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First Order Reactant in MHD Turbulence Before the Final Period of Decay for the Case of Multi-point and Multi-time in Presence of Dust Particles

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ABSTRACT

Following Deissler's theory, the decay for the concentration fluctuation of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence at times before the final period in presence of dust particle for the case of multi-point and multi-time is studied and have considered correlations between fluctuating quantities at two and three point. Two and three point correlation equations are obtained and the set of equations is made to determinate by neglecting the quadruple correlations in comparison to the second and third order correlations. The correlation equations are converted to spectral form by taking their Fourier transforms. Finally we obtained the decay law of magnetic energy for the concentration fluctuations before the final period in presence of dust particle for the case of multi-point and multi-time by integrating the energy spectrum over all wave numbers.

Keywords: MHD Turbulence, First order Reactant, Dust particle, Decay before the final period

1. Introduction

The relative motion of dust particle and the air will dissipate energy because of the drag between dust and air, and extract energy from turbulent intensity is reduced than the Reynolds stresses will be decreased and the force required to maintain a given flow rate will likewise be reduced. Sarker [1] discussed the vorticity covariance of dusty fluid turbulence in a rotating frame.

. The behavior of dust particles in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid. Saffman [2] derived and equation that describe the motion of a fluid containing small dust particle, which is applicable to laminar flows as well as turbulent flow. Using the Saffman's equations Michael and Miller [3] discussed the motion of dusty gas occupying the semi-infinite space above a rigid plane boundary. Sarker and Rahman [4] considered dust particles on their won works. Sinha [5] studied the effect of dust particles on the acceleration covariance of ordinary turbulence. Kishore and Sinha [6] also studied the rate of change of vorticity covariance in dusty fluid turbulence.

Deissler [7,8] developed a theory "decay of homogeneous turbulence for times before the final period". Using Deissler's theory, Loeffler and Deissler [9] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. In their approach they considered the two and three-point correlation equations and solved these equations after neglecting fourth and higher order correlation terms. Using Deissler theory, Kumar and Patel [10] studied the first-order reactant in homogeneous turbulence before the final period
of decay for the case of multi-point and single-time correlation. Kumar and Patel [11] extended their problem [10] for the case of multi-point and multi-time concentration correlation. Patel [12] also studied in detail the same problem to carry out the numerical results. Sarker and Kishore [13] studied the decay of MHD turbulence at time before the final period using chandrasekher's relation [14]. Sarker and Islam [15] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Azad and Sarker [16] studied the Decay of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Islam and Sarker [17] also studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time.

Following Deissler's theory we studied the magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction in MHD turbulence before the final period of decay for the case of multi-point and multi-time in presence of dust particle. Here, we have considered the two-point, two-time and three-point, three-time correlation equations and solved these equations after neglecting the fourth-order correlation terms. Finally we obtained the decay law for magnetic field energy fluctuation of concentration of dilute contaminant undergoing a first order chemical reaction in MHD turbulence for the case of multi-point and multi-time in presence of dust particle is obtained. If the fluid is clean , the equation reduces to one obtained earlier by Islam and Sarker[17].

2. Basic Equations

The equations of motion and continuity for viscous, incompressible dusty fluid MHD turbulent flow are given by

$$\frac{\partial u_i}{\partial t}+\frac{\partial}{\partial x_k}(u_i u_k - h_i h_k) = -\frac{\partial w}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k \partial x_k}+f(u_i - v_i) \tag{1}$$

$$\frac{\partial h_i}{\partial t}+\frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \lambda\frac{\partial^2 h_i}{\partial x_k \partial x_k} \tag{2}$$

$$\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_K} = -\frac{K}{m_s}(v_i - u_i) \tag{3}$$

with $\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0$. (4)

Here, u_i, turbulence velocity component; h_i, magnetic field fluctuation component; v_i, dust particle velocity component; $w(\hat{x},t) = \frac{P}{\rho} + \frac{1}{2}\langle h^2 \rangle$ total MHD pressure $p(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; ν, Kinematic viscosity; $\lambda = \nu / P_M$, magnetic diffusivity; P_M, magnetic prandtl number; x_k, space co-ordinate; the subscripts can take on the values 1, 2 or 3 and the repeated subscripts in a term indicate a summation; $\in_{mkl}$, alternating tensor; $f = \frac{KN}{\rho}$, dimension of frequency ; N, constant number density of dust particle $m_s = \frac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s; ρ_s, constant density of the material in dust particle.

