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An Analytical Study of Temperature Control in Hyperthermia by Microwave

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ABSTRACT

An analytical study was considered on the strategy of attaining a desired rise of temperature χ^* at a particular point of location of the tumour $x = x_1$ situated inside the muscle during a fixed time T by controlling optimally time dependent heating power $Q_2(t)$ (wm^{-2}) applied on the surface of the tissue consisting of skin , fat, muscle, point tumour, muscle given in Fig A when the surface cooling temperature is taken as constant throughout the entire tissue with the aid of finite difference method and Maximal Principle under calculus of variation.

Keyword: Optimal control, microwave, heating power, surface cooling temperature, scattering coefficient, tumour, hyperthermia .

Notations:

χ = temperature $({}^0c)$

χ_1 = arterial temperature $({}^0c)$

χ_0 = initial temperature $({}^0c)$

ρ_i = density of tissue $(kg\ m^{-3})$ in i^{th} layer.

c_i = specific heat of tissue $(J\ kg^{-1}\ k^{-1})$ in i^{th} layer.

k_i = thermal conductivity of tissue $(w\ m^{-1}\ k^{-1})$ in i^{th} layer.

b_i = product of flow and heat capacity of blood $(w\ m^{-3}\ k^{-1})$ in i^{th} layer.

χ^* = desired rise of temperature $({}^0c)$ of tumour.

A_1, A_2 = constraints of control $Q(wm^{-2})$

Q_2 = laser heating power (wm^{-2}) at the skin surface.

q_i = temperature $({}^0c)$ of tissue in i^{th} layer.

T = specific time (s).

Q_1 = spatial heat per unit volume (wm^{-3})

t_1 = switching time (s).

L = depth of tissue (m)

$h = \frac{L}{p}$ = length of equal subintervals of tissue segment $(0, L)$ (m).

p = number of division.

λ = heat convection coefficient between skin and surface.

u = surface cooling temperature.

β = scattering coefficient (m^{-1}).

1. Introduction

Microwave diathermy has provided a growing interest in qualitifying both the hazards and beneficial effects of electro-magnetic radiation on tissue consisting of skin , fat, muscle, and tumour layers. An ideal hyperthermia treatment may be considered to heat a tumour so that the temperature in all malignant tissue is to attain a certain value 43^0c, while the temperature of the surrounding normal tissue stays below this value to avoid the damage of healthy tissue.

In practice induced microwave power level and cooling temperature on the surface of the tissue are the only variables which are accessible to direct control Kolias et al [1] studied the effect of blood flow cooling on ultrasonic formation.

In [2] Shih et al investigated analytically cooling effect of thermally significant blood vessels on the extent of thermal lesion during heating treatments. Craciunescu et al in [3] studied the effect of pulsatile blood on temperature distribution and heat transfer in rigid vessels which may act as an important initiation the treatment of hyperthermia on tissues. Analytical study on bio-heat transfer problems by induced surface heating on skin was performed by Deng et al [5] playing a significant role in cancer treatments. [6] of Golub considered fundamentals of optimal control problems.

In [7] Dhar et al considered a theoretical study on the strategy for achieving rise of temperature on the tissue consisting of skin , fat, muscle , and tumour layers by artificial surface cooling using finite difference technique. Wager in [8] studied a optimization procedure to calculate transient temperature profiles in plane tissue by multiple electro-magnetic applications. Khanafer et al [11] conducted a numerical study to determine the influence of pulsatile laminar flow and heating distribution in a single blood vessel and tumour in hyperthermia. Dhar et al in [12] investigated analytically a system described by bio-heat equation for a plane tissue

so as to attain a desired temperature throughout the entire tissue by induced heat source after least possible time.

In [13] Gex-fabry et al have investigated thermal response of two compartment model composed of a spherical tumour embedded in a cylindrical normal tissue subjected to a magnetic induction heating. A computational technique for temperature distribution for fast hyperthermia treatment which minimizes a certain goal function was investigated by Das et al [14] .

In [15] Wren et al considered a hybrid equation study that included both an increased thermal conductivity and a heat sink. A method of optimization of an electro-magnetic APA was studied employing noninvasive temperature estimation by Kowalski et al [16] . Loulou et al [17] analyzed thermal dose optimization in hyperthermia treatment under the conjugate gradient method.

Kowalski et al [18] presented a hybrid proportional integral-in-time and cost minimization problem in space feedback control system for electro-magnetic, deep regional hyperthermia treatment . The problem of two finite regions treating inner diseased tissue by generating heat with the aid of application of alternating magnetic field was the subject analytical study in Bagaria et al [19]. A numerical study of 3-dimensional bio-heat problem with different spatial heating by discritizing with the aid of finite difference method was presented in Karaa et al [20]. On some recent developments in hyperthermia one can cite a paper where the facilitation of real-time magnetic resonance image on the background of patient treatment of hyperthermia for heating system with large number of physical sources (e.g. antennas) was presented by Cheng et al [21].

In the work Liu et al [22] analyzed the temperature distribution in biological tissues under magnetic hyperthermia treatment within the duel-phase-lag model. A comprehensive analytical study was carried through the biological tissues ,vascular system, convective heat exchange of blood, metabolic system and imposed heat flux in Mahjoob et al [23]. In paper [24] Yuan investigated temperature distribution in bio-heat transfer equation and equation of porous model with the aid of finite difference method.

