Journal of Physical Sciences, Vol. 12, 2008, 23-32

Statistical Analysis of the Mixed Accelerated Life Test for the Type II Progressively Censored Sample

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Received September 17, 2008; accepted October 27, 2008

ABSTRACT

This paper studied statistical analysis of a new kind of accelerated life testing which combined progressively and constantly life test. The point estimations of the parameters in Exponential distribution and acceleration equation are obtained based on type II progressively censored data. Furthermore, we give the interval estimations of the parameters by Bootstrap sample. Besides, the estimations of the failure rate, the reliability functions and mean life are given at normal stress level, as well as the approximated unbiased estimations of failure rate and mean life. In the end, an illustrative numerical example is examined by means of the Monte-Carlo simulation.

***Keywords**: Exponential distribution; Type II Progressively censored; accelerated life test; parameter estimation*

1. Introduction

It is difficult to getting failure data from products with high quality and long life at normal stress. So people often use accelerate life testing to save time and cost. Comparing to constant stress life testing, progressively life testing can get failure data in shorter time, so it is an economical and effective method to analysis products with high quality. At present, some scholars have researched statistical analysis and Bayes analysis from progressively life testing. For example, Yincai Tang(2006) has investigated Bayes analysis on progressively life testing terminated by type I and type II censoring based on Weibull distribution. Wang and Fei (2004) gained the MLEs of parameters from Weibull distribution based on TFR model. And the statistical analysis of combined data from the progressively and constantly life tests under power-weibull model by Fei(2000).

In order to estimate parameters more efficient, progressively life testing need two groups stress with different change pace at least. It may expend too much cost and time. Moreover, most accelerated life testing analysis based on the assumption as follows: the failure mechanism is not change in high stress and normal stress. But in progressively life testing, stress will probably exceed the reasonable range. Therefore, Bingxing Wang bring forward a new model in which combined the progressively life testing and constantly life testing. In this test, n productions was put in progressively life testing for some time, then put the surviving productions in constantly life testing. But Wang only discussed the situation of full sample. In engineering science, we should remove some survival productions randomly at each failure time and analysis them because of many reasons, for example, in order to understand failure mechanism of productions, degenerate case and continual improvement in manufacturing. This censoring scheme are progressive Type-II censoring, it can save time and cost to a great extent [5-7].

In this paper, we consider a new model in which the stress increases linearly with time t before a pre-fixed time (say T_0), and it will constant after the time T_0. Data obtained are progressive Type-II censored from exponential distribution. The model considered here is discussed in detail in section 2 and the MLEs of unknown parameters and some reliability items are derived. We then discussed the asymptotic unbiased estimation and asymptotic interval estimations of the mean life in section 3. In section 4, we derived the interval estimations of all the unknown parameters by using Bootstrap methods. Monte Carlo simulation results are presented in section 5, respectively. Finally, we make some concluding remarks in section 6.

2. Maximum likelihood estimation

The discussion of the statistical analysis of the ALT is based on the following basic assumptions:

Assumptions 1. Under a stress $V (V > 0)$ the life time follows the exponential distribution with the cumulative distribution function (CDF)

$$F_T(t) = 1 - \exp\{-t/\theta\}, t \ge 0$$

here $\theta > 0$.

Assumptions 2. The relationship between the characteristic life time $\theta > 0$ and the stress V satisfies the inverse power low, i.e., $\ln\theta = -\ln dV^c = -\ln d - c\ln V$, where the parameters $d > 0$ and $c > 0$.

Assumptions 3. The remaining life of the productions depends only on the current cumulative fraction failed and the current stress, regardless how the fraction accumulated. Mathematically it means that the accumulated failure probability of the product after working t_1 under stress $V_1, F_1(t_1)$,is the same as that of the product after working t_2 under stress V_2, $F_2(t_2)$ .i.e.$F_1(t_1)=F_2(t_2)$,thus $t_1/\theta_1 = t_2/\theta_2$.

