, type 1, type 2, and type 3 formalisms with a table indication the automata and grammar for the languages. 2+3+10[CO3]

14. Design a Turing machine and a multi-tape Turing machine that accepts $L = \{a^n b^{2n}, n > 0\}$. List all the transitions for each. 7½+7½[CO3]

15. Prove the following :

(i) The intersection of recursive and recursively enumerable languages is recursively enumerable.

(ii) If L is a recursive language, then $\Sigma^* - L$ is also recursive.

(iii) The virus detection is undecidable. 5+5+5[CO4]

16. Explain the properties of NP complete and NP Hard problems with examples. Is it substantial to claim P = NP in the computation of complexity? Explain.

10+5[CO4]

Internal Assessment - 30 marks