Wagter[25] considered a numerical procedure to calculate and control transient temperature profiles in plane multilayered biological tissue by means of microwave –induced hyperthermia treatment. Here [9] and [10] of Butkovosky and Pontrayagin et al gave the basic formation of optimal control problems

Strohbehn[26] reviewed physical and technical aspects in hyperthermia treatments. Kuznetsov [27] studied analytically optimal control problem for bio-heat equation in a homogeneous tissue on the temperature distribution and thermal dose on a located centre point of the tumour due to spatial heating with the aid of 'Maximal Principle' calculating Lagrange multiplier by solving total constant volumetric heat generation over the duration of the process.

In this paper, with the aid of Maximal Principle, we would like to attain a desired temperature χ^* at any point of location $x = x_1$ during a fixed time by controlling optimally time dependent heating power $Q_2(t)\ (wm^{-2})$ described by bio-heat equation in a multilayered tissue consisting of skin, fat, muscle, tumour

layers taking constant surface cooling temperature throughout the entire process with the help of finite difference method. Here power absorbed from laser electro-magnetic field per unit volume $Q_1(x,t)$ (wm^{-3}) has been considered according to well known Beer's Law [5], given by,

$$Q_1(x,t) = \beta e^{-\beta x} Q_2(t) \text{ where } Q_2(t)\ (wm^{-2})$$

signifies time dependent laser heating power and β is scattering coefficient.

2. Basic Equation

The one-dimensional bio-heat equation [8]

$$pc\frac{\partial \chi}{\partial t} = k\frac{\partial^2}{\partial x^2}\chi - b(\chi - \chi_1) + Q_m(x) + Q_1(x,t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where; according to well-known Beer's Law, $Q_1(x,t) = \beta e^{-\beta x} Q_2(t)$ [5]

Here $Q_2(t)$ signifies time dependent laser heating power (wm $^{-2}$)

Boundary conditions:

$$K\frac{\partial \chi}{\partial x} = \lambda\{\chi - u(t)\} \text{ on } x = 0 \qquad \ldots\ldots\ldots\ldots (2)$$

$$\chi(x,t) = \chi_1 \text{ on } x = L \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Initial condition:

$$\chi(x,o) = \chi_O \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Let us reduce the system with distributed parameters to a lumped parameters using finite difference method by dividing the segment (O,L) on the X- axis into p equal intervals of length h for h = L/p. where we designated the temperature of the tissue at each point by quantities q_i=(i=o, 1, ... , p) on the basis of considering the parameters corresponding to current layers as soon as the solution point crosses the interface between the previous layer and the current layer [7].

$$\frac{d\,q_1}{dt} = {}^1F_{j,s} \qquad \ldots\ldots\ldots\ldots (5)$$

$$j = 1, s = 1$$

$$\frac{d\,q_j}{dt} = {}^2F_{j,s} \qquad \ldots\ldots (6)$$

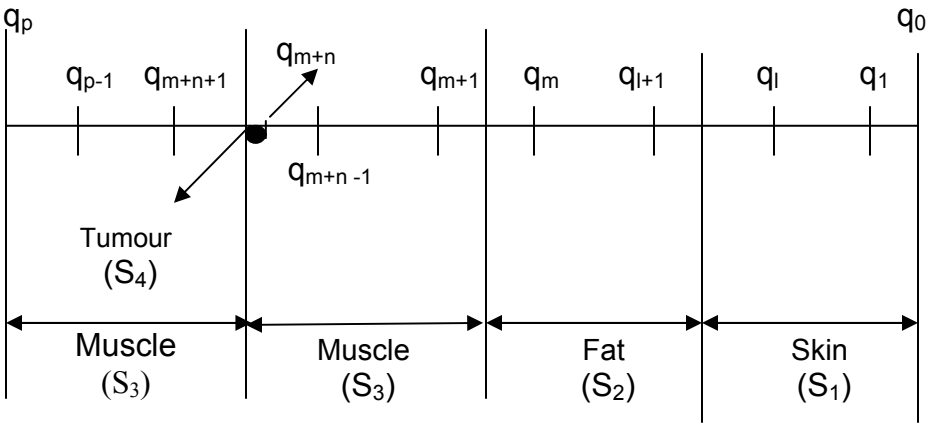

Fig – A

$j = 2,.....,l-1; \quad j = l+2,.....,m-1, \quad j = m+2,.....m+n-2$

$s = 1 \qquad s = 2 \qquad s = 3$

$j = m+n+2,....., p-1$

$s = 3$

$$\frac{d\,q_j}{dt} = {}^3F_{j,s} \qquad \ldots\ldots\ldots\ldots (7)$$

$j = l \;\; ; \;\; j = m \;\; ; \;\; j = m+n-1 \;\; ;$

$s = 2 \quad s = 3 \quad s = 4$

$$\frac{d\,q_j}{dt} = {}^4F_{j,s} \qquad \ldots\ldots\ldots\ldots (8)$$

$j = l+1 \;\; ; \;\; j = m+1 \;\; ;$

$s = 2 \qquad s = 3$

$$\frac{d\,q_j}{dt} = {}^5F_{j,s} \qquad \ldots\ldots\ldots\ldots (9)$$

$j = m+n+1$

$s = 4$

$$\frac{d}{dt}q_j = {}^6F_{j,s} \qquad \ldots\ldots\ldots\ldots (10)$$

$j = m+n,\ \ s = 4$

where,

$${}^2F_{j,s} = \frac{k_s}{\rho_s c_s h^2}\left(q_{j+1} - 2q_j + q_{j-1}\right) - \frac{b_s}{\rho_s c_s}\left(q_j - \chi_1\right) + \frac{Qm_j}{\rho_s c_s} + \frac{\beta e^{-j\beta h}}{\rho_s c_s}Q_2(t)$$