An ensemble of n productions is initially placed on stress kt (say V_1, where k is a predetermined positive constant),and run until a predetermined time T_0 ,then the experiment translate to constantly life testing with stress kT_0 (say V_2). Supposes there are r failures in the experiment, the failure times are denoted by $t_1, t_2 \cdots t_r, (t_1 \le t_2 \le \cdots \le t_r)$. At the failure time t_i , k_i surviving productions are censored from the $n - \sum_{j=0}^{i-1}(k_j+1), (k_0 = 0, i = 1,2,\cdots r)$ unfailing productions. Where $r, k_1, k_2, \cdots, k_r$ are pre-fixed nonnegative constant. Here, it is obvious that $n = r + \sum_{i=1}^{r} k_r$ .This is progressive Type-II censoring and Type-II censoring corresponds to the special case when $k_i = 0, i = 1,2,\cdots r$.

According to the assumptions 3 and the conclusion of progressively life testing, we know that CDF of productions' life T in this experiment is:

$$F_T(t) = \begin{cases} 1 - \exp\{-dk^c t^{c+1}/(c+1)\},\ 0 < t \le T_0 \\ 1 - \exp\{-dV_2^c [t - cT_0/(c+1)]\},\ t > T_0 \end{cases}$$

Set $X=T/T_0$, the CDF and PDF of X are:

$$F_X(x) = \begin{cases} 1 - \exp\{-x^{c+1}/\sigma\},\ 0 < x \le 1, \\ 1 - \exp\{-[(c+1)x - c]/\sigma\},\ x > 1 \end{cases}$$

$$f_X(x) = \begin{cases} \sigma^{-1}(c+1)x^c \exp\{-x^{c+1}/\sigma\}, 0 < x \le 1 \\ \sigma^{-1}(c+1)\exp\{-[(c+1)x - c]/\sigma\},\ x > 1 \end{cases} \quad (1)$$

here $\sigma = (c+1)/dV_2^c T_0$.

Suppose the failure number under the stress V_1 is r_1 and under stress V_2 is r_2 , $r_1 + r_2 = r$.($r_1 r_2 \ne 0$ in this paper) .According to Lawless J F(1982),the likelihood function about $x_1, x_2, \cdots x_r, (x_i = t_i/T_0, i = 0,1,\cdots,r)$ to be maximized for obtaining the MLEs of c ,d is giving by

$$\begin{aligned} L(c,\sigma) &= A\sum_{i=1}^{r} f(x_i)[1 - F(x_i)]^{k_i} \\ &= A(c+1)^r/\sigma^r \prod_{i=1}^{r_1} x_i^c \exp\{-\sum_{i=1}^{r_1}(k_i+1)x_i^{c+1}/\sigma\}\exp\{-\sum_{j=r_1+1}^{r}(k_j+1)[(c+1)x_j - c]/\sigma\} \end{aligned}$$

where $A = \prod_{i=1}^{r}[n - \sum_{j=1}^{i-1}(k_j+1)] > 0$ (same in the following) is the normalizing constant.

Noting that σ is a function about *c*, *d*, so the MLEs of *c*, *d* is giving by

$$\sigma = r^{-1}\{\sum_{i=1}^{r_1}(k_i+1)x_i^{c+1} + \sum_{j=r_1}^{r}(k_j+1)[(c+1)x_j - c]\} \tag{2}$$

$$r\ln V_2 + \sum_{i=1}^{r_1}\ln x_i - \sigma^{-1}\sum_{i=1}^{r_1}(k_i+1)x_i^{c+1}(\ln x_i - (c+1)^{-1} + \ln V_2)$$
$$- \sigma^{-1}\sum_{j=r_1+1}^{r}(k_j+1)\cdot\{\ln V_2[(c+1)x_j - c] - (c+1)^{-1}\} = 0 \tag{3}$$

By substituting for σ in (3), we can get a function which only has one variable *c*. So the MLE of *c* can be obtained easily. Then MLE of *d* can be obtained by using (2).