3. Two-Point, Two-Time Correlation and Spectral Equations

Under the conditions that (I) the turbulence and the concentration magnetic field are homogeneous (ii) the chemical reaction has no effect on the velocity field and (iii) the reaction rate and the magnetic diffusivity are constant, the induction equation of a magnetic field fluctuation of concentration of a dilute contaminant undergoing a first order chemical reaction at the points p and p' separated by the vector $\hat{r}$ could be written as

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \lambda \frac{\partial^2 h_i}{\partial x_k \partial x_k} - R h_i \quad (5)$$

and $$\frac{\partial h'_j}{\partial t'} + u'_k \frac{\partial h'_j}{\partial x'_k} - h'_k \frac{\partial u'_j}{\partial x'_k} = \lambda \frac{\partial^2 h'_j}{\partial x'_k \partial x'_k} - R h'_j \ . \quad (6)$$

Multiplying equation (5) by h'_j and equation (6) by h_i and taking ensemble average, we get

$$\frac{\partial \langle h_i h'_j \rangle}{\partial t} + \frac{\partial}{\partial x_k}\left[\langle u_k h_i h'_j \rangle - \langle u_i h_k h'_j \rangle\right] = \lambda \frac{\partial^2 \langle h_i h'_j \rangle}{\partial x_k \partial x_k} - R\langle h_i h'_j \rangle \quad (7)$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t'}+\frac{\partial}{\partial x'_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial x'_k\partial x'_k}-R\langle h_i h'_j\rangle \ . \qquad (8)$$

Angular bracket $\langle -----\rangle$ is used to denote an ensemble average.

Using the transformations

$$\frac{\partial}{\partial x_k}=-\frac{\partial}{\partial r_k},\ \frac{\partial}{\partial x'_k}=\frac{\partial}{\partial r_k},\ \left(\frac{\partial}{\partial t}\right)t'=\left(\frac{\partial}{\partial t}\right)\Delta t-\frac{\partial}{\partial \Delta t},\ \frac{\partial}{\partial t'}=\frac{\partial}{\partial \Delta t} \qquad (9)$$

into equations (7) and (8), we obtain

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)-\frac{\partial}{\partial r_k}\left[\langle u_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right](\hat{r},\Delta t,t)$$

$$=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad (10)$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u'_j h_i h'_k\rangle\right](\hat{r},\Delta t,t)=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle \qquad (11)$$

Using the relations of Chandrasekhar [14]

$$\langle u_k h_i h'_j\rangle=-\langle u'_k h_i h'_j\rangle,\quad \langle u'_j h_i h'_k\rangle=\langle u_i h_k h'_j\rangle .$$

Equations (10) and (11) become

$$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+2\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=2\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-2R\langle h_i h'_j\rangle \qquad (12)$$

and $$\frac{\partial\langle h_i h'_j\rangle}{\partial t}+\frac{\partial}{\partial r_k}\left[\langle u'_k h_i h'_j\rangle-\langle u_i h_k h'_j\rangle\right]=\lambda\frac{\partial^2\langle h_i h'_j\rangle}{\partial r_k\partial r_k}-R\langle h_i h'_j\rangle \ . \qquad (13)$$

Now we write equations (12) and (13) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms:

$$\langle h_i h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \psi_i \psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad (14)$$

and $$\langle u_i h_k h'_j\rangle(\hat{r},\Delta t,t)=\int_{-\infty}^{\infty}\langle \alpha_i \psi_k \psi'_j\rangle(\hat{K},\Delta t,t)\exp\left[\hat{i}(\hat{K}.\hat{r})\right]d\hat{K} \qquad (15)$$

Interchanging the subscripts i and j then interchanging the points p and p′ gives

$$\langle u'_k h_i h'_j \rangle (\hat{r}, \Delta t, t) = \langle u_k h_i h'_j \rangle (-\hat{r}, -\Delta t, t + \Delta t)$$

$$= \int_{-\infty}^{\infty} \langle \alpha_i \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \exp\left[\hat{i}\left(\hat{K}.\hat{r}\right)\right] d\hat{K} \tag{16}$$

where $\hat{K}$ is known as a wave number vector and $d\hat{K}$ =dK_1 dK_2 dK_3. The magnitude of $\hat{K}$ has the dimension 1/length and can be considered to be the reciprocal of an eddy size. Substituting of equation (14) to (16) in to equations (12) and (13) leads to the spectral equations

$$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial t} + 2\left[\lambda k^2 + R\right] \langle \psi_i \psi'_j \rangle = 2ik_k \left[\langle \alpha_i \psi_k \psi'_j \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \tag{17}$$

and $$\frac{\partial \langle \psi_i \psi'_j \rangle}{\partial \Delta t} + \left[\lambda K^2 + R\right] \langle \psi_i \psi'_j \rangle = ik_k \left[\langle \alpha_i \psi_k \psi'_j \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_j \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \tag{18}$$

The tensor equations (17) and (18) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial t} + 2\left[\lambda K^2 + R\right] \langle \psi_i \psi'_i \rangle = 2ik_k \left[\langle \alpha_i \psi_k \psi'_i \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \tag{19}$$

and $$\frac{\partial \langle \psi_i \psi'_i \rangle}{\partial \Delta t} + \left[\lambda K^2 + R\right] \langle \psi_i \psi'_i \rangle = ik_k \left[\langle \alpha_i \psi_k \psi'_i \rangle (\hat{K}, \Delta t, t) - \langle \alpha_k \psi_i \psi'_i \rangle (-\hat{K}, -\Delta t, t + \Delta t) \right] \tag{20}$$

The terms on the right side of equations (19) and (20) are collectively proportional to what is known as the magnetic energy transfer terms.