$\ldots\ldots\ldots\ldots$ (11)

$j = 2,\ldots\ldots,l-1;\quad j = l+2,\ldots\ldots,m-1,\quad j = m+2,\ldots\ldots m+n-2,$

$s = 1 \qquad s = 2 \qquad s = 3$

$j = m+n+2,\ \ldots\ldots,\ p-1$

$s = 3$

$${}^3F_{j,s} = \frac{k_s}{\rho_s c_s h^2}q_{j+1} + \frac{k_{s-1}}{\rho_{s-1} c_{s-1} h^2}\left(q_{j-1} - 2qj\right) - \frac{b_{s-1}}{\rho_{s-1} c_{s-1}}\left(q_j - \chi_1\right) + \frac{Qm_j}{\rho_{s-1} c_{s-1}} + \frac{\beta e^{-j\beta h}}{\rho_{s-1} c_{s-1}}Q_2(t)$$

$\ldots\ldots\ldots\ldots\ldots..(12)$

$j = l \quad ; \quad j = m \quad ; \quad j = m+n-1$

$s = 2 \qquad s = 3 \qquad s = 4$

$${}^4F_{j,s} = \frac{k_s}{\rho_s c_s h^2}\left(q_{j+1} - 2q_j\right) + \frac{k_{s-1}}{\rho_{s-1} c_{s-1} h^2}q_{j-1} - \frac{b_s}{\rho_s c_s}\left(q_j - \chi_1\right) + \frac{Qm_j}{\rho_s c_s} + \frac{\beta e^{-j\beta h}Q_2(t)}{\rho_s c_s}$$

$\ldots\ldots....(13)$

$j = l+1 \quad ; \quad j = m+1 \quad ;$

$s = 2 \qquad s = 3$

$${}^5F_{j,s} = \frac{k_{s-1}}{\rho_{s-1} c_{s-1} h^2}\left(q_{j+1} - 2q_j\right) + \frac{k_s}{\rho_s c_s h^2}q_{j-1} - \frac{b_{s-1}}{\rho_{s-1} c_{s-1}}\left(q_j - \chi_1\right) + \frac{Qm_j}{\rho_{s-1} c_{s-1}} + \frac{\beta e^{-j\beta h}Q_2(t)}{\rho_{s-1} c_{s-1}}$$

$\ldots\ldots\ldots..(14)$

$j = m+n+1$

$s = 4$

$$^{6}F_{j,s} = \frac{k_3}{\rho_3 c_3 h^2}\left(q_{m+n+1} + q_{m+n-1}\right) - \frac{2k_4 q_{m+n}}{\rho_4 c_4 h^2} - \frac{b_4}{\rho_4 c_4}\left(q_{m+n} - \chi_1\right) +$$

$$\frac{Qm(m+n)}{\rho_4 c_4} + \frac{\beta e^{-(m+n)\beta h}}{\rho_4 c_4} Q_2(t) \qquad [j = m+n, s = 4] \qquad (15)$$

The equation (2) , (3) and (4) thus stands,

$$k_1 \frac{q_1 - q_O}{h} = \lambda\{q_O - u(t)\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

$$q_p = \chi_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

and

$$q_j(o) = \chi_O, \quad j = 0,1,2,\ldots., p \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

With the help of equation (16) we get,

$$^{1}F_{j,s} = \frac{1}{\rho_1 c_1}\left[\frac{k_1^2}{h^2(k_1 + \lambda h)} - \frac{2k_1}{h^2} - b_1\right] q_1 + \frac{k_1}{\rho_1 c_1 h^2} q_2 + \frac{k_1 \lambda h u(t)}{\rho_1 c_1 h^2 (k_1 + \lambda h)} + \frac{Qm_1}{\rho_1 c_1} + \frac{\beta e^{-\beta h} Q_2(t)}{\rho_1 c_1} + \frac{b_1}{\rho_1 c_1} \chi_1$$

$j = 1, s = 1$

$\ldots\ldots\ldots\ldots\ldots\ldots..(19)$

3. Statement of the optimal control problem

With the aid of maximal principle, we would like to attain desired temperature χ^* at a particular point of location of the tumor $x=x_1$ situated inside the muscle during a fixed time T by controlling optimally time dependent heating power $Q_2(t)$ applied on the surface of the skin of tissue consisting of skin, fat, muscle, point tumour and muscle layers given in Fig A. When the surface cooling temperature $u(t)$ is constant throughout the tissue.

After discritizing the functional [6] stands, $\frac{1}{2}\int_0^T \left\{\chi^* - q(t)_{m+n}\right\}^2 dt$ ……(20) is to be minimized.

Here the particular point of the tumour is taken as the centre of a very small tumour in length [27].

