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Transformations in Fuzzy Planes

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ABSTRACT

In this paper, basic transformation such as complement, standard negation, cut, and shifting has been defined and by using of their figure, they have been explained. Then the integrals have been proved. At the end, falling and rising planes have been studied and a theorem about it has been proved.

1. Introduction

H.J. ohlbach [1] has shown same operations about fuzzy time intervals in 2004 that we argue a series of operations on fuzzy time planes by using it and [3],[5] and by [18-20] we show hulls and standard complement by using their forms and we show by images which are used in [21] Time planes usually don't appear from nowhere, but they are constructed from other time planes. we discussed about relationship and basic concepts of fuzzy with dimension more than one (with time dimension) , quadrangular and triangular in side of cycle is assumed as subspace that each quadrangular side is imaged as figure of planes.

we define summary of formula of basic unary transformation such as extent, scaleup, cut and time.after it,we study them by showing their figure .then we continue to argue about integrals and we prove some theorems. in fact,our gold for presenting of this paper is that there are fuzzy planes which can be defined 2-dimension basic transformation for them,be drawn their figure and be defined some theorems for them.

2. Transformations

Definition 1(Basic Unary Transformations**)** Let $p \in F_{R^k}$ be a fuzzy plane. We define the following (parameterized) plane operators:

$\hat{S} = \sup(p(x,y))$

$$\mathrm{f_m} = \text{first maximom}$$

$$l_m = last\ maximom$$

$$identity(p) \stackrel{\text{def}}{=} p$$

$$extend^{+}(p)f(x) \stackrel{\text{def}}{=} \begin{cases} sup_{f(y)\le f(x)} p(f(y)) & if\ f(x) \le p^{f_m} \\ (1,1) & otherwise \end{cases}$$

$$extend^{-}(p)f(x) \stackrel{\text{def}}{=} \begin{cases} sup_{f(y)\ge f(x)} p(f(y)) & if\ f(x) \ge p^{l_m} \\ (1,1) & otherwise \end{cases}$$

$$scaleup(p)f(x) \stackrel{\text{def}}{=} \begin{cases} \dfrac{pf(x)}{\hat{\mathrm{S}}} & if\ \hat{\mathrm{S}} \neq 0 \\ (0,0) & otherwise \end{cases}$$

$$cut_{(x_1,y_1),(x_2,y_2)}(p)(f(x)) \stackrel{\text{def}}{=} \begin{cases} (0,0) & if\ f(x) < (x_1,y_1)\ or\ f(x) \ge (x_1,y_1) \\ p(f(x)) & otherwise \end{cases}$$

$$cut_{(x_1,y_1),+}(p)(f(x)) \stackrel{\text{def}}{=} \begin{cases} (0,0) & if\ f(x) < (x_1,y_1) \\ p(f(x)) & otherwise \end{cases}$$

$$cut_{(x_1,y_1),-}(p)(f(x)) \stackrel{\text{def}}{=} \begin{cases} (0,0) & if\ f(x) \ge (x_1,y_1) \\ p(f(x)) & otherwise \end{cases}$$

$$shift_n(p)(f(x)) \stackrel{\text{def}}{=} p(f(x) - n)$$

$$times_a(p)(f(x)) \stackrel{\text{def}}{=} min\left(1, a.p(f(x))\right) \quad a \ge 0$$

$$exp_e(p)(f(x)) \stackrel{\text{def}}{=} (p)(f(x))^e \quad e \ge 0$$

$$integrate^{+}(p)(f(x)) \stackrel{\text{def}}{=} \lim_{a\to\infty} \lim_{b\to\infty} \frac{\int_{-a}^{x}\int_{-b}^{x} p(f(y_1,y_2))\, dy_1\, dy_2}{\int_{-a}^{+a}\int_{-b}^{+b} p(f(y_1,y_2))\, dy_1\, dy_2}$$

$$integrate^{-}(p)(f(x)) \stackrel{\text{def}}{=} \lim_{a\to\infty}\lim_{b\to\infty}\frac{\int_{x}^{+a}\int_{x}^{+b} p(f(y_1,y_2))\, dy_1\, dy_2}{\int_{-a}^{+a}\int_{-b}^{+b} p(f(y_1,y_2))\, dy_1\, dy_2}$$

3. Extend

$extend^{+}(p)$ follows the left part of the monotone hull of the plane until the left maximum p^{lm} is reached and then stays at fuzzy value 1. $extend^{-}(p)$ is the symmetric version of $extend^{+}(p)$.

