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An Optimal Control Problem on Air Pollution Due to Emission From Moving Vehicle on Road

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ABSTRACT

In this paper we have investigated an integrated volume concentration of pollutants due to the emission from moving vehicle in an considered air shed on the basis of optimal control of emission from the vehicle so that the integrated concentration of pollution can be attained in compliance with air quality standard of health.

Notations:

c(x, y, z, t) : concentration of pollutants μg m^{-3}
u: Velocity of the wind, ms^{-1}
Q(x, y, z, t) : emission of the pollutant from the vehicle, μgs^{-1}
Q(x, t) : Integrated pollution emission from the vehicle only y-z plane, μgs^{-1}
δ : dirac - detta function
v_1 : Uniform velocity of the vehicle, ms^{-1}
v_2 :Velocity of deposition, ms^{-1}
c_o : Initial concentration, μgm^{-3}
c_2 : Concentration of entrainment pollutants, at x=0 μgm^{-3}
k_2, k_3 : Coefficients of eddy-diffusivity, m^2s^{-1}
c_1 : Integrated concentration an y-z idane, μgm^{-1}
m: Number of vehicles
H: Height of inversion layer, m
T: Total time, S
Ω : Total area of y-z plane, m^2
$f(\frac{x}{L})$ = Reduction of pollutant for vehicle
L= Length of concerned street within two traffic signals, m
x = horizontal length,m

$f(\frac{v_1 t}{L})$ = Reduction of pollutant for each vehicle.

$\chi(t)$ = integrated concentration of pollutant over the considered airshed, μg

χ^* = concentration of health air quality standard for an individual taking as a point receptor, μg

$\psi(t)$ = auxiliary function.

1. **Introduction**

During the last decade there has been a growing concern on chemically reactive pollutants emitted from automotive vehicles in cities. The EPA's first concern in this case is with hydrocarbons, carbon monoxide and nitrogen oxides. From an air quality standpoint, NO and NO_2 are important because of the role they play in the chemistry of photochemical smog. The dangers with carbon monoxide occur because of it's strong affinity with blood Caroxyhomoglobin which causes abnormalities on cardiac and brain function. Carbon monoxide is not toxic in itself and is non- cumulative. At high concentration carbonmonoxide is lethal to most animal species, pulmonary edema.

Thus the need for development of suitable techniques for reducing the concentration of gaseous pollutants like CO and NO_2 cannot be ruled out. Out of many techniques such as (i) modification of combustion system, (ii) induction system, (iii) ignition system, (iv) exhaust- gas re-circulation, (v) catalytic converters systems can generally be specified [1, p51] in respect of reducing carbon monoxide.

In average motor vehicle with no emission control devices, the estimated values of annual production of the pollutants are about 770 kg of carbon monoxide, 240kg of unburned hydrocarbons and 40kg of nitrogen oxides [1, p50].

In course of study [1, p51] it has been found that catalytic converter fitted in the exhaust lines has the capacity to reduce the pollutant carbon monoxide by 60 -70 percent under different load conditions at various speeds and mixture ratio. The introduction of secondary air and its heating have considerably increased the conversion efficiency of the pollutants. Although some power loss has been observed due to the introduction of the converter, it is not very high compared to the reduction of the pollutants. The cost of such devices with indigenous catalysts is much lower compared to other devices[p51].

Some recent studies have been made on vehicular pollution where, in most of the cases, practical and experimental considerations have been taken into account. Besides the vast consideration on emission of pollution from vehicle, some very recent important studies on this topic can be mentioned [6,7,8,9,10] and also studies on it can well be cited [12].

In this paper, we would like to investigate the integrated concentration of pollutants emitted from vehicle moving with constant velocity within a certain air-shed so as to attain an expected reduced value (considering the aspect of air quality standard) by controlling the emission source of the moving vehicle. Thus this theoretic study will provide a guideline on the control emission of moving vehicle pollutants by means of technical devices, though it is to some extent most difficult to control or better to say to reduce the vehicle emission continuously with respect to time. But it may be expected that in view of our theoretic study, some modification on the approach of the present techniques can he advanced on the guideline of our theoretic study and also some modern computerized techniques may find a way out in near computer stimulated future to come.

Here we would like to concentrate on a particular physical situation where there are building complexes on the both sides of the streets along which the vehicles are moving. This situation is frequently observed specially in cities.

In this case, diffusion-dispersion are basically influenced by the physical structure of the building complexes, for example, large objects can play as obstructions to produce aero-dynamies downwash, causing higher concentrations in their immediate vicinity [3, p 109].