Thus according to [6,9,28], the Hamiltanian stand with the help of calculus of variation under ‘Maximal Principle’ :

$$H = -\frac{1}{2}\left\{\chi^* - q_{m+n}(t)\right\}^2 + \sum_{j=2}^{l-1} \varphi_j \, {}^2F_{j,1} + \varphi_l \, {}^3F_{l.2} + \varphi_1 \, {}^1F_{1,1}$$

$$+ \varphi_{l+1} \, {}^4F_{\overline{l+1},2} + \sum_{j=l+2}^{m-1} \varphi_j \, {}^2F_{j,2} + \varphi_m \, {}^3F_{m,3} +$$

$$+ \varphi_{m+1} \, {}^4F_{\overline{m+1},2} + \sum_{j=m+2}^{m+n-2} \varphi_j \, {}^2F_{j,3} + \varphi_{m+n-1} \, {}^3F_{\overline{m+n-1},4} +$$

$$+ \varphi_{m+n} \, {}^6F_{\overline{m+n},4} + \varphi_{m+n+1} \, {}^5F_{\overline{m+n+1},4}$$

$$+ \sum_{m+n+2}^{p-1} \varphi_j \, {}^2F_{j,3}$$

……(21)

where the auxiliary function φ_j Gulub [6] is given by,

$$\frac{d}{dt}\varphi_j(t) = -\frac{\partial H}{\partial q_j},\ j = 1,2,\ldots,p-1 \qquad \text{……(22)}$$

with terminal condition $\varphi_j(T) = 0, j = 1,2,\ldots\ldots,p-1$ ……(23)

From the relation $\frac{\partial H}{\partial Q_2} = 0$, one can construct the optimal control $Q_2(t)$ adjusting by simulation so as to satisfy the condition, given by,

$$Q_2(t) = \frac{\beta}{\rho c} sign\left[\sum_{j=1}^{p-1} \varphi_j e^{-j\beta h}\right] \qquad \text{……(24)}$$

Taking the average value of ρc as 3.075×10^6 $Jm^{-3}k^{-1}$ (Table -1)

Which for one switching time $t = t_1$ stands $\sum_{j=1}^{p-1} \varphi_j e^{-j\beta h} = 0$ ……(24a)

Where $\varphi_j(t)$ can be found out from the solution of the eqn (22) under condition eqn (23) after obtaining $q_j\ (j = 1,2,\ldots\ldots,p-1)$

We have assumed time dependent controllable incident power density $Q_2(t)$ (wm^{-2}) is piecewise constant function changing it's value at certain discrete instants (switching points) when the cooling temperature is constant throughout the entire process as in equation [25].

4. Numerical Calculation

For numerical calculations we have taken the data as given in [7,25]

	ρ c($Jm^{-3}k^{-1}$)	k($wm^{-1}k^{-1}$)	b(wm^{-3} k^{-1})	Qm(wdm^{-3})	Depth in tissue (mm)
Skin	3.4 x 10^6	.44	7500	1	1
Fat	2.5 x 10^6	.23	0	0	2
Muscle	3.4 x 10^6	.50	5550	3	2
Tumour	3.00 x 10^6	.30	2000	3	(point)
Muscle	3.4 x 10^6	.50	5550	3	5

h= length of each division of tissue layer = 0.5mm, $\chi_0 = 25^oC$ (initial temperature), $\chi_1 = 37^oC$, $\lambda = 200$ Wm^{-1} K^{-1}, $\chi^* = 43^oC$, $\beta = 100m^{-1}$

Table 1

All the results obtained for the problem was simulated in the MATLAB 7 environment and in particular, the bio- heat condition equation and auxiliary equation (22) have been simulated by using the standard Runga Kutta MATLAB function ODE 45.

For the sake of simplicity, we assume one specified switching time t = t_1 when the only controllable variable $Q_2(t)$ is given by,

$$\begin{aligned} Q_2(t) &= A_1\ (wm^{-2}),\ 0 \le t \le t_1 \\ &= A_2\ (wm^{-2}),\ t_1 < t \le T \end{aligned} \qquad (24b)$$

and the surface cooling temperature is specified as,

$$u(t) = 5(^0c),\ 0 \le t \le T \qquad \ldots\ldots\ldots(24c)$$

Here for the sake of singular controls Q_2 (t) attains specified extremum values A_1 and A_2.

For numerical calculations the data given in Table1 was considered with respect to depth of skin, fat, muscle, point tumour, muscle, given by 1 mm,2 mm,3 mm, tumour situated at the point $x = 6$ mm and 4 mm respectively as indicated in Fig A for length of each division $h = 0.5$ mm and $\chi_1 = 37^0 c$, $\lambda = 200 wm^{-1}k^{-1}$, $\chi^* = 43^0 c$, $\beta = 100m^{-1}$, $\chi_0 = 25^0 c$, $u(t) = 5^0 c$ throughout the process. To facilitate calculation by simulation, we have taken $Q_2(t) = 1.01\ Q(t)$.

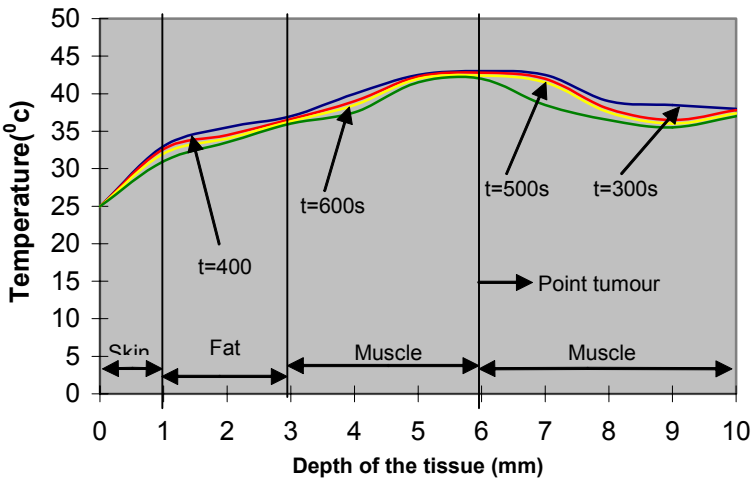

Fig 1: Temperature(^{0}c) vs depth of the tissue(mm) for Q(t)=3275 wm^{-2}, 0<=t<=300;

