

2025

M.Sc. 3rd Semester Examination

PHYSICS

Paper : PHS - 301

(Quantum Mechanics-III)

Full Marks : 25

Time : One Hour

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

1. Answer any *two* questions :

2×2=4

- (a) Compute the total cross section for an electron of energy E scattering elastically from a nucleus with phase shifts of $\delta_0 = \pi/3$, $\delta_1 = \pi/6$ and $\delta_l = 0$ for all $l \geq 2$.
- (b) Write down the Klein-Gordon equation and show that in the non-relativistic limit the equation reduces to the Schrödinger equation.
- (c) For an Helium atom, suppose that the ground state wave function in the central field approximation is

given by $\psi(\vec{x}_1, \vec{x}_2) = \frac{Z^3}{\pi a_0^3} e^{-Z(\eta_1 + \eta_2)/a_0} \chi_{\text{spin}}$. Giving

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reasons, write down the explicit expression for χ_{spin} in terms of spins of the individual electrons.

- (d) Explain stimulated emission and absorption with diagrams and write the corresponding Dirac delta functions in the corresponding transition probabilities.

2. Answer any *two* questions :

4×2=8

- (a) Consider a system of two electrons in a one dimensional infinite potential well with $V(x) = 0$ for $0 \leq x \leq L$ and $V(x) = \infty$ otherwise. Write down the eigenfunctions including spin and eigenvalues corresponding to the ground and first excited state of the system.
- (b) Consider scattering of a particle from a potential $V(r) = V_0 e^{-(r^2/R^2)}$, where V_0 and R are constants. Compute the differential cross section in the Born approximation (up to the first order).
- (c) Write down the Dirac equation in terms of $\bar{\alpha}$ and β matrices. Further using the Dirac representation of $\bar{\alpha}$ and β matrices, find the energy eigenvalues and eigenvectors in the rest frame of the particle.
- (d) Consider a Hamiltonian of the form

$$H = H_0 + V(t), \text{ where } H_0 = \frac{1}{(2m)} \hat{p}^2 + \frac{1}{2} m \omega^2 \hat{x}^2$$

(3)

and $V(t) = \frac{\hat{x}}{(\sqrt{\pi\tau})} e^{-(t/\tau)^2}$. A particle initially

(i.e., $t \rightarrow -\infty$) is in the ground state of H_0 . Determine the transition probability from the ground state of to an excited state of H_0 as $t \rightarrow \infty$, up to first order in perturbation theory.

3. Answer any *one* question :

8×1=8

- (a) (i) Show that the Lippmann-Schwinger equation in the position basis can be written as

$$\langle \bar{x} | \psi^{(\pm)} \rangle = \langle \bar{x} | \phi \rangle + \frac{2m}{\hbar^2} \int d^3x' G_{\pm}(\bar{x}, \bar{x}') \langle \bar{x} | V | \psi^{(\pm)} \rangle$$

where m is the mass of a particle. Further, find the expression for $G_{\pm}(x, x')$ for a particle of energy E .

- (ii) Consider a system of two spin 1/2 particles and suppose that the system is in the state $|+-\rangle$ for $t \leq 0$. The Hamiltonian for the system for $t > 0$ is given by

$$H = \left(\frac{4\Delta}{\hbar^2} \right) \bar{S}_1 \cdot \bar{S}_2. \text{ Assume that the } H = 0$$

for $t \leq 0$. Find the probability as a function of time for finding the particles in the $|+-\rangle$ state using first order perturbation theory, treating H as a perturbation.

4+4

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- (b) (i) An electron is subjected to a time varying magnetic field given by

$$\vec{B}(t) = B_0(\hat{x} \cos \omega t + \hat{y} \sin \omega t)$$

If at $t = 0$, the electron is in the eigenstate $|+\rangle_z$. Find the probability of finding the electron in the state $|+\rangle_z$ as a function of time. [Use the Rabi's formula].

- (ii) Compute the total phase shift for the low-energy scattering of a particle of mass m from an attractive square well potential, $V = -V_0$ for $r < a$ and $V = 0$ for $r > a$ ($V_0 > 0$). 4+4

List of Formulae

1. Gaussian integrals :

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\pi/a};$$

$$\int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx = (-1)^n \frac{\partial^n}{\partial a^n} \int_{-\infty}^{\infty} e^{-ax^2} dx;$$

$$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = e^{b^2/4a} \sqrt{\pi/a}$$

2. Rabi's formula :

$$|c_2(t)|^2 = \frac{\gamma^2/\hbar^2}{\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4} \sin^2 \left\{ \left[\gamma^2/\hbar^2 + (\omega - \omega_{21})^2/4 \right] t \right\}$$

(5)

3. Wave functions of 1-D harmonic oscillator :

$$\psi(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} e^{-\frac{m\omega x^2}{2\hbar}} H_n \left(\sqrt{\frac{m\omega}{\hbar}} x \right);$$

$$H_n(x) = (-1)^n e^{x^2} \cdot \frac{d^n}{dx^n} (e^{-x^2})$$

Internal Assessment : 5 marks