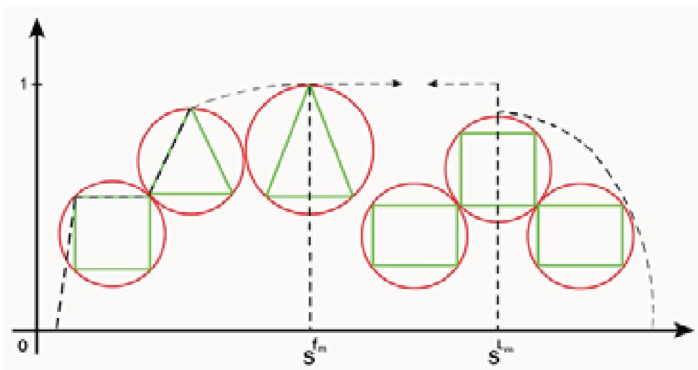

Figure 1. $extend^{+}(p)$, $extend^{-}(p)$

$extend^{+}(p)$ is useful for implementing a `before'-relation because only the left part of p is relevant for evaluating `before'. $extend^{-}(p)$ on the other hand, can be used for an `after'-relation.

4. Scale up

The scaleup-function is different to the identity function only if the height $\hat{S}$ is not 1. In this Case it scales the membership function up such that scaleup $\hat{S}$ = 1.

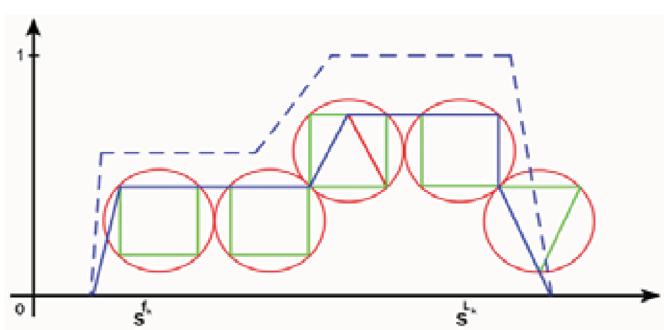

Figure 2. Scaleup

5. Cut

$cut_{(x_1,y_1),(x_2,y_2)}(p)(f(x))$ just cuts the piece between (x_1,y_1) and (x_2,y_2) out of the plane s. The resulting plane is closed at (x_1,y_1) and half open at (x_2,y_2).

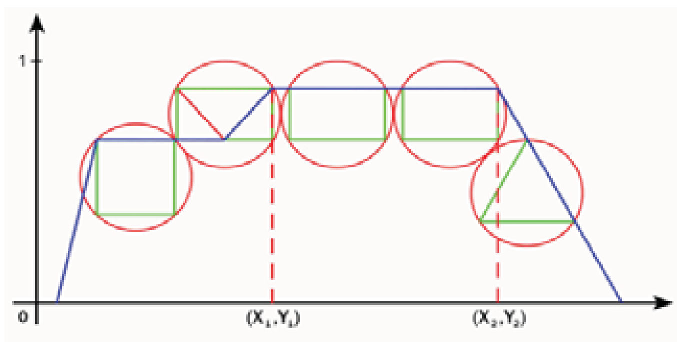

Figure 3. $cut_{(x_1,y_1),(x_2,y_2)}$

$cut_{(x_1,y_1),+}(p)(f(x))$ Cuts the part out of p before (x_1,y_1) where as $cut_{(x_1,y_1),-}(p)(f(x))$ cuts the part out of p after (x_1,y_1).

6. Shift

$shift_n$ just moves the plane by n time units.