4. Three-Point, Three-Time Correlation and Spectral Equations

Similar procedure can be used to find the three-point correlation equations. For this purpose we take the momentum equation of dusty fluid MHD turbulence at the point P and the induction equations of magnetic field fluctuations, governing the concentration of a dilute contaminant undergoing a first order chemical reaction at p′ and p′′ separated by the vector $\hat{r}$ and $\hat{r}'$ as

$$\frac{\partial u_l}{\partial t} + u_k \frac{\partial u_l}{\partial x_k} - h_k \frac{\partial h_l}{\partial x_k} = -\frac{\partial w}{\partial x_l} + \nu \frac{\partial^2 u_l}{\partial x_k \partial x_k} + f(u_l - v_l) \quad \text{------- (21)}$$

$$\frac{\partial h'_i}{\partial t'} + u'_k \frac{\partial h'_i}{\partial x'_k} - h'_k \frac{\partial u'_i}{\partial x'_k} = \lambda \frac{\partial^2 h'_i}{\partial x'_k \partial x'_k} - Rh'_i \quad \text{-------- (22)}$$

$$\frac{\partial h''_j}{\partial t''} + u''_k \frac{\partial h''_j}{\partial x''_k} - h''_k \frac{\partial u''_j}{\partial x''_k} = \lambda \frac{\partial^2 h''_j}{\partial x''_k \partial x''_k} - Rh''_j \,. \quad \text{------- (23)}$$

Multiplying equation (21) by $h'_i\ h''_j$, equation (22) by $u_l h''_j$ and equation (23) by $u_l h'_i$, taking ensemble average, one obtains

$$\frac{\partial\langle u_l h'_i h''_j\rangle}{\partial t}+\frac{\partial}{\partial x_k}\left[\langle u_k u_l h'_i h''_j\rangle-\langle h_k h_l h'_i h''_j\rangle\right]=\frac{\partial\langle w h'_i h''_j\rangle}{\partial x_l}+\nu\frac{\partial^2\langle u_l h'_i h''_j\rangle}{\partial x_k\partial x_k}$$

$$+f(\langle u_l h'_i h''_j\rangle-\langle v_l h'_i h''_j\rangle) \qquad \text{----- (24)}$$

$$\frac{\partial\langle u_l h'_i h''_j\rangle}{\partial t'}+\frac{\partial}{\partial x'_k}\left[\langle u_l u'_k h'_i h''_j\rangle-\langle u_l u'_i h'_k h''_j\rangle\right]=\lambda\frac{\partial^2\langle u_l h'_i h''_j\rangle}{\partial x'_k\partial x'_k}-R\langle u_l h'_i h''_j\rangle \qquad \text{-------- (25)}$$

and
$$\frac{\partial\langle u_l h'_i h''_j\rangle}{\partial t''}+\frac{\partial}{\partial x''_k}\left[\langle u_l u''_k h'_i h''_j\rangle-\langle u_l u''_j h'_i h''_k\rangle\right]=\lambda\frac{\partial^2\langle u_l h'_i h''_j\rangle}{\partial x''_k\partial x''_k}-R\langle u_l h'_i h''_j\rangle \qquad \text{------- (26)}$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r'_k}\right),\ \frac{\partial}{\partial x'_k}=\frac{\partial}{\partial r_k},\quad \frac{\partial}{\partial x''_k}=\frac{\partial}{\partial r'_k},$$

$$\left(\frac{\partial}{\partial t}\right)t',t''=\left(\frac{\partial}{\partial t}\right)\Delta t,\Delta t'-\frac{\partial}{\partial\Delta t}-\frac{\partial}{\partial\Delta t'},$$

$$\frac{\partial}{\partial t'}=\frac{\partial}{\partial\Delta t},\ \frac{\partial}{\partial t''}=\frac{\partial}{\partial\Delta t'}$$ into equations (24) to (26), we have

$$\frac{\partial\langle u_l h'_i h''_j\rangle}{\partial t}-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r'_k}\right)\left[\langle u_k u_l h'_i h''_j\rangle-\langle h_k h_l h'_i h''_j\rangle\right]+\frac{\partial}{\partial r_k}\left[\langle u_l u'_k h'_i h''_j\rangle-\langle u_l u'_i h'_k h''_j\rangle\right]$$

$$+\frac{\partial}{\partial r'_k}\left[\langle u_l u''_k h'_i h''_j\rangle-\langle u_l u''_j h'_i h''_k\rangle\right]=-\left(\frac{\partial}{\partial r_l}+\frac{\partial}{\partial r'_l}\right)\langle w h'_i h''_j\rangle+\nu\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r'_k}\right)^2\langle u_l h'_i h''_j\rangle$$

$$+\lambda\left[\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k \partial r_k}+\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k' \partial r_k'}\right]+f\left(\left\langle u_l h_i' h_j''\right\rangle-\left\langle v_l h_i' h_j''\right\rangle\right)$$