According to the conclusions above, we can get the MLEs of some reliability items at the normal stress. The estimation of mean life is $\hat{\theta}_0 = 1/(\hat{d}V_0^{\hat{c}})$, the estimation of reliability life with reliability $1-\alpha (0<\alpha<1)$ is $\hat{t}_\alpha = -\hat{\theta}_0 \ln\alpha = -\ln\alpha / \hat{d}V_0^{\hat{c}}$, the estimation of accelerated coefficient about stress V relative to the normal stress is $\tau_{V\sim V_0} = (V/V_0)^{\hat{c}}$,the estimation of reliability function is $\hat{R}(t) = \exp\{-t\hat{d}V_0^{\hat{c}}\}$,the estimation of failure rate is $\lambda_0 = \hat{d}V_0^{\hat{c}}$.

3. Asymptotic unbiased estimation and asymptotic interval estimation of θ

Here, we present a basic result about progressive Type-II censored sample based on exponential distribution.

Lemma 1. Suppose life time of product *T* obey exponential distribution $\exp(1/\sigma)$, $t_1, t_2, \cdots, t_r$ are the progressive Type-II censored sample from sample size *n*. The meaning of $k_1, k_2, \cdots, k_r$ is the same as above. Noting that $w_i = (n-k_i+1)(t_i - t_{i-1})$, $i = 1, 2, \cdots, r, t_0 = 0$, so all the w_i are identically obey exponential distribution $\exp(1/\sigma)$, and $\sum_{i=1}^{r} w_i = \sum_i^r (n-k_i+1)(t_i - t_{i-1}) = \sum_{i=1}^{r}(k_i+1)t_i$ obey Γ distribution $\Gamma(r, 1/\sigma)$.

Proof. The unite probability density function (UPDF) are giving by

$$f(t_1,\cdots,t_r) = A\prod_{i=1}^{r} f(t_i)[1-F(t_i)]^{k_i} = A\sigma^{-r}\exp\{-\sum_{i=1}^{r}(k_i+1)t_i/\sigma\} = A\sigma^{-r}\exp\{-\sum_{i=1}^{r} w_i/\sigma\}$$

Then the Jacobian determinant of $w_i = (n-k_i+1)(t_i - t_{i-1})$ is $|J| = A^{-1}$ .So the UPDF of $w_1, w_2, \cdots w_r$ is

$$g(w_1, w_2, \cdots w_r) = f(t_1, t_2, \cdots, t_r)\,|J| = \sigma^{-r}\exp(-\sum_{i=1}^{r} w_i/\sigma) .$$

Consequently, $w_1, w_2, \cdots w_r$ obey the exponential distribution with expectation σ .We thus obtain $\sum_{i=1}^{r} w_i$ obey the Γ distribution $\Gamma(r, 1/\sigma)$.

From $\sigma = (c+1)/dV_2^c T_0 = (c+1)\exp(-\ln d)/(dV_2^c T_0)$,we have $\dfrac{\partial \sigma}{\partial(\ln d)} = -\sigma$ .So

$$\frac{\partial^2 \ln L}{\partial c^2} = -B + 2(C+D)[(c+1)^{-1} - \ln V_2] - G\sigma^{-1}[(c+1)^{-2} + ((c+1)^{-1} - \ln V_2)^2] ,$$

$$\frac{\partial^2 \ln L}{\partial c \partial(\ln d)} = G\sigma^{-1}[(c+1)^{-1} - \ln V_2] - C - D, \quad \frac{\partial^2 \ln L}{\partial(\ln d)^2} = -G/\sigma \text{£¬}$$

where

$$B = \sigma^{-1}\sum_{i=1}^{r_1}(k_i+1)x_i^{c+1}(\ln x_i)^2, \quad C = \sigma^{-1}\sum_{i=1}^{r_1}(k_i+1)x_i^{c+1}\ln x_i,$$

$$D = \sigma^{-1}\sum_{j=r_1+1}^{r}(k_j+1)(x_j-1), \quad G = \sum_{i=1}^{r_1}(k_i+1)x_i^{c+1} + \sum_{j=r_1+1}^{r}(k_j+1)[(c+1)x_j - c],$$

and

$$\begin{aligned} B &= (c+1)^{-2}\sum_{i=1}^{r_1}(k_i+1)(x_i^{c+1}/\sigma)(\ln x_i^{c+1})^2 \\ &= (c+1)^{-2}\sum_{i=1}^{r_1}(k_i+1)(x_i^{c+1}/\sigma)[\ln(x_i^{c+1}/\sigma) + \ln\sigma]^2 \\ &= (c+1)^{-2}(B' + 2C'\ln\sigma + H(\ln\sigma)^2) \text{£¬} \end{aligned}$$