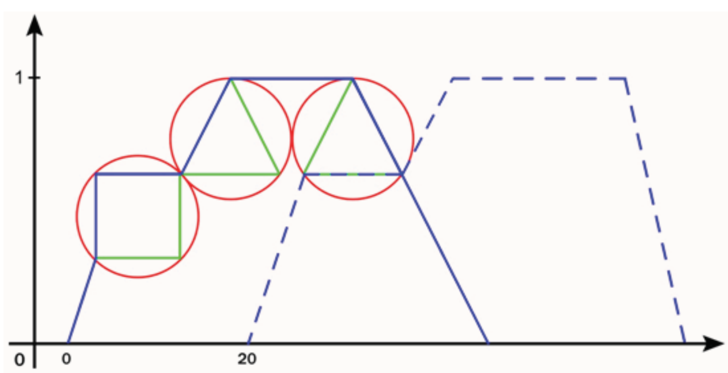

Figure 4. $shift_{20}$

7. Times

$times_a$ multiplies the membership function by a, but keeps the result smaller or equal 1. $times_a$ has no effect on crisp planes.

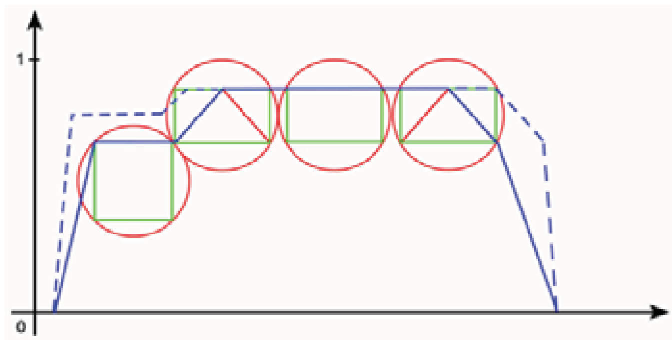

Figure 5. $times_2$

8. Exp

exp_e takes the membership function to the exponent e. It can be used to damp increases or decreases. exp_e has also no effect on crisp planes. exp_e is non-linear in the sense that straight lines are turned into curved lines.

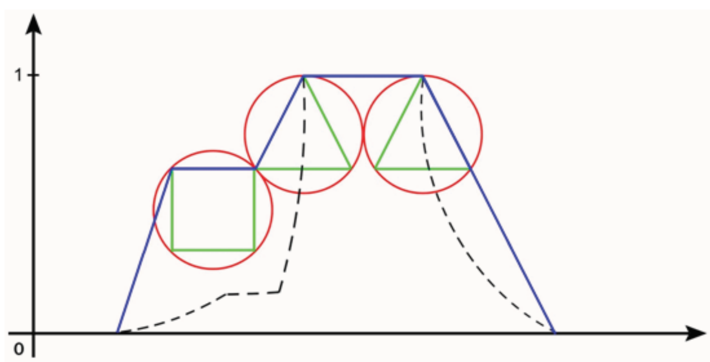

Figure 6. $\exp_3$

9. Integrate

This operator integrates over the membership function and normalizes the integral to values ≤ 1. The two integration operators $integrate^+$ and $integrate^-$ can be simplified for finite fuzzy time planes.

Proposition 1.(Integration for Finite planes) If the fuzzy plane p is finite then

$$integrate^{+}(p)(f(x)) \stackrel{\text{def}}{=} \frac{\int_{-a}^{x}\int_{-b}^{x} p(f(y_1,y_2))\, dy_1\, dy_2}{|p|}$$

And

$$integrate^{-}(p)(f(x)) \stackrel{\text{def}}{=} \frac{\int_{x}^{+a}\int_{x}^{+b} p(f(y_1,y_2))\, dy_1\, dy_2}{|p|}.$$

The proofs are straightforward [1].

Example 1. For $integrate^{+}$ and $integrate^{-}$:

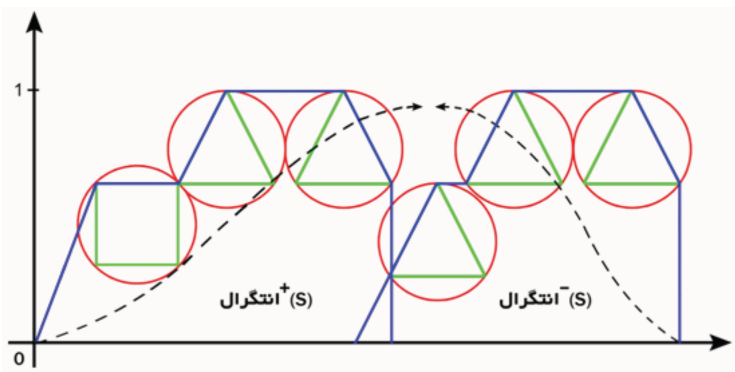

Figure 7. $integrate^{+}$ and $integrate^{-}$

The integration operator for infinite planes p with finite kernel turns the plane into a constant function which does no longer depend on the finite part of p.