---------- (27)

$$\frac{\partial\left\langle u_l h_i' h_J''\right\rangle}{\partial \Delta t}+\frac{\partial}{\partial r_k}\left[\left\langle u_l u_k' h_i' h_j''\right\rangle-\left\langle u_l u_i' h_k' h_j''\right\rangle\right]=\lambda\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k \partial r_k}-R\langle u_l h_i' h_j''\rangle$$

------ (28)

and $$\frac{\partial\left\langle u_l h_i' h_j''\right\rangle}{\partial \Delta t'}+\frac{\partial}{\partial r_k'}\left[\left\langle u_l u_k'' h_i' h_j''\right\rangle-\left\langle u_l u_j'' h_i' h_k''\right\rangle\right]=\lambda\frac{\partial^2\left\langle u_l h_i' h_j''\right\rangle}{\partial r_k' \partial r_k'}-R\langle u_l h_i' h_j''\rangle$$

. ----------(29)

In order to convert equations (27)–(29) to spectral form, we can define the following six dimensional Fourier transforms:

$$\left\langle u_l h_i' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

----(30)

$$\left\langle u_l u_k' h_i' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \phi_K' \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

-----(31)

$$\left\langle w h_i' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

--------(32)

$$\left\langle u_k u_l h_i' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_K \phi_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}'$$

-------(33)

$$\left\langle h_k h_l h_i' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_K \beta_l \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' -$$

---- (34)

$$\left\langle u_l u_i' h_K' h_j''\right\rangle\left\langle \hat{r},\hat{r}',\Delta t,\Delta t',t\right\rangle=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_l \phi_i' \beta_i' \beta_j''\right\rangle\left(\hat{K},\hat{K}',\Delta t,\Delta t',t\right)\exp\left[\hat{i}\left(\hat{K}.\hat{r}+\hat{K}'.\hat{r}'\right)\right]d\hat{K}d\hat{K}' -$$

-----(35)

$$\langle v_l h_i' h_j'' \rangle \langle \hat{r}, \hat{r}, \Delta t, \Delta t', t \rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \langle \mu_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) \exp\left[\hat{i}\left(\hat{K}.\hat{r} + \hat{K}'.\hat{r}'\right)\right] d\hat{K} d\hat{K}' \text{ -}$$

------- (36)

Interchanging the points P′ and P′′ along with the indices i and j result in the relations

$$\left\langle u_l u_k'' h_i' h_j'' \right\rangle = \left\langle u_l u_k' h_i' h_j'' \right\rangle .$$

By use of these facts and the equations (30)-(36), we can write equations (27)-(29) in the form

$$\frac{\partial}{\partial t}\langle \phi_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda\left[(1+P_M)\left(k^2 + k'^2\right) + 2P_M kk' + \frac{2R}{\lambda} - \frac{1}{\lambda} f\right]$$

$$\times \left\langle \phi_l \beta_i' \beta_j'' \right\rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) = \left[i\left(k_k + k_k'\right)\left\langle \phi_k \phi_l \beta_i' \beta'' \right\rangle - i\left(k_k + k_k'\right)\left\langle \beta_k \beta_l \beta_i' \beta_j'' \right\rangle\right.$$

$$- i\left(k_k + k_k'\right)\left\langle \phi_l \phi_k' \beta_i' \beta_j'' \right\rangle + i\left(k_k + k_k'\right)\left\langle \phi_l \phi_i' \beta_k' \beta_j'' \right\rangle - i\left(k_l + k_l'\right)\left\langle \gamma \beta_i' \beta_j'' \right\rangle$$

$$\left. - f\left\langle \mu_l \beta_i' \beta_j'' \right\rangle \right]\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) \quad \text{--------- (37)}$$

$$\frac{\partial}{\partial \Delta t}\langle \phi_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda\left[K^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

$$= -ik_k \left\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \right\rangle \left(\hat{K}, \hat{K}' \Delta t, \Delta t', t\right) + ik_k \left\langle \phi_l \phi_i' \beta_k' \beta_j'' \right\rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