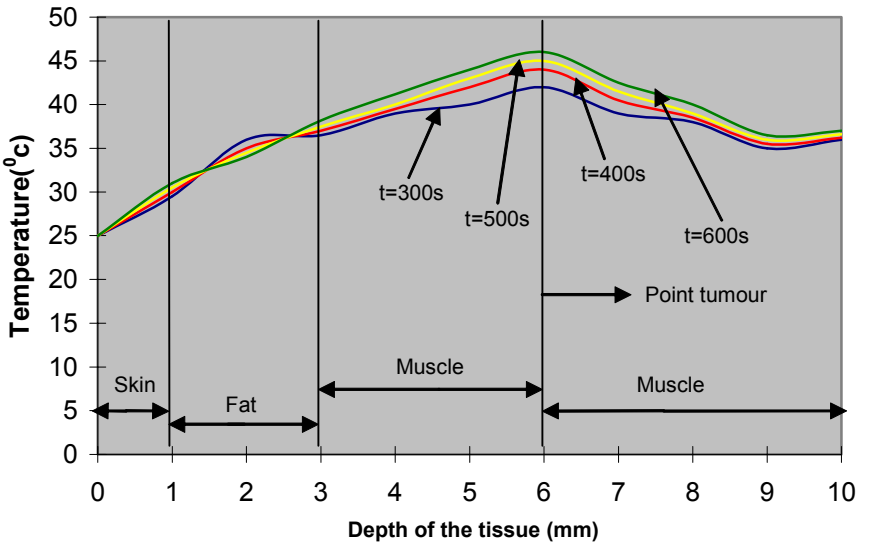

Fig 2: Temperature(^{0}c) vs depth of the tissue(mm) for Q(t)=2950 wm^{-2}, 0<=t<=300; Q(t)=452 wm^{-2}, 300<=t<=600

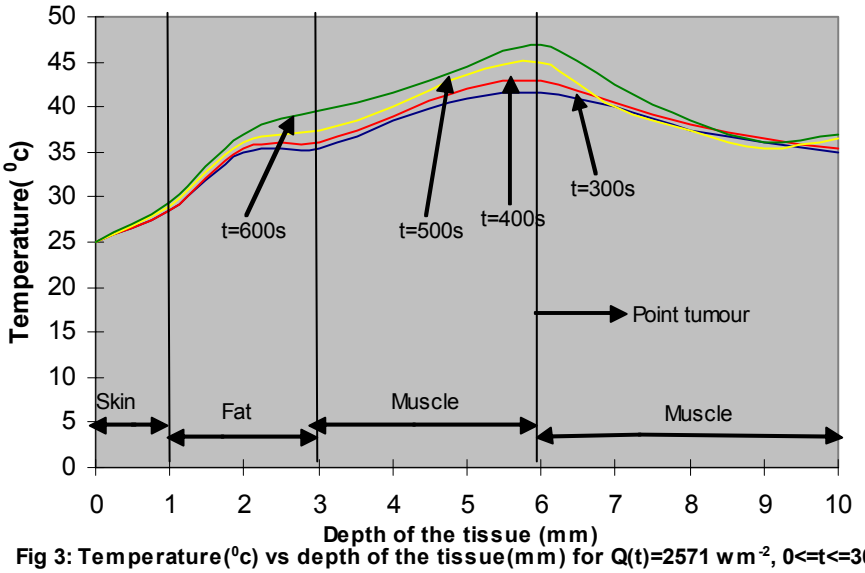

Fig 3: Temperature(^{0}c) vs depth of the tissue(mm) for Q(t)=2571 wm^{-2}, 0<=t<=300; Q(t)=379 wm^{-2}, 300<=t<=600

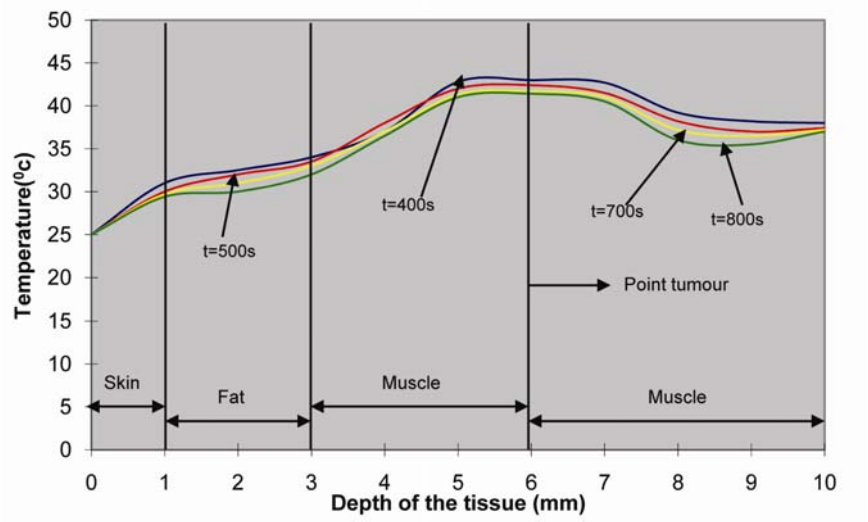

Fig 4: Temperature(^{0}c) vs depth of the tissue(mm) for Q(t)=2489 wm^{-2}, 0<=t<=400; Q(t)=0, 400<=t<=800