In the similar manner, we have

$$C = (c+1)^{-1}(C' + H\ln\sigma), \quad D = (c+1)^{-1}[D' - \sigma^{-1}\sum_{j=r_1+1}^{r}(k_j+1)],$$

(the same in the following),here

$$B' = \sum_{i=1}^{r_1}(k_i+1)(x_i^{c+1}/\sigma)[\ln(x_i^{c+1}/\sigma)]^2, \quad C' = \sum_{i=1}^{r_1}(k_i+1)(x_i^{c+1}/\sigma)\ln(x_i^{c+1}/\sigma),$$

$$H = \sum_{i=1}^{r_1}(k_i+1)(x_i^{c+1}/\sigma), \; D' = \sum_{j=r_1+1}^{r}(k_j+1)[(c+1)x_j - c]/\sigma.$$

From (1) we see that $x_1^{c+1}/\sigma, x_2^{c+1}/\sigma, \cdots, x_{r_1}^{c+1}/\sigma, ((c+1)x_{r_1+1} - c)/\sigma, \cdots, ((c+1)x_r - c)/\sigma$ are progressive Type-II censored sample obey the exponential distribution $\exp(1/\sigma)$ from sample size n. According to lemma 1, $G \sim \Gamma(r, 1/\theta), EG = r\sigma$,and A', B', C', D', H are statistics. Therefore, we can use Monte-Carlo simulation to get their expectation. This gives Fisher information matrix F .

$$F = \begin{bmatrix} -E(\dfrac{\partial^2 \ln L}{\partial c^2}) & -E(\dfrac{\partial^2 \ln L}{\partial c \partial(\ln d)}) \\ -E(\dfrac{\partial^2 \ln L}{\partial(\ln d)\partial c}) & -E(\dfrac{\partial^2 \ln L}{\partial(\ln d)^2}) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & r \end{bmatrix} \tag{4}$$

here,

$A_{11}=(c+1)^{-2}EB'+(2\ln\sigma/(c+1)^2-I)EC'+[\ln\sigma/(c+1)^2-I]\ln\sigma\cdot EH-I[ED'-\sigma^{-1}E\sum_{j=r_1+1}^{r}(k_j+1)]$,

$A_{12}=(c+1)^{-1}[EC'+ED'+\ln\sigma\cdot EH-\sigma^{-1}E\sum_{j=r_1+1}^{r}(k_j+1)]-r[(c+1)^{-1}-\ln V_2]$.

$I=2[(c+1)^{-2}-(c+1)^{-1}\ln V_2]$,

By substituting for c,d,σ with their expectation in (4), we can obtain approximation of F. So approximate asymptotic variances and approximate asymptotic covariance of the MLEs of c,d are given by $D\hat{c}=r/|F|, D(\ln\hat{d})=A_{11}/|F|, Cov(\hat{c},\ln\hat{d})=-A_{12}/|F|$.Therefore the MLEs of the mean life and the failure rate at the normal stress are

$$\theta_0=\exp(-\ln\hat{d}-\hat{c}\ln V_0),\quad \lambda_0=\exp(\ln\hat{d}+\hat{c}\ln V_0),$$

and approximate asymptotic variance of $\ln\theta_0$ and $\ln\lambda_0$ is

$$\delta^2=D(\ln\theta_0)=D(\ln\lambda_0)=[A_{11}+r\cdot(\ln V_0)^2-2A_{12}\ln V_0]/|F|.$$