--------- (38)

and $\frac{\partial}{\partial \Delta t'}\langle \phi_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda\left[K^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_j'' \rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$

$$= -ik_k' \left\langle \phi_l\, \phi_k' \beta_i' \beta_j'' \right\rangle \left(\hat{K}, \hat{K}' \Delta t, \Delta t', t\right) + ik_k' \left\langle \phi_l \phi_i' \beta_k' \beta_j'' \right\rangle \left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

--------- (39)

If the derivative with respect to x_l is taken of the momentum equation (21) for the point P, the equation multiplied by $h_i' h_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2 \left\langle w h_i' h_j'' \right\rangle}{\partial x_l \partial x_l} = \frac{\partial^2}{\partial x_l \partial x_k}\left(\left\langle u_l u_k h_i' h_j'' \right\rangle - \left\langle h_l h_k h_i' h_j'' \right\rangle\right) \quad \text{---------- (40)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_l \partial r_l} + 2\frac{\partial^2}{\partial r_l \partial r_l'} + \frac{\partial^2}{\partial r_l' \partial r_l'}\right]\langle w h_i' h_j'' \rangle = \left[\frac{\partial^2}{\partial r_l \partial r_k} + \frac{\partial^2}{\partial r_l' \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k'} + \frac{\partial^2}{\partial r_l' \partial r_k'}\right]\times$$

$$\left(\langle u_l u_k h_i' h_j'' \rangle - \langle h_l h_k h_i' h_j'' \rangle\right) \qquad \text{---------(41)}$$

Taking the Fourier transforms of equation (28)

$$-\langle \gamma \beta_i' \beta_j'' \rangle = \frac{\left(k_l k_k + k_l' k_k + k_l k_k' + k_l' k_k'\right)\left(\langle \phi_l \phi_k \beta_i' \beta_j'' \rangle - \langle \beta_l \beta_k \beta_i' \beta_j'' \rangle\right)}{k_l k_l + 2k_l k_l' + k_l' k_l'} .$$

--------(42)

Equation (42) can be used to eliminate $\langle \gamma \beta_i' \beta_j'' \rangle$ from equation (37)

The tensor equations (37) to (39) can be converted to scalar equation by contraction of the indices i and j and inner multiplication by k_l

$$\frac{\partial}{\partial t} k_l \langle \phi_l \beta_i'' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda\left[(1+P_M)\left(k^2 + k'^2\right) + 2P_M k k' + \frac{2R}{\lambda} - \frac{1}{\lambda} f\right]\times$$

$$\langle \phi_l \beta_i'' \beta_i'' \rangle\left(\hat{K}, \hat{K}, \Delta t, \Delta t', t\right) = i\left(k_k + k_k'\right)\langle \phi_k \phi_l \beta_i' \beta_j'' \rangle$$

$$\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) - i\left(k_k + k_k'\right)\langle \beta_K \beta_l \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) - i\left(k_k + k_k'\right)$$

$$\langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + i\left(k_k + k_k'\right)\langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) - i\left(k_l + k_l'\right)$$

$$\langle \gamma \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) - f\langle \mu_l \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) \qquad \text{--------- (43)}$$

$$\frac{\partial}{\partial \Delta t} k_L \langle \phi_l \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + \lambda\left[K^2 + \frac{R}{\lambda}\right]\langle \phi_l \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

$$= -ik_k \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right) + ik_k \langle \phi_l \phi_i' \beta_k' \beta_i'' \rangle\left(\hat{K}, \hat{K}', \Delta t, \Delta t', t\right)$$

--------- (44)

and $\frac{\partial}{\partial \Delta t'} k_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[K'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t)$

$$= -ik_k' \langle \phi_l \phi_k' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + ik_k' \langle \phi_l \phi_i' \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) \,. \qquad \text{---------- (45)}$$

5. Solution for Times Before the Final Period

It is known that the equation for final period of decay is obtained by considering the two-point correlations after neglecting third-order correlation terms. To study the decay for times before the final period, the three-point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. The term $\langle \gamma \beta_i' \beta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equations (43) to (45)

$$\frac{\partial}{\partial t} K_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' + \frac{2R}{\lambda} - \frac{1}{\lambda} fs \right] \times \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \qquad \text{---------- (46)}$$

$$\frac{\partial}{\partial \Delta t} K_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[k^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0 \qquad \text{----- (47)}$$

and $\frac{\partial}{\partial \Delta t'} K_l \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) + \lambda \left[k'^2 + \frac{R}{\lambda} \right] \langle \phi_l \beta_i' \beta_i'' \rangle (\hat{K}, \hat{K}', \Delta t, \Delta t', t) = 0$ ------- (48)

where $\langle \mu_l \beta_i' \beta_i'' \rangle = C \langle \phi_l \beta_i' \beta_i'' \rangle$ and 1-C=S, here C and S are arbitrary constant.