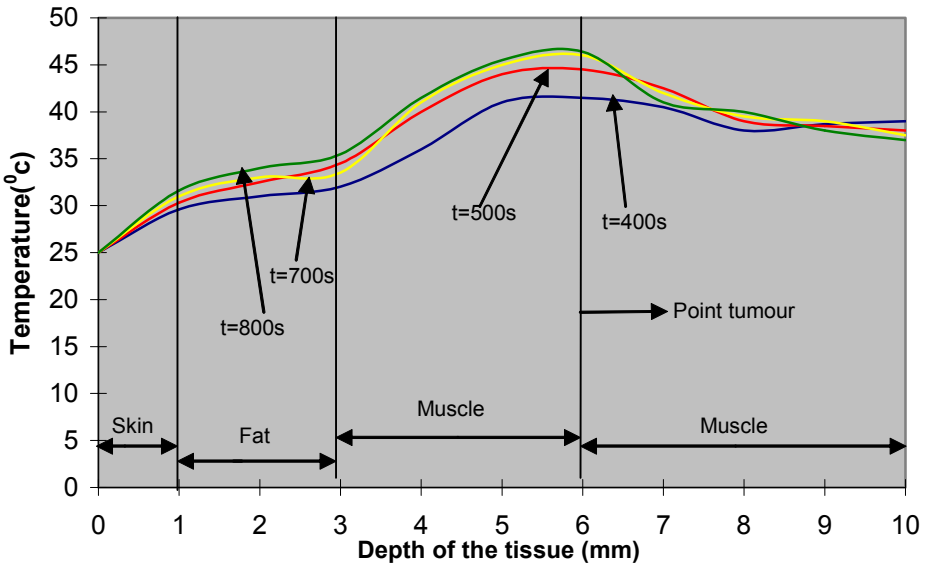

Fig 5: Temperature(^{0}c) vs depth of the tissue (mm) for Q(t)=2072 wm^{-2}, 0<=t<=400; Q(t)=556 wm^{-2}, 400<=t<=800

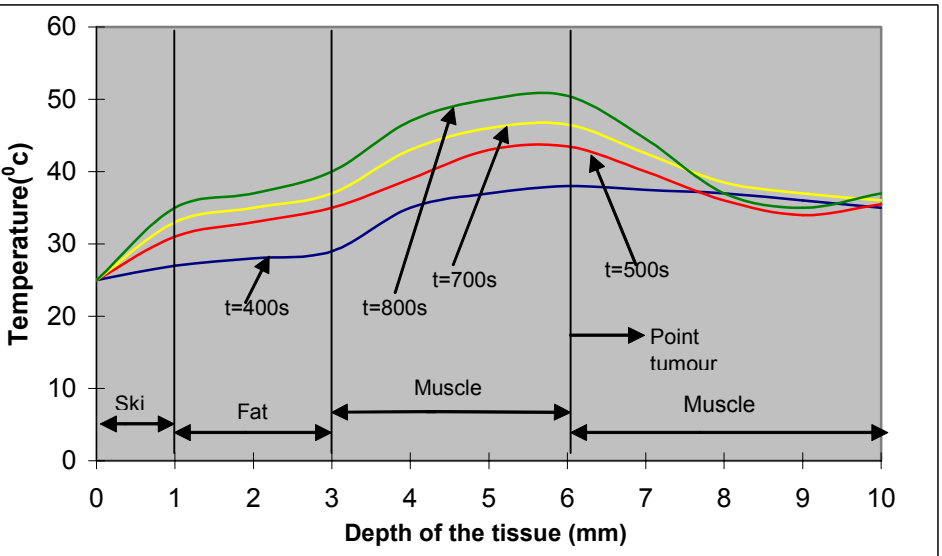

Fig 6: Temperature(^{0}c) vs depth of the tissue (mm) for Q(t)=1723 wm^{-2}, 0<=t<=400; Q(t)=1021 wm^{-2}, 400<=t<=800

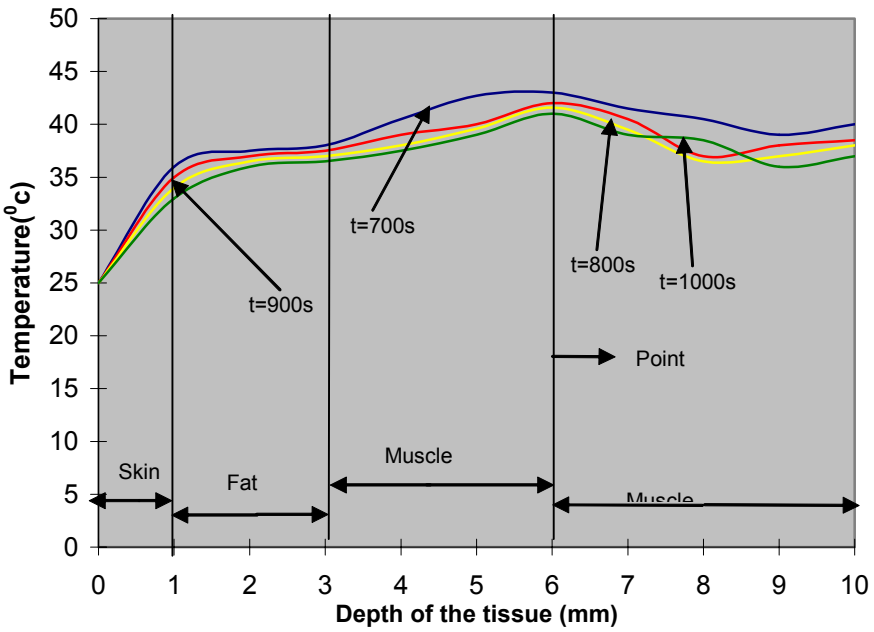

Fig 7: Temperature(^{0}c) vs depth of the tissue (mm) for Q(t)=2145 wm^{-2}, 0<=t<=700; Q(t)=0, 700<=t<=1000

