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MHD Natural Convection Flow of Viscous Incompressible Fluid From a Vertical Flat Plate

[1]*S. F. Ahmmed and M. S. Alam Sarker*

Department of Applied Mathematics
Rajshahi University
Rajshahi-6205, Bangladesh
sfahmmed@yahoo.com

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ABSTRACT

A two-dimensional natural convection flow of a viscous incompressible and electrically conducting fluid past a vertical impermeable flat plate is considered in presence of a uniform transverse magnetic field. The governing equations are reduced to ordinary differential equations by introducing appropriate coordinate transformations. We solve that ordinary differential equations and find the velocity profiles, temperature profile, the skin friction and nusselt number. The effects of Grashof number (Gr), Hartmann number(M) and Prandtl number(Pr) on velocity profiles and temperature profiles are shown graphically.

***Keywords*: Natural convection, Grashof number(Gr), Hartmann number(M) and Prandtl number(Pr)**

1. Introduction

Studies of forced, free and mixed convection flow of a viscous incompressible fluid, in the absence of magnetic field, along a vertical surface have extensively been conducted by Sparrow and Gregg [1], Merkin [2], Loyed and Sparrow [3]. Hunt and Wilks [4] introduced a group of continuous transformations computation for the boundary layer equations between the similarity regimes for mixed convection flow. In the case of similarity regimes Hunt and Wilks [4] recognized $\zeta(=Gr_x/Re_x^2$, where Gr_x is the local Grashof number and Re_x is the local Reynolds number), a governing parameter for the flow from a vertical plate. Forced convection exists as when ζ goes to zero, which occurs at the leading edge, and the free convection limit, can be reached at large values of ζ. Perturbation solutions have been developed in both the cases, since both the forced convection and free convection limits admit similarity solution. Empirical patching of two perturbation solutions have also been carried out to provide a uniformly valid solution by Raju et al [5] which covers the whole range of the values of ζ. They obtained a finite difference solution applying an algebraic transformation $Z=1/(1+\zeta^2)$. Considering the free convection as a perturbation quantity has developed many solutions. Tingwi et al [6] have also studied the effect of forced and free convection along a vertical flat plate with uniform heat flux by considering that the buoyancy parameter ζ_p to be $Gr_x/Re_x^{5/2}$. The solutions were obtained for the small buoyancy parameter taking into the account of the perturbation technique.

Because of its application for MHD natural convection flow in the nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors, such as Sparrow and Cess [7], Reley [8] and Kuiken [9]. Simultaneous occurrence of buoyancy and magnetic field forces in the flow of an electrically conducting fluid up a hot vertical flat plate in the presence of a strong cross magnetic field was studied by Sing and Cowling [10] who had shown that regardless of strength of applied magnetic field there will always be a region in the neighborhood of the leading edge of the plate where electromagnetic forces are unimportant. Creamer and Pai [11] presented a similarity solution for the above problem with uniform heat flux by formulating it in terms of both a regular and inverse series expansions of characterizing coordinate that provided a link between the similarity states closed to and far from the leading edge. Hossain and Ahmed [13] studied the combined effect of the free and forced convection with uniform heat flux in the presence of strong magnetic field. Hossain et al [14] also investigated the MHD free convection flow along a vertical porous flat plate with a power law surface temperature in the presence of a variable transverse magnetic field employing two different methods namely (i) perturbation methods for small and large values of the scaled stream-wise transpiration velocity variable ξ_s ($=V_0\sqrt{(2x/\nu U_\infty}$, where V_0 is the transpiration velocity) and (ii) the finite difference together with the Keller box method [15]. Wilks [12] recognized a parameter ξ, defined by $\xi = (\sigma H_0^2/\rho_\infty)^2 x/g\beta(T_0-T_\infty)$ to investigate the MHD free convection flow about a semi-infinite vertical plate in a strong cross magnetic field. The work of that follows reformulates the problem in terms of coordinates expansions with respect to a non-dimensional characteristic length which is fundamental to the problem in its reflection to the relative magnitudes of buoyancy and magnetic forces at varying locations along the plate. A step by step numerical solution has been obtained to supplement the series solutions for small and large ξ.