Integrating equations (46) to (48) between t_o and t, we obtain

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = f_l \exp\left\{ -\lambda \left[(1 + P_M)(k^2 + k'^2) + 2P_M kk' \cos\theta + \frac{2R}{\lambda} - \frac{1}{\lambda} fs (t - t_o) \right\} \right.$$

$$k_l \langle \phi_l \beta_i' \beta_i'' \rangle = g_l \exp\left[-\lambda \left(K^2 + \frac{R}{\lambda} \right) \Delta t \right]$$

and $k_l \langle \phi_l \beta_i' \beta_i'' \rangle = q_l \exp\left[-\lambda \left(k'^2 + \frac{R}{\lambda} \right) \Delta t' \right]$.

For these relations to be consistent, we have

$$k_l\langle\phi_l\beta_i'\beta_i''\rangle = k_l\langle\phi_l\beta_i'\beta_i''\rangle_o \exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+k'^2\Delta t'$$
$$+2P_M kk'\cos\theta(t-t_o)+\frac{2R}{\lambda}\left(t-t_0+\frac{\Delta t+\Delta t'}{2}\right)-\frac{fs}{\lambda}(t-t_o)]\} \quad \text{-------- (49)}$$

where θ is the angle between $\hat{K}$ and $\hat{K}'$ and $\langle\phi_l\beta_i'\beta_i''\rangle_o$ is the value of $\langle\phi_l\beta_i'\beta_i''\rangle$ at $t = t_o$, $\Delta t = \Delta t' = 0$, $\lambda = \frac{\nu}{P_M}$

By letting $\hat{r}' = 0$, $\Delta t' = 0$ in the equation (30) and comparing with equations (15) and (16) we get

$$\langle\alpha_i\psi_k\psi_i'\rangle(\hat{K},\Delta t,t) = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}' \quad \text{----------- (50)}$$

and $\langle\alpha_i\psi_k\psi_i'\rangle(-\hat{K},-\Delta t,t+\Delta t) = \int_{-\infty}^{\infty}\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},\hat{K}',\Delta t,o,t)d\hat{K}'$.

---------- (51)

Substituting equation (49) to (51) into equation (19), one obtains

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t) = \int_{-\infty}^{\infty}2ik_l\,[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}',\Delta t,0,t)$$
$$-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}',\Delta t,0,t)\ ]_o \exp[-\lambda\{(1+P_M)(k^2+k'^2)(t-t_o)$$
$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t)-\frac{fs}{\lambda}(t-t_o)\}]\,d\hat{k}$$

------------(52)

Now, $d\hat{K}'$ can be expressed in terms of k′ and θ as $-2\pi k' d(\cos\theta) dk'$ (cf. Deissler [8])

i.e. $d\hat{K}' = -2\pi k' d(\cos\theta)dk'$ --------- (53)

Substituting of equation (53) in equation (52) yields

$$\frac{\partial}{\partial t}\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda\left[k^2+\frac{R}{\lambda}\right]\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t) = 2\int_0^{\infty}2\pi ik_l\,[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')$$
$$-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_o k'^2\left[\int_{-1}^{1}\exp\{-\lambda[(1+P_M)(k^2+k'^2)(t-t_o)\right.$$
$$\left.+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta+\frac{2R}{\lambda}(t-t_0+\Delta t/2)-\frac{fs}{\lambda}(t-t_o)]\}d(\cos\theta)\right]dk'$$

----------(54)

The quantity $[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')]_0$ depends on the initial conditions of the turbulence.

In order to find the solution completely and following Loeffler and Deissler [9] we assume that

$$ik_l\left[\langle\phi_l\beta_i'\beta_i''\rangle(\hat{K},\hat{K}')-\langle\phi_l\beta_i'\beta_i''\rangle(-\hat{K},-\hat{K}')\right]_o=\frac{-\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \quad \text{---------- (55)}$$

where δ_0 is a constant determined by the initial conditions. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave numbers for positive value of δ_0.
Substituting equation (55) into equation (54) we get

$$\frac{\partial}{\partial t}2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)+2\lambda\left[k^2+R/\lambda\right]2\pi\langle\psi_i\psi_i'\rangle(\hat{K},\Delta t,t)=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k'^2$$

$$\left[\int_{-1}^{1}\exp\left\{-\lambda\left[(1+P_M)(k^2+k'^2)(t-t_o)+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta\right.\right.\right.$$

$$\left.\left.\left.+\frac{2R}{\lambda}(t-t_0+\Delta t/2)-\frac{fs}{\lambda}(t-t_o)\right]\right\}d(\cos\theta)\right]d\hat{k}' \quad \text{--------- (56)}$$