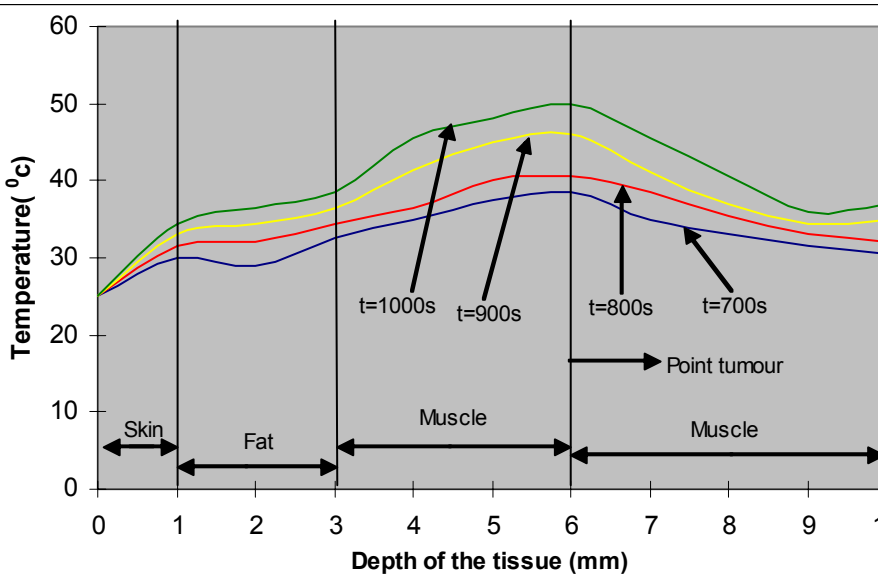

Fig 8: Temperature(^{0}c) vs depth of the tissue (mm) for Q(t)=403 wm^{-2}, 0<=t<=700; Q(t)=2072 wm^{-2}, 700<=t<=1000

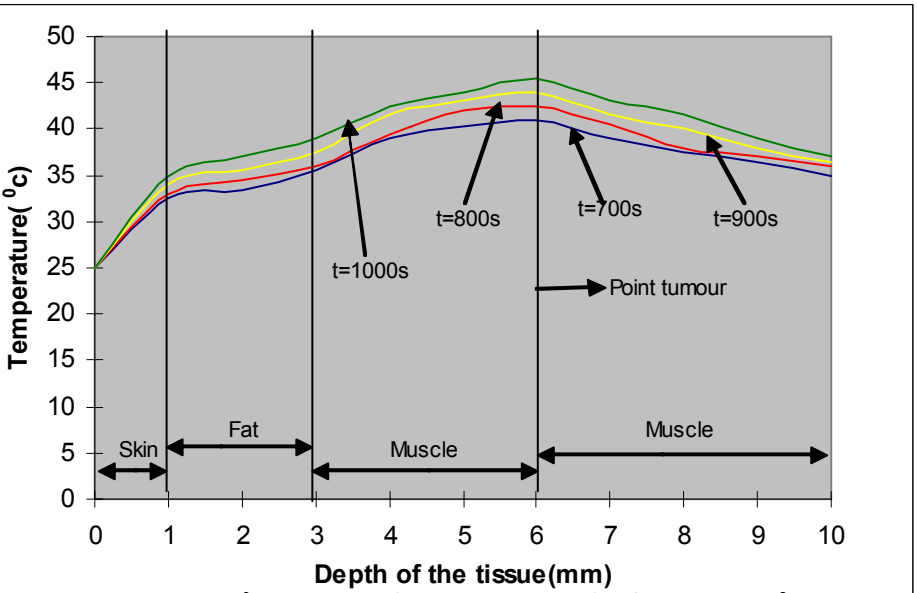

Fig 9: Temperature(^{0}c) vs depth of the tissue (mm) for Q(t)=1421 wm^{-2}, 0<=t<=700; Q(t)=861 wm^{-2}, 700<=t<=1000

5.

5. Results and Discussion

In drawing all the figures we have considered set of input variable $Q(t)wm^{-2}$ as certain desired choices where $Q(t)wm^{-2}$ has been obtained by adjustment through simulation under the condition $\sum_{j=1}^{p-1} \varphi_j e^{-j\beta h} \le \varepsilon$, where ε is very small number $10^{-2} \sim 10^{-3}$.