From the asymptotic normality of MLEs, we know that $\ln\hat{\theta}_0$ approximately subordinate to Normal distribution $N(\ln\theta_0,\delta^2)$ and $E\hat{\theta}_0=\theta_0\exp\{\delta^2/2\}$.Therefore, the asymptotic unbiased estimation of mean life θ_0 at the normal stress is $\hat{\theta}_{0U}=\hat{\theta}_0\exp\{-\delta^2/2\}$.Similarly, the asymptotic unbiased estimation of failure rate λ_0 at the normal stress is $\hat{\lambda}_{0U}=\hat{\lambda}_0\exp\{-\delta^2/2\}$.With confidence level $1-\alpha$, two-side interval estimation of mean life θ_0 and failure rate λ_0 are

$$(\hat{\theta}_0\exp(-U_{1-\alpha/2}\delta),\hat{\theta}_0\exp(U_{1-\alpha/2}\delta)),(\hat{\lambda}_0\exp(-U_{1-\alpha/2}\delta),\hat{\lambda}_0\exp(U_{1-\alpha/2}\delta))$$

where U_α is the lower side fractile of standard normal distribution.

4. Interval estimations based on Bootstrap sample

Generally, Bootstrap-t interval estimation is better than Bootstrap percentile interval estimation on the aspects of stability, length and cover rate (see Debasis Kundu [9]).In this section, we present Bootstrap-t method to construct CIs for unknown parameters. The detail steps are as follows:

1) Given $n,r,k_i(i=1,\cdots r),T_0$ and progressive Type-II censored sample $t_i(i=1,\cdots,r)$, we obtain $\hat{c},\hat{d},\hat{\theta}$ using (2) and (3).

2) We generate a random progressive Type-II censored sample of size r from Uniform (0,1) distribution like Uditha Balasooriya and N. Balakrishnan (2000)[10].

(i) Generate a random sample of size r from Uniform(0,1) distribution $s_1,s_2,\cdots s_r$,

(ii) For given k_i, set $s_i'=s_i^{1/(i+k_r+k_{r-1}+\cdots+k_{r-i+1})}$, $i=1,2,\cdots,r$,

(iii)Noting $u_i=1-(s_r's_{r-1}'\cdots s_{r-i+1}')$, $i=1,2,\cdots r$, then $u_1,u_2,\cdots,u_r$ are progressive

Type-II censored sample from Uniform (0, 1) distribution.

3) Generate progressive Type-II censored sample from exponential distribution using adverse transform method. From Mao,Wang and Pu [11], we know that $v_i = -\ln(1-u_i), (i = 1,2,\cdots r)$ are from exponential distribution exp(1).Setting $t_i' = [(1+\hat{c})v_i / (\hat{d}k^{\hat{c}})]^{1/(\hat{c}+1)}, i = 1,\cdots,r$,So r_1 was given by $t_{r_1}' < T_0 < t_{r_1+1}'$ .For $i \le r_1$,setting $x_i^* = t_i'/T_0$,and for $r_1 < i \le r$,setting $x_i^* = c/(c+1) + T_0^{-1}d^{-1}V_2^{-c}v_i$.

4) Using (1)-(3) and $\{x_1^*, x_2^*, \cdots, x_r^*\}$ above, obtain $\hat{c}^*, \hat{d}^*$, then gain $\hat{\theta}^*, \hat{\lambda}^*$ using accelerated equation and $\theta = 1/\lambda$.

5) Calculate $\hat{V}(\hat{c}^*) = \hat{c}^{*2}/r$ and determine the statistic $T^* = (\hat{c}^* - \hat{c}) / \sqrt{\hat{V}(\hat{c}^*)}$.

6) Repeat Step 2)-5) N times.

Let $\hat{F}^*(x) = P(\hat{c}^* \le x)$ be the cumulative distribution function of $\hat{c}^*$ .for a given x, define $\hat{c}_{B-t}(x) = \hat{c} + \sqrt{\hat{V}(\hat{c}^*)}\hat{F}^{*-1}(x)$ .The approximate $100(1-\alpha)\%$ confidence interval for c is given by $(c_L, c_U) = (\hat{c}_{B-t}(\alpha/2), \hat{c}_{B-t}(1-\alpha/2))$ .Similarly, the approximate $100(1-\alpha)\%$ confidence intervals for d,θ,λ are given by $(\hat{d}_{B-t}(\alpha/2), \hat{d}_{B-t}(1-\alpha/2)), (\hat{\theta}_{B-t}(\alpha/2), \hat{\theta}_{B-t}(1-\alpha/2))$£$(\hat{\theta}_{B-t}^{-1}(1-\alpha/2), \hat{\theta}_{B-t}^{-1}(\alpha/2))$, for $\tau_{V\sim V_0} = (V/V_0)^c$, is given by $((V/V_0)^{c_L}, (V/V_0)^{c_U})$.