Multiplying both sides of equation (56) by k^2, we get

$$\frac{\partial E}{\partial t}+2\lambda k^2E=F \quad \text{---------- (57)}$$

where, $E=2\pi k^2\langle\psi_i\psi_i'\rangle$, E is the magnetic energy spectrum function and F is the magnetic energy transfer term and is given by

$$F=-2\delta_0\int_0^\infty\left(k^2k'^4-k^4k'^2\right)k^2k'^2\times\left[\int_{-1}^{1}\exp\left\{-\lambda\left[(1+P_M)(k^2+k'^2)(t-t_o)\right.\right.\right.$$

$$+k^2\Delta t+2P_M(t-t_o)kk'\cos\theta$$

$$\left.\left.\left.+\frac{2R}{\lambda}(t-t_0+\Delta t/2)-\frac{fs}{\lambda}(t-t_o)\right]\right\}d(\cos\theta)\right]dk' \quad \text{---------(58)}$$

Integrating equation (58) with respect to cosθ and k' we have

$$F=-\frac{\delta_oP_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o)^{3/2}(1+P_M)^{5/2}}\exp\left\{\left(\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_M k^4}{4P_M^2\lambda^2(t-t_o)^2(1+P_M)}\right.$$
$$+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8$$
$$-\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(t-t_o+\Delta t)^{3/2}(1+P_M)^{5/2}}\exp\left\{\left(\frac{fs}{\lambda}\right)(t-t_o)\right\}\times$$
$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta t/2)\right]\times\left[\frac{15P_M k^4}{4v^2(t-t_o+\Delta t)^2(1+P_M)}\right.$$
$$\left.+\left\{\frac{5P_M^2}{(1+P_M)^2}-\frac{3}{2}\right\}\frac{k^6}{P_M\lambda(t-t_o+\Delta t)}+\left\{\frac{P_M^3}{(1+P_M)^3}-\frac{P_M}{1+P_M}\right\}k^8\right] \quad .$$

------ (59)

The series of equation (59) contains only even power of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three-point and three-times.

If we integrate equation (59) for Δt=0 over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \qquad \text{---------- (60)}$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity. Physically it was to be expected as F is a measure of the energy transfer and the total energy transferred to all wave numbers must be zero.

The linear equation (57) can be solved to give

$$E=\exp\left[-2\lambda k^2(t-t_o+\Delta t/2)\right]\int F\exp\left[2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]dt$$
$$+J(k)\exp\left[-2\lambda(k^2+R/\lambda(t-t_o+\Delta t/2)\right] \qquad \text{---------- (61)}$$

where $J(k)=\frac{N_o k^2}{\pi}$ is a constant of integration and can be obtained as by Crrsin[18].

Substituting the values of F from equation (59) into equation (61) gives the equation

$$E=\frac{N_o k^2}{\pi}\exp\left[-2\lambda(k^2+R/\lambda)(t-t_o+\Delta t/2)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\times\exp\left[fs(t-t_o)\right]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{1+P_M}{1+2P_M}\Delta t\right)-2R(t-t_0+\Delta\, t/2)\right]$$
$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0)^{3/2}}-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0)^{1/2}}\right.$$
$$\left.+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9}{3(1+P_M)^{5/2}}F(\omega)\right]+\frac{\delta_o P_M\sqrt{\pi}}{4\lambda^{3/2}(1+P_M)^{7/2}}\ \exp[fs(t-t_o)]$$

$$\exp\left[\frac{-k^2\lambda(1+2P_M)}{1+P_M}\left(t-t_o+\frac{P_M}{1+P_M}\Delta t\right)-2R(t-t_0+\Delta\, t/2)\right]$$
$$\left[\frac{3k^4}{2P_M\lambda^2(t-t_0+\Delta t)^{5/2}}+\frac{(7P_M-6)k^6}{3\lambda(1+P_M)(t-t_0+\Delta t)^{3/2}}\right.$$
$$\left.-\frac{4(3P_M^2-2P_M+3)k^8}{3(1+P_M)^2(t-t_0+\Delta t)^{1/2}}+\frac{8\sqrt{\lambda}(3P_M^2-2P_M+3)k^9F(\omega)}{(1+P_M)^{5/2}P_M^{1/2}}\right] \qquad \text{---------} (62)$$

where $F(\omega)=e^{-\omega^2}\int_0^{\omega}e^{x^2}dx$, $\omega=k\sqrt{\frac{\lambda(t-t_0)}{1+P_M}}$ or $k\sqrt{\frac{\lambda(t-t_0+\Delta t)}{1+P_M}}$

By setting $\hat{r}=0$, j=i, $d\hat{k}=-2\pi k^2 d(\cos\theta)d\hat{k}$ and $E=2\pi k^2\langle\psi_i\psi_j'\rangle$ in equation (14) we get the expression for magnetic energy decay law as

$$\frac{\langle h_i h_i'\rangle}{2}=\int_0^{\infty}E\,dk \qquad \text{---------} (63)$$