Fig 1,Fig 2, and Fig 3 display the distribution of temperature in skin, fat ,muscle, point tumour at 6 mm and muscle layers as specified in Fig A, at some particular time $t = 300s, 400s, 500s,$ and $600s$, for different choices of time-dependent heating power $Q(t)wm^{-2}$ by microwave obtained by suitable adjustment through simulation so as to satisfy the condition given in equation (35a) given by :

Fig 1: $Q(t) = 3275\ wm^{-2}, \quad 0 \le t \le 300;$
$= 0, \quad 300 < t \le 600;$

Fig 2: $Q(t) = 2950\ wm^{-2}, \quad 0 \le t \le 300;$
$= 452\ wm^{-2}, \quad 300 < t \le 600;$

Fig 3: $Q(t) = 2571\ wm^{-2},\ 0 \le t \le 300;$

$= 379\ wm^{-2},\ 300 < t \le 600;$ respectively.

Here the total time of operation $T = 600s$ having switching time $t = 300s$.

In Fig 1, it is observed that the temperature of the point tumour located at $x = 6$mm rises to it's maximum $43^0 c$ at $300s$ after that it decreases to till the end of the process $t = 600s$.
In Fig 2, one of the notable feature is that the temperature of the point tumour attains the value $46^0 c$ at $600s$. In Fig 3, the temperature of the point tumour rises to $47^0 c$ at $t = 600s$.

Fig 4, Fig 5, and Fig 6 display the distribution of temperature of the point tumour in skin, fat ,muscle, tumour (point) and muscle layers at some particular time when time-dependent heating power $Q(t)wm^{-2}$ in Fig 4, Fig 5, and Fig 6 are specified as :

Fig 4: $Q(t) = 2489\ wm^{-2},\ 0 \le t \le 400;$

$= 0,\ 400 < t \le 800;$

Fig 5: $Q(t) = 2072\ wm^{-2},\ 0 \le t \le 400;$

$= 556\ wm^{-2},\ 400 < t \le 800;$

Fig 6: $Q(t) = 1723\ wm^{-2},\ 0 \le t \le 400;$

$= 1021\ wm^{-2},\ 400 < t \le 800;$ respectively which are obtained satisfying the condition given in equation (25).

Here the total time of operation of the process changes to $T = 800s$ having switching time at $t = 400s$. Considering all those figures it is observed that point tumour temperature at $x = 6$mm attains maximum value $43^0 c$ at $t = 400s$ in Fig 4, rises to $44.5^0 c, 46^0 c$ and $46.4^0 c$ at $T = 600s, 700s$ and $800s$ respectively in Fig 5. In Fig 6, it is found that point tumour temperature rises to $43.5^0 c, 46.5^0 c$ and $50.5^0 c$ at $600s, 700s$ and $800s$ respectively.
Fig 7, Fig 8, and Fig 9 display the distribution of rise of temperature in the given specified layers and at the point tumour at $x = 6$mm due to different choices of heating power $Q(t)wm^{-2}$ by microwave are given by :

Fig 7: $Q(t) = 2145\ wm^{-2},\ 0 \le t \le 700;$

$= 0,\ 700 < t \le 1000;$

Fig 8: $Q(t) = 403\ wm^{-2}, \quad 0 \le t \le 700;$

$= 2072\ \ wm^{-2}, \quad 700 < t \le 1000;$

Fig 9: $Q(t) = 1421\ wm^{-2}, \quad 0 \le t \le 700;$

$= 861\ wm^{-2}, \quad 700 < t \le 1000;$ respectively.

The total time of operation changes to $T = 1000s$ with the switching time $t = 700s$. It has been noted that the tumour attains temperature $43^0 c$ (maximum) during the process in Fig 7, attains maximum temperature $50^0 c$ at $t = 1000s$ in Fig 8 and in Fig 9 , the maximum point tumour temperature is $46.5^0 c$ at $t = 1000s$.

Here the most preferable cases may be considered as depicted in Fig 1, Fig 4, and Fig 7 where the heating power $Q(t)wm^{-2}$ is switched off in the second segment of the total time of operation of the process concerned in each case which spontaneously decreases the temperature of the tissue after attaining the desired temperature $43^0 c$ of the point tumour. This case can have more importance since the temperature of the normal tissue generally less than $43^0 c$ taking into account of the overheating of the normal tissue. Further, it is to note that with the increase of the total time of operation of the process the first segment of the of operation, when the power is switched off in the second segment ,increases with the corresponding decrease of $Q(t)wm^{-2}$ which can be taken as most practicable from the consideration of attaining optimal rise of temperature $43^0 c$ at the point tumour.

6. Conclusion

It is important to note that the different figures having the same switching point as well as for the same time of operation of the process are probably due to simulation of certain desired choices of $Q(t)\ (wm^{-2})$, where $Q(t)\ (wm^{-2})$ have been adjusted by simulation so as to satisfy the condition given by $\sum_{j=1}^{p-1} \varphi_j e^{-j\beta h} \le \varepsilon$ for different values of ε, lies between $10^{-2} \sim 10^{-3}$, that comes in the way of simulation, which demonstrates non-uniqueness of the solution.

But it is observed that about 98.7% of the desired temperature $43^0 c$ can be attained if one considers the strategy as given in Fig 1, Fig 4 and Fig 7. However, it can be taken into granted that with the increase of the number of switching points as to satisfy the condition given by $\sum_{j=1}^{p-1} \varphi_j e^{-j\beta h} \le \varepsilon$ by simulation for $\varepsilon \approx 10^{-5}$, this analytical study may provide an initiating background in course of drawing

almost a final strategy which can be applied in one, two and three dimensional structures of tissue having the different located tumour point with different time of operation.

Thus one can take this primary analytical study as test cases form of a preferable study in hyperthermia treatment .But for practical applications of this theoretical investigation, it must be considered in terms of clinical trials.

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