In the above analysis, the solutions for the problem, Wilks [12] used only series solutions method. But here the governing equations are reduced to ordinary differential equations by introducing appropriate coordinate transformations. We solve that ordinary differential equations and find the velocity profiles, the temperature profile, and the skin friction and nusselt number. The effects of Grashof number (Gr), Hartmann number (M) and Prandtl number (Pr) on velocity profiles and temperature profiles are shown graphically.

2. The governing equations

The basic equations steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscosity depending on temperature and also thermal conductivity depending on temperature past a semi-infinite vertical impermeable flat plate in the presence

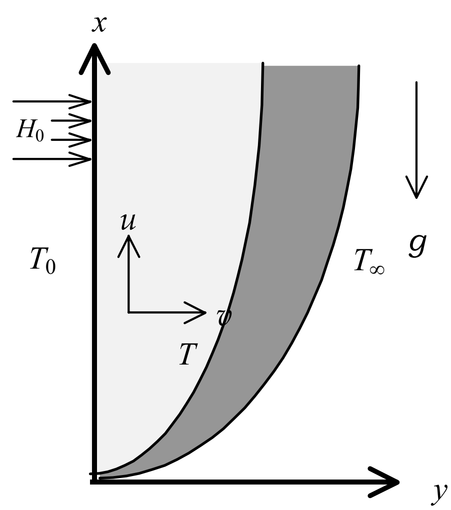


Figure 1: The flow configuration and coordinates system

of a uniformly distributed transverse magnetic field of strength H_0 are as given below

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma {H_0}^2 u}{\rho_\infty} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho_\infty c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u=v=0,\quad T=T_0 \quad \text{at } y=0\\ &u\to 0,\quad T\to T_\infty \quad \text{as } y\to\infty \end{aligned} \tag{4}$$

Here *u, v* is the velocity components associated with the direction of increase of coordinates *x* and *y* measured along and normal to the vertical plate. *T* is the temperature of the fluid in the boundary layer, *g* is the acceleration due to gravity, β is the coefficient of thermal expansion, κ is the thermal conductivity, ρ_∞ is the density of the fluid, c_p is the specific heat at constant pressure and T_∞ is the temperature of the ambient fluid and ν the kinematics viscosity of the fluid. From the continuity equation (1) we consider the velocity normal t the plate is of the form $v = -V_0$.

Now we introduce the following transformations to the equation (2) and (3)

$$Y = \frac{V_0 y}{\nu},\ U = \frac{u}{U_0},\ \theta = \frac{T - T_\infty}{T_0 - T_\infty},\ Gr = \frac{g\beta(T_0 - T_\infty)\nu}{U_0 V_0^2},$$

$$\Pr = \frac{\mu c_p}{\kappa},\ M = \frac{\sigma H_0^2 \nu}{\rho_\infty V_0^2} \tag{5}$$

and we get the following equations

$$\frac{d^2U}{dY^2} + \frac{dU}{dY} - MU + Gr\theta = 0 \tag{6}$$

$$\frac{d^2\theta}{dY^2} + \Pr\frac{d\theta}{dY} = 0 \tag{7}$$

with the boundary conditions

$$U = 0;\quad \theta = 1 \quad \text{at } Y = 0$$

$$U \to 0;\quad \theta \to 0 \quad \text{as} \quad Y \to \infty \tag{8}$$

3. Result

The equations (6) and (7) with the boundary condition (8) are simply ordinary differential equation. We can find the solution of that equation (6) and (7) as the following form equation (9) and (10) respectively

$$U = \frac{Gr}{\Pr^2 - \Pr - M}\left[\exp\left(-\frac{1+\sqrt{1+4M}}{2}Y\right) - \exp(-\Pr Y)\right] \tag{9}$$