Further, an area, generally an urban or industrial complex, where temperature is higher than that of its normally surroundings, heat island effect may lead to a local circulation pattern is which air pollution can get trapped [3, p271].

1.1. Formation of the problem

We would like to investigate the integrated concentration of the pollutant emitted from vehicles moving with a constant velocity in a congested area of a city like Kolkata (India). It is assumed that the vehicle stops at end where there are traffic signal which controls the traffic as is generally observes in certain area in the city. Further, one can assume that in the concerned street there are two traffic signals. From the first signal, the vehicle just starts as the signal becomes blue.

Here Q(x, t) designates the emission of the pollutants from the vehicle moving with a constant velocity. It is to note that in course of motion of the vehicle moving with, a constant velocity the emission of the pollutants will generally be same (equal) in absence of any technical devices of reducing the emission of the pollutants in the vehicle as the engine works at the same and equal rate. In this case Q(x, t) = Q (say) as constant for all values of x and t.

Thus thinking on the aspect of regulation of the emission of the pollutants from the vehicle in the atmosphere one has to think on the another aspect, more probably, on the distribution of tile pollutants in the atmosphere after a certain reduction which can be assumed as $f\left(\frac{x}{L}\right)$ by means of technical devices inside the vehicle.

Let us assume:

i) The inversion is low in an adverse meteorological situation.
ii) The level of the road of moving vehicle is plane and horizontal.
iii) The diffusion is considered small in regards to the wind vector in the direction of x-axis of moving vehicle, i.e., the diffusivity constant along x-direction equals to zero.
iv) The wind velocity along cross- wind and vertical directions are absent.
v) The pollutants get trapped on the cross- wind direction by the building complexes such that $\frac{\partial c}{\partial y} = 0$ on the boundaries $Y = \pm Y_1$(say).
vi) The Gaussian plume is taken into consideration from the physical standpoint and simplicity.

1.2. Basic equation

Differential equation of the concentration distribution of gaseous pollutants c(x, y, z, t) can be written as [3]

$$\frac{\partial c}{\partial t}+u\frac{\partial c}{\partial x}=k_2\frac{\partial^2 c}{\partial y^2}+k_3\frac{\partial^2 c}{\partial z^2}+mQ(x,y,z,t)\delta(x-v_1t)\delta(y)\delta(z) \qquad \text{.........(1)}$$

where Q (x,t) designates the strength of emission of pollutants from a vehicle considered as a point source moving under uniform velocity v_1.

The initial and boundary conditions, according to the formation of the problem can be written as, [3,4]

$c = c_o$ at t = 0 ....(2)

$v_2 c = k_3\frac{\partial c}{\partial z}$ on z = 0 ....(3)

$\frac{\partial c}{\partial z}=0$ on z = H ....(4)

$c = c_2$ on x = 0 (a constant entrainment of the pollutant) ....(5a)

$\frac{\partial c}{\partial y}=0$ on y = ± y_1 ....(5b)

We would like to make additional assumption that the vehicle emission, the distance of the building complex along the side of the road and the inversion height are such that within the considered airshed in the stipulated time, the atmosphere has the characteristic of being homogeneous.

The integrated concentration distribution an y=z plane $-y_1 \le y \le, y_1, 0 \le \mathrm{Z} \le H$ can be obtained by integrating equation (1) which with the help of equation (3), (4) and (5) stands,

$$\frac{\partial c_1}{\partial t}+u\frac{\partial c_1}{\partial x}=mQ(x,t)\delta(x-v_1t)-v_2\bar{c}_1 \qquad \text{..................(6)}$$

$$c_1=\int_{-y_1}^{y_2}\int_0^H c\,dy\,dz, \qquad \overline{c_1}=\int_{-y_1}^{y_2} c\,dy \qquad \text{................(7)}$$

Let us consider the integrated concentration in (0,L). Then integrating equation (6) in x-direction (0, L), equation (6) can be written as,

$$\frac{d\chi}{dt}+u(\chi_1-\chi_o)=\int_0^L mQ(x,t)\delta(\chi-v_1t)dx-v_2\,\bar{\chi} \qquad \text{..................(8)}$$

$$\text{for } \chi(t)=\int_0^L c_1dx,\ \ \bar{\chi}(t)=\int_0^L \bar{c}_1\,dx \qquad \text{..................(9)}$$

where

$$\chi_o=c_2\Omega, \chi_1=c_0\Omega \quad \text{on x = L} \qquad \text{..................(10)}$$

and $\Omega(area)=2y_1H,\ y_1=\sqrt{2k_2T}$

Let us assume $Q(x,t) = Q\left\{1\text{-}f\left(\frac{x}{L}\right)\right\}$

where Q signifies the constant emission of the pollutant from the vehicle in absence of reduction with no emission control devices.