Proposition 2.(Integration for planes with (Finite Kernel[1])) If the infinite fuzzy plane p has a finite kernel with $p_1 \stackrel{\text{def}}{=} p(-\infty,-\infty)$ and $p_2 \stackrel{\text{def}}{=} p(+\infty,+\infty)$ then $integrate^{+}(p)(f(x)) = \frac{p_1}{p_1+p_2}$ and $integrate^{-}(p)(f(x)) = \frac{p_2}{p_1+p_2}$.

Proof: [2].

10. Rising and Falling Fuzzy planes

A fuzzy set p is rising iff for its membership function $p(x,y) = (1,1)$ for all $(x,y) > p^{fm}$. P is falling $\boldsymbol{iff}$ for its membership function $p(x,y) = (1,1)$ for all $(x,y) < p^{lm}$.

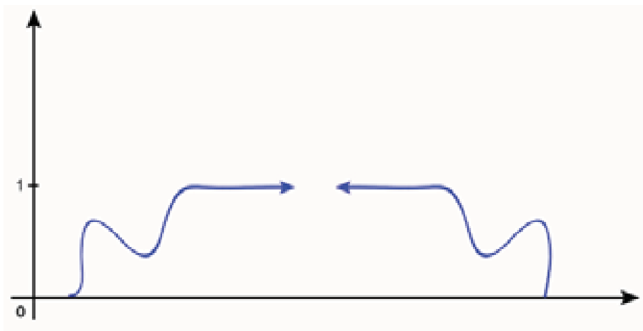

Figure 8. Rising and falling planes

Proposition 3. The basic unary transformations $extend^{+}$ and int^{+} are rising plane operators and the unary transformations $extend^{-}$ and int^{-} are falling plane operators.

Proof: [2].

11. Complement of Fuzzy Time plane.

The complement operator for fuzzy time plane is to be understood in the following sense: if for a particular plane segment (x, y) the probability to belong to a set μ is (x', y') then the probability to belong to the complement of μ is $n(x', y')$ where n is a so called negation function.

Definition 2. (Negation Function) A function $n \in Y - FCT^{1}$ satisfying the conditions:

- $n(0,0) = (1,1)$ and $n(1,1) = (0,0)$;
- n is non-increasing, i.e. $\forall((x_1, y_1), (x_2, y_2)) \in ((0,0), (1,1))$:

$$f(x_1, y_1) \leq f(x_2, y_2) \Rightarrow n(x_1, y_1) \geq n(x_2, y_2)$$

is called a negation function.

Let NF be the set of all negation functions.

Example 2. (Standard Negation and λ-Complement) The function

$$n(f(y)) \stackrel{\text{def}}{=} (1 - f(y))$$

is the standard fuzzy negation.

For any $\lambda > -1$ the so called λ-complement is the function

$$n_\lambda(y) = \frac{1-y}{1+\lambda x} \Rightarrow \begin{cases} x, y \in E, (y - x < r) \\ \quad E \subseteq R^k \end{cases}$$

$$= \frac{1-y}{1+\lambda x} \leq \frac{1-y}{1+\lambda x - \lambda y + \lambda y} \leq \frac{1-y}{1+\lambda(x-y)+\lambda y} \geq \frac{1-y}{1+\lambda r+\lambda y}$$

$$\geq \frac{1-y}{1+\lambda(r+y)}$$

$$\Rightarrow n_\lambda(y) \stackrel{\text{def}}{=} \frac{1-y}{1+\lambda y}$$

Both functions n and n_λ are negation functions in the sense:

$N(S)(f(x)) \stackrel{\text{def}}{=} n\Big(S\big(f(x)\big)\Big)$ is the standard complement operator,

$N_\lambda(S)(f(x)) \stackrel{\text{def}}{=} n_\lambda\Big(S\big(f(x)\big)\Big)$ is the λ-complement operator.

If S is a crisp plane then $N(S) = N_\lambda(S)$.