$$\theta = \exp(-\Pr Y) \tag{10}$$

We also find the skin friction and the rate of heat transfer as follows

$$C_f = \left(\frac{dU}{dY}\right)_{Y=0} = \frac{Gr}{\Pr^2 - \Pr - M}\left[\Pr - \frac{1+\sqrt{1+4M}}{2}\right] \tag{11}$$

$$Nu = -\left(\frac{d\theta}{dY}\right)_{Y=0} = \Pr \tag{12}$$

4. Discussion

In this section we discuss the results obtained from the solution of the equations governing the MHD free convection flow of a viscous incompressible and electrically conducting fluid with uniform viscosity and uniform thermal conductivity, in the presence of uniform transverse magnetic field along an impermeable vertical flat plate. For the solutions of the governing non-similar equations, a group of transformations is used to get a group of ordinary differential equations. Here we consider the low Prandl number (Pr) liquid metals. We have pursued solutions for Pr equals 0.92 for ammonia, 0.72 for air, 0.05 for lithium and 0.004 for sodium at 649^0c.

We have calculated the skin friction and the rate of heat transfer in the equation no. (13) and (14). For increasing values of Prandtl number, the local skin friction decreases monotonically .The skin friction increase at the decreasing values of the magnetic field parameter, M and increasing values of Grashof number Gr.

The velocity profiles for Gr=1, 5, 10, 15, Gr= -1, -5, -10, -15, M=2, 3, 4, 5, M= -2, -3, -4, -5, Pr=0.92, 0.72, 0.05 and 0.004 are depcited in the figures from Fig.2 to Fig.5. In the Fig.2(A) and Fig.2(B), the velocity profiles for Pr=0.72 and M=1.5 with Gr=1, 5, 10, 15 and Gr= -1, -5, -10, -15 are plotted. Here we see that the velocity profile increases with the increasing values of Grashof number (Gr). These effects are significant near the surface of the plate. In the downstream region these profiles go to a limiting point. In the Fig.3(A) and Fig.3(B), the velocity profiles for Pr=0.72 and M=2, 3, 4, 5 with Gr= 5, -5 are plotted. These velocity profile increases with the increasing values of magnetic field parameter (M). We see that for Y=1 these effect is significant and for large values of Y these profiles go to a limiting point.

The velocity profiles for Gr=5 and M=5 are shown in the Fig.4(A) and 4(B) for Pr equals 0.92 for ammonia, 0.72 for air, 0.05 for lithium and 0.004 for sodium at 649^0c. Again the temperature profiles for for Gr=5 and M=5 are shown in the Fig.5 for Pr = 0.004, 0.05, 0.72 and 0.92. Here for increasing values of Pr, the velocity profiles as well as temperature profiles decreases. In the upstream regime the effect of Pr on the velocity profiles is not remarkable. The velocity profile and temperature profiles that we obtained is similar to that of Wilks. The effects of different Pr are significant near the surface of the plate.

5. Conclusion

In this paper, the problem of magnetohydrodynamic free convection flow along a vertical flat plate is investigated. The local non-similarity equations governing the flow for the case of uniform viscosity and thermal conductivity are developed. The numerical computations were carried out only for the case of assisting flow for the fluids having low Prandtl number appropriate for liquid metals (Pr 0.92 for ammonia, 0.72 for air, 0.05 for lithium and 0.004 for sodium at 649^0c).

The results thus we obtained for skin friction and the rate of heat transfer coefficient are presented in tabular form in the case of different properties of the liquid metals. The velocity profiles and the thermal conductivity profiles are given graphically in the in the case of constant viscosity. Finally, followings may be concluded from the throughout present investigations:

1. For increasing values of Prandtl number, the local skin friction decreases monotonically.
2. The skin friction increase at the decreasing values of the magnetic field parameter, M and increasing values of Grashof number Gr.
3. Profiles for the velocity as well as the thermal conductivity decrease due to the increasing values of the Prandtl number, Pr.