Substituting equation (62) into equation (63) and after integration, we get

$$\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}(T+\Delta T/2)^{3/2}}\exp[-2R(T+\Delta\, T/2)]$$
$$+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[-2R(T+\Delta\, T/2)]\exp[(fs)]$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)T^{3/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{7/2}}+\frac{5P_M(7P_M-6)}{16(1+2P_M)(T+\Delta T)^{3/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{7/2}}$$

$$+\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8(1+2P_M)T^{1/2}\left(T+\frac{1+P_M}{1+2P_M}\Delta T\right)^{9/2}}+$$

$$\frac{35P_M\left(3P_M^2-2P_M+3\right)}{8(1+2P_M)(T+\Delta T)^{1/2}\left(T+\frac{P_M}{1+2P_M}\Delta T\right)^{9/2}}$$

$$+\frac{8P_M\left(3P_M^2-2P_M+3\right)(1+2P_M)^{5/2}}{3.2^{23/2}(1+P_M)^{11/2}}\sum_{n=0}^{\infty}\frac{1.3.5............(2n+9)}{n!(2n+1)2^{2n}(1+P_M)^n}\times$$

$$\left.\left\{\frac{T^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}+\frac{(T+\Delta T)^{(2n+1)/2}}{\left(T+\Delta T/2\right)^{(2n+1)/2}}\right\}\right] \qquad \text{--------- (64)}$$

where T=t-t_0 .

For $T_m=T+\Delta T/2$, equation (64) takes the form

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6(1+P_M)(1+2P_M)^{5/2}}\exp[fs]\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2(1+2P_M)}\right)^{5/2}}\right.$$

$$+\frac{5P_M(7P_M-6)}{16(1+2P_M)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2(1+2P_M)}\right)^{7/2}}$$

$$+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}+---\Bigg]\Bigg] \qquad \text{-------(65)}$$

This is the decay law of magnetic energy fluctuations of concentration of a dilute contaminant undergoing a first order chemical reaction of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle.

6. Results and Discussion

In equation (65) we obtained the decay law of magnetic energy fluctuations of a dilute contaminant undergoing a first order chemical reaction before the final period considering three-point correlation terms for the case of multi-point and multi-time in MHD turbulence in presence of dust particle. If the fluid is clean then f=0, the equation (65) becomes

$$\frac{\langle h^2\rangle}{2}=\exp[-2RT_m]\left[\frac{N_0}{8\sqrt{2\pi}\lambda^{3/2}T_m^{3/2}}+\frac{\pi\delta_0}{4\lambda^6\left(1+P_M\right)\left(1+2P_M\right)^{5/2}}\right.$$

$$\times\left[\frac{9}{16\left(T_m-\Delta T/2\right)^{5/2}\left(T_m+\frac{\Delta T}{1+2P_M}\right)^{5/2}}+\frac{9}{16\left(T_m+\Delta T/2\right)^{5/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{5/2}}\right.$$

$$+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m-\frac{\Delta T}{2}\right)^{3/2}\left(T_m+\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}$$

$$+\frac{5P_M\left(7P_M-6\right)}{16\left(1+2P_M\right)\left(T_m+\frac{\Delta T}{2}\right)^{3/2}\left(T_m-\frac{\Delta T}{2\left(1+2P_M\right)}\right)^{7/2}}+---\Bigg]\Bigg] \qquad \text{----------(66)}$$

which was obtained earlier by Sarker and Islam [17].

If we put ΔT=0, R=0, in equation (66) we can easily find out

$$\frac{\langle h^2\rangle}{2}=\frac{\langle h_i h_i'\rangle}{2}=\frac{N_0T^{-3/2}}{8\sqrt{2\pi}\lambda^{3/2}}+\frac{\pi\delta_0}{4\lambda^6\left(1+P_M\right)\left(1+2P_M\right)^{5/2}}T^{-5}\left\{\frac{9}{16}+\frac{5}{16}\frac{P_M(7P_M-6)}{1+2P_M}+-----\right\} \qquad \text{-----(67)}$$

which is same as obtained earlier by Sarker and Kishore [13].

This study shows that due to the effect of rotation of fluid in the flow field with chemical reaction of the first order in the concentrarion the magnetic field fluctuation in MHD turbulence in presence of dust particle for the case of multi-point and multi-time i,e.the turbulent energy decays more slowly than the energy for clean fluid and the rate is governed by $\exp[fs]$. Here the chemical reaction ($R \neq 0$) in dusty fluid MHD turbulence for the case of multi-point and multi-time causes the concentration to decay more they would for clean fluid and it is governed by $\exp\left[-\left\{2RT_M - fs\right\}\right]$

The first term of right hand side of equation (65) corresponds to the energy of magnetic field fluctuation of concentration for the two-point correlation and the second term represents magnetic energy for the three-point correlation. In equation (65), the term associated with the three-point correlation die out faster than the two-point correlation. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (65). For large times the last term in the equation (65) becomes negligible, leaving the -3/2 power decay law for the final period.

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