Proposition 4. (Idempotency of the negation functions) For every $(f(x), f(y)) \in ((0,0),(1,1))$ we have for the standard negation

(i): $n\Big(n\big(f(x)\big)\Big) = f(x)$ and for the λ-complement,

(ii): $n_\lambda\Big(n_\lambda\big(f(x)\big)\Big) = f(x)$.

Proof:

(i): $n\Big(n\big(f(x)\big)\Big) = f(x)$

We have $n_\lambda(y) \stackrel{\text{def}}{=} \frac{1-y}{1+\lambda y} \Rightarrow$

$$= \frac{1 - n_\lambda(f(x))}{1+\lambda\Big(n_\lambda\big(f(x)\big)\Big)} = \frac{1 - \dfrac{1-f(x)}{1+\lambda(f(x))}}{1+\lambda\left(\dfrac{1-f(x)}{1+\lambda(f(x))}\right)} = \frac{\dfrac{1+\lambda(f(x)) - 1 + f(x)}{1+\lambda(f(x))}}{\dfrac{1+\lambda(f(x)) + \lambda - \lambda f(x)}{1+\lambda(f(x))}}$$

$$= \frac{\lambda(f(x)) + f(x)}{1+\lambda} = \frac{f(x)(1+\lambda)}{1+\lambda} = f(x).$$

This property need not hold for other negation functions.

We give some examples for standard and λ-complement, The dashed lines indicate the complement

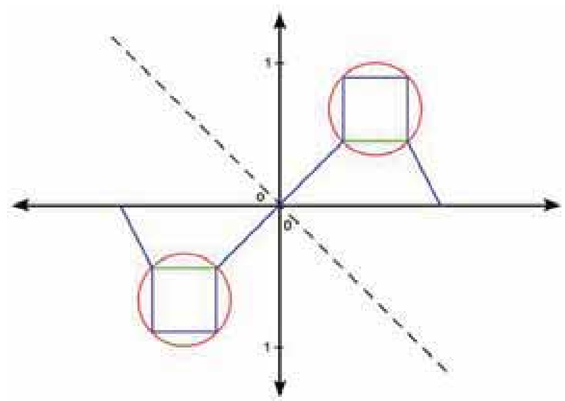

Figure 9. Standard Complement and λ −Complement for a Crisp plane

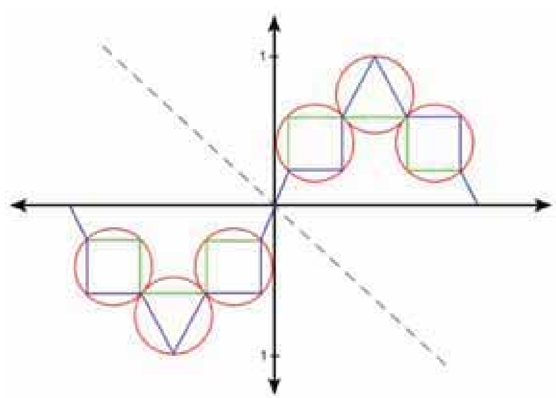

Figure 10. Standard Complement for a Fuzzy plane

If we define `tonight' as a fuzzy plane, rising from 0 at 6pm to 1 at 8pm, we could use the standard complement for `before tonight'. The term `long before tonight' must of course be represented differently to `before tonight'. A λ-complement version with $\lambda = 2$ looks as follows:

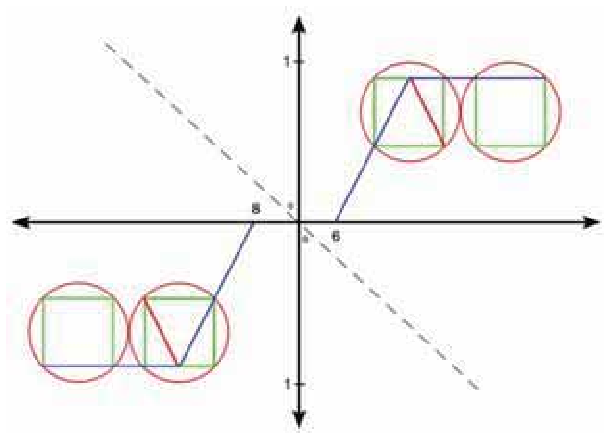

Figure 11. λ -Complement for $\lambda = 2$

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