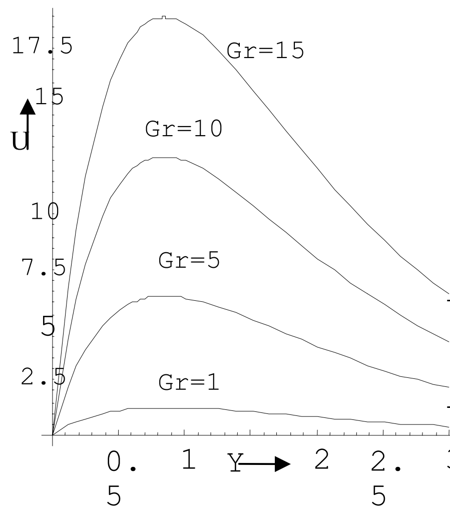


Fig.2(A). Velocity profiles for different values of Gr with Pr=0.72 and M=1.5

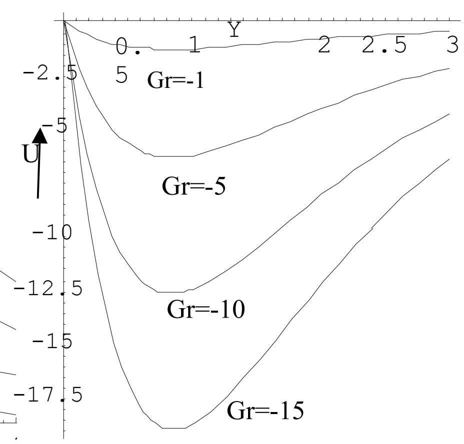
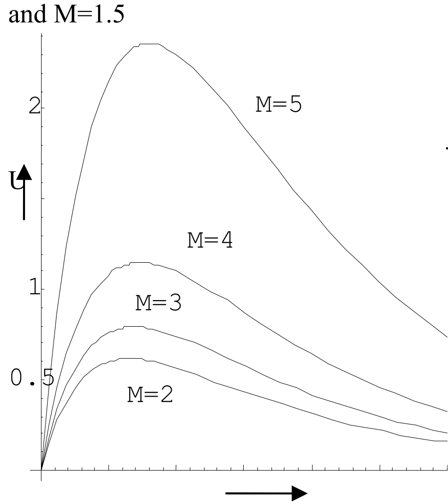
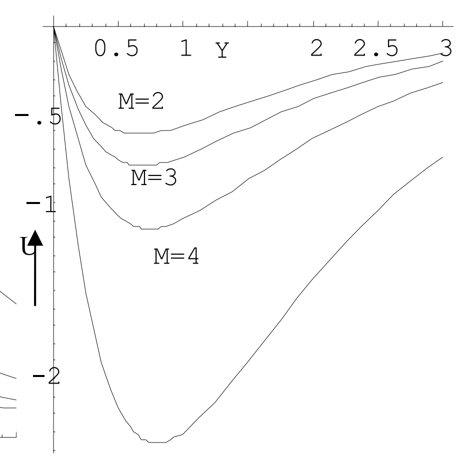
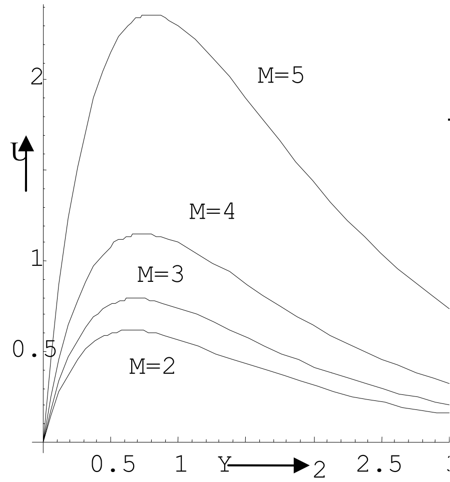


