PG/3rd Sem /PHS/25(Old)

2025

M.Sc. 3rd Semester Examination

PHYSICS

Paper : PHS - 301.1 & 301.2

(Old Syllabus)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

PHS - 301.1

(Quantum Mechanics-III)

1. Answer any *two* questions : 2×2=4

   (a) Compute the total cross section for an electron of energy E scattering elastically from a nucleus with phase shifts of $\delta_0 = \pi/3, \delta_1 = \pi/6$ and $\delta_l = 0$ for all $l \geq 2$.

   (b) Write down the Klein-Gordon equation and show that in the non-relativistic limit the equation reduces to the Schrödinger equation.

   (c) For an Helium atom, suppose that the ground state wave function in the central field approximation is

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given by $\psi(\vec{x}_1, \vec{x}_2) = \dfrac{Z^3}{\pi a_0^3} e^{-Z(r_1+r_2)/a_0} \chi_{\text{spin}}$. Giving reasons, write down the explicit expression for χ_{spin} in terms of spins of the individual electrons.

(d) Explain stimulated emission and absorption with diagrams and write the corresponding Dirac delta functions in the corresponding transition probabilities.

2. Answer any *two* questions : 4×2=8

(a) Consider a system of two electrons in a one dimensional infinite potential well with $V(x) = 0$ for $0 \le x \le L$ and $V(x) = \infty$ otherwise. Write down the eigenfunctions including spin and eigenvalues corresponding to the ground and first excited state of the system.

(b) Consider scattering of a particle from a potential $V(r) = V_0 e^{-(r^2/R^2)}$, where V_0 and R are constants. Compute the differential cross section in the Born approximation (up to the first order).

(c) Write down the Dirac equation in terms of $\vec{\alpha}$ and β matrices. Further using the Dirac representation of $\vec{\alpha}$ and β matrices, find the energy eigenvalues and eigenvectors in the rest frame of the particle.

(d) Consider a Hamiltonian of the form $H = H_0 + V(t)$, where $H_0 = \frac{1}{(2m)}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$ and $V(t) = \frac{\hat{x}}{\left(\sqrt{\pi}\tau\right)}e^{-(t/\tau)^2}$. A particle initially (i.e., $t \to -\infty$) is in the ground state of H_0. Determine the transition probability from the ground state of to an excited state of H_0 as $t \to \infty$, up to first order in perturbation theory.

3. Answer any *one* question : 8×1=8

(a) (i) Show that the Lippmann-Schwinger equation in the position basis can be written as

$$\left\langle \vec{x} \mid \psi^{(\pm)} \right\rangle = \langle \vec{x} \mid \phi \rangle + \frac{2m}{\hbar^2} \int d^3x' G_{\pm}(\vec{x}, \vec{x}') \left\langle \vec{x} \middle| V \middle| \psi^{(\pm)} \right\rangle$$

where m is the mass of a particle. Further, find the expression for $G_{\pm}(x, x')$ for a particle of energy E.

(ii) Consider a system of two spin 1/2 particles and suppose that the system is in the state $|+-\rangle$ for $t \leq 0$. The Hamiltonian for the system for $t > 0$ is given by $H = \left(4\Delta/\hbar^2\right)\vec{S}_1.\vec{S}_2$. Assume that the $H = 0$ for $t \leq 0$. Find the probability as a function of

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time for finding the particles in the $|-+\rangle$ state using first order perturbation theory, treating H as a perturbation. 4+4

(b) (i) An electron is subjected to a time varying magnetic field given by

$$\vec{B}(t) = B_0(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$$

If at $t = 0$, the electron is in the eigenstate $|+\rangle_z$. Find the probability of finding the electron in the state $|+\rangle_z$ as a function of time. [Use the Rabi's formula].

(ii) Compute the total phase shift for the low-energy scattering of a particle of mass m from an attractive square well potential, $V = -V_0$ for $r < a$ and $V = 0$ for $r > a$ $(V_0 > 0)$. 4+4

List of Formulae

1. Gaussian integrals :

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\pi / a};$$

$$\int_{-\infty}^{\infty} x^{2n} e^{-x^2}\,dx = (-1)^n \frac{\partial^n}{\partial a^n}\int_{-\infty}^{\infty} e^{-ax^2}\,dx;$$

$$\int_{-\infty}^{\infty} e^{-ax^2+bx}\,dx = e^{b^2/4a}\sqrt{\pi / a}$$

2. Rabi's formula :

$$|c_2(t)|^2 = \frac{\gamma^2/\hbar^2}{\gamma^2/\hbar^2 + (\omega-\omega_{21})^2/4}\sin^2\left\{\left[\gamma^2/\hbar^2 + (\omega-\omega_{21})^2/4\right]t\right\}$$

3. Wave functions of 1-D harmonic oscillator :

$$\psi(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} c^{\frac{-m\omega x^2}{2\hbar}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right);$$

$$H_n(x) = (-1)^n e^{x^2}\frac{d^n}{dx^n}(e^{-x^2})$$

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PHS - 301.2

(Statistical Mechanics-I)

1. Answer any *two* questions : 2×2=4

   (a) In Liouville's theorem, if ρ does not depend on q and p explicitly, then prove that ρ = constant.

   (b) For ideal gas, find an expression of microstates $\Omega(E, N, V : \delta E)$.

   (c) How concept of negative temperature is introduced?

   (d) For spin ½ particle $\hat{\rho} = a\hat{I} + b\vec{\sigma}$ show that $\hat{\rho} = \frac{1}{2}\left(\hat{I} + \vec{P}\cdot\vec{\sigma}\right)$.

2. Answer any *two* questions : 4×2=8

   (a) If Hamiltonian of a single particle

   $$H = \frac{p_\theta^2}{2mr^2} + \frac{p_\phi^2}{2mr^2 \sin^2\theta}.$$

   Find an expression of equation of state.

   (b) Deduce an expression of canonical partition function for 3D quantum Harmonic oscillator.

   (c) If $\hat{H} = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{pmatrix}$.

   Prove that diagonal elements are stationary in time and the off-diagonal elements oscillate with frequency $(\varepsilon_1 - \varepsilon_2)/\hbar$.

(d) Find an expression of Fermi-Dirac distribution function from grand canonical ensemble.

3. Answer any ***one*** question : 8×1=8

(a) Prove that $\rho_{\bar{r},\bar{r}'} = \frac{1}{V} e^{-\frac{mk_B T}{2\hbar^2}\left(\bar{r}-\bar{r}'\right)^2}$ for a particle in a box of infinite potential.

(b) Consider a box containing an ideal classical gas at pressure P and temperature T. The walls of the box have N_0 absorbing sites, each of which can absorb one molecule of the gas. Let $-\varepsilon$ be the energy of an absorbed molecule.

(i) Find the fugacity of the gas in terms of T and P.

(ii) Find the average number of absorbed molecules at low and high-pressure limits.

Internal Assessment : 10 marks