5. Numerical Example

Since the performance of the different method cannot be compared theoretically, we use Monte Carlo simulations to compare different methods for sampling schemes and different censored number. We consider $c = d = 1, T_0 = 1, k = 1, V_0 = 0.4$,then the mean life and failure rate at normal stress are $\theta_0 = 2.5, \lambda_0 = 0.4$ .There schemes are used as follows:

(I) $n = 20, r = 15, k_2 = k_5 = k_{13} = 1, k_{10} = 2, k_i = 0, i = 1,\cdots 15, i \ne 2, 5, 10, 13$£

(II) $n = 25, r = 20, k_2 = k_7 = k_{12} = k_{15} = k_{17} = 1, k_i = 0, i = 1,\cdots 20, i \ne 2, 7, 12, 15, 17$£

(III) $n = 30, r = 25, k_1 = k_4 = k_9 = k_{15} = k_{19} = 1, k_i = 0, i = 1,\cdots 25, i \ne 1, 4, 9, 15, 19$£

We generate 1000 samples to gain the bias and mean square error for each scheme, take their mean value as estimate value. The bias and mean square error (MSE) of the estimations in every scheme are as follows:

Table 1: MLEs of parameters

Parameter	c		d		θ_0		λ_0	
scheme	bias	MSE	bias	MSE	bias	MSE	bias	MSE
I	0.0777	0.1440	0.0844	0.1101	0.2324	1.1166	0.0104	0.0209
II	0.0763	0.1231	0.0741	0.0879	0.2167	0.8378	0.0008	0.0177
III	0.0528	0.1227	0.0540	0.0605	0.1689	0.8291	0.0004	0.0150

Table 2: interval estimations based on Booотstrap-t method

Parameter	c		d		θ_0		λ_0	
scheme	$\hat{c}_L$	$\hat{c}_U$	$\hat{d}_L$	$\hat{d}_U$	$\hat{\theta}_{0L}$	$\hat{\theta}_{0U}$	λ_{0L}	λ_{0U}
I	0.4689	1.2665	0.6442	1.3485	1.4312	2.7767	0.2148	0.6291
II	0.3820	1.2758	0.6802	1.3384	1.2526	3.4652	0.2774	0.5535
III	0.7081	1.5793	0.6424	1.3213	1.2924	3.3533	0.2890	0.5512

Table 3: interval estimations based on fisher information

Parameter	θ_0		λ_0	
scheme	$\hat{\theta}_{0L}$	$\hat{\theta}_{0U}$	λ_{0L}	λ_{0U}
I	1.8257	3.3567	0.2979	0.5477
II	1.9872	3.3525	0.2983	0.5032
III	2.0311	3.0513	0.3277	0.4923

In addition, we can gain the approximately unbiased estimation of mean life θ_0 and the failure rate λ_0 at the normal stress using the methods above, from scheme I to III are 2.4332, 2.5485, 2.4704, and 0.3970, 0.3825, 0.3986.

From above three tables, we can see that methods in this paper are correct and feasible. It shows that as sample size increased from 20 to30, precision of most point and interval estimations are improve on different degree. Among them, interval estimations based on Fisher information are most obvious (see table 3).

While comparing them, point estimations based on fisher information are better than MLEs, interval estimations based on fisher information are also better than which based on Bootstrap-t method. But Bootstrap-t method can give interval estimations of all the parameters, while fisher information cannot. In practical problems people can choose suitable method with practical needs.

6. Conclusions

This paper gives point and interval of unknown parameters from exponential distribution and accelerated equation based on progressive Type-II censoring sample. Particularly, unbiased approximate estimation of mean life and failure rate at the normal stress are obtained. More over, we introduced Bootstrap-t method to accelerated life testing. This method different from traditional ones, and it is convenient and easy to application. The simulation shows that methods in this paper are correct and feasible.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No.70471057) and Natural Science Foundation of the Education Department of Shaanxi Province (No. 03JK065)

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