Fig.2(B). Velocity profiles for different values of Gr with Pr=0.72 and M=1.5







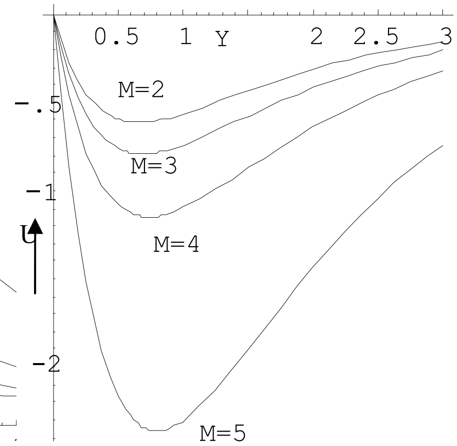
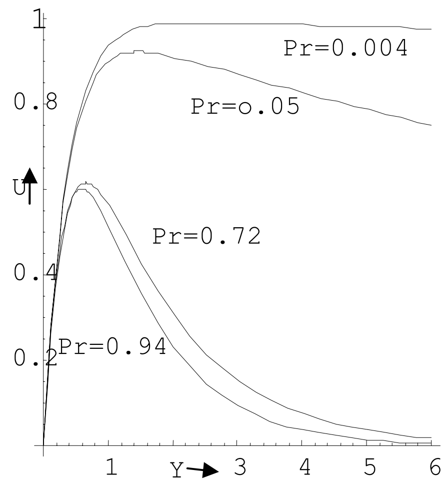


Fig.3(A). Velocity profiles for different values of M with Pr=0.72 and Gr=5

Fig.3(B). Velocity profiles for different values of M with Pr=0.72 and Gr=-5



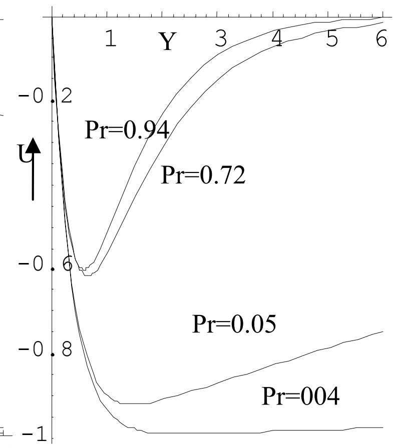
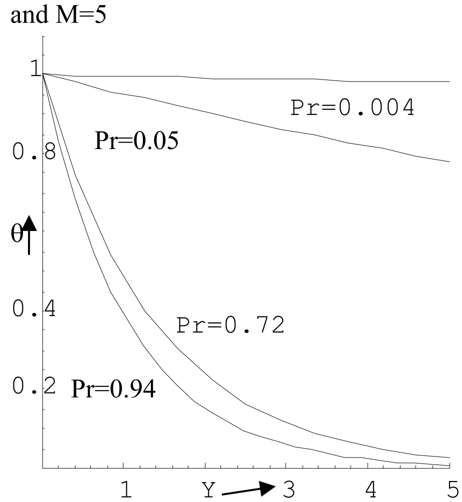


Fig.4(A). Velocity profiles for different values of Pr with Gr=5 and M=5

Fig. 4(B). Velocity profiles for different values of Pr with Gr=-5 and M=5



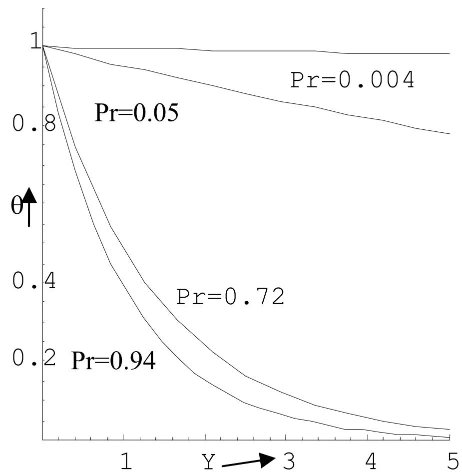


Fig. 5.Temperature profiles for different values of Pr with Gr=5 and M=5

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