Then eqn(8) stands,

$$\frac{d\chi}{dt}+u(\chi_1-\chi_o)=mQ\left\{1-f\left(\frac{v_1t}{L}\right)\right\}-v_2\,\bar{\chi} \qquad \text{............(11)}$$

and $f\left(\frac{v_1t}{L}\right)$ signifies reduction by means of technical devices inside the vehicle as mentioned specially in [1].

In this connection it has been assumed in eq. (9) that as the vehicles stops on x = L the emission of the pollutant from vehicle engine will be stopped, nevertheless, since the velocity of the vehicles is greater than the velocity of the wind (v_1>u), as in general case, the pollution plume will not reach at the instant when the vehicles are on x = L. Thus when the vehicles will reach an x=L only the initial pollutant prevail and hence in equation (9),

$$\chi_1=c_o\Omega. \qquad \text{.........(12)}$$

However, it is worthwhile to mention that we have considered a group of vehicles moving along the path side by side in two/three horizontal layers as seen generally in the city streets and for the sake of simplicity as well as for nearest approximated solution of our concerned problem one may assume these two/three layers as a single layer.

1.3. Statement of the optimal control problem

To obtain control $f\left(\frac{v_1 t}{L}\right)$ for the process described by equation (8) under condition equation (9) for $0 \le t \le T$ so that the integrated pollution concentration over the whole arished, bounded by (0,L), $(-y_{1,}y_1), (0, H)$ attains a concentration below a desired concentration of air quality standard χ^* where $f\left(\frac{v_1 t}{L}\right)$ is constrained such that $f\left(\frac{v_1 t}{L}\right)$ maintains a least value during the time T which means the possible least reduction of the pollutant emitted from each vehicle and consequently in turn it will facilitate in calculating the approximate cost for possible reduction of the pollutant.

Thus one may write (12)

$$J = \frac{1}{2}\int_0^T \left[\left\{ \frac{\chi^* - \chi(t)}{\chi^*} \right\}^2 + \left\{ f\left(\frac{v_1 t}{L}\right)^2 \right\} \right] dt \qquad \text{to be minimized} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

Following [5], with the aid of Maximal principle, one can construct the Hamiltonian given by,

$$H = -\frac{1}{2}\left[\left\{ \frac{\chi^* - \chi(t)}{\chi^*} \right\}^2 + \left\{ f\left(\frac{v_1 t}{L}\right)^2 \right\} \right] + \psi(t)\left[mQ\left\{1 - f\left(\frac{v_1 t}{L}\right)\right\} - v_2\,\bar{\chi} - u(\chi_1 - \chi_o) \right]$$

…………………..(14)

and the auxiliary function $\psi(t)$ can be obtained [5]

$$\frac{d\psi}{dt} = \frac{-\partial H}{\partial \chi} \quad for \quad \psi(T) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

The optimal control thus from $\frac{\partial H}{\partial \mathrm{f}} = 0$ stands as,

$$f\left(\frac{v_1 t}{L}\right) = -mQ\psi(t) \qquad \text{..................(16)}$$

Let us consider a stuffy atmosphere when $c_2 = 0$ (no entrainment) and for the sake of simplicity the atmosphere is completely clean from any pollution at the on set of the process (when the vehicle just starts having green signal) thus $c_0 = 0$.

General health standard of carbon monoxide pollutant emitted from motor vehicle can be taken as $40\,\mu g\, m^{-3}$ over 1 hour [2, p205]. As we would like to consider the health standard of an individual person which can be taken, for the sake of generality, as a point receptor and also there is at present no health standard for very short period (according to our problem concerned), we may assume for near approximate solution of our problem, $40\,\mu g$ over 1 hour as our individual health standard, that is, on a point receptor. Thus the health standard of carbon monoxide along the path of length 1000 m. may be taken as $4000\,\mu g$ over the period for our problem concerned.

1.4. Numerical calculation

Here we have assumed (i) no velocity of dry deposition of gaseous pollutants is taken considering the aspect that generally important city streets are covered throughout by pitch that is $v_2 = 0$, (ii) the system is considered in very early morning when the atmosphere is almost clean from pollutant and no entrainment of the Carbon monoxide is expected from the neighboring surroundings, that is, $\chi_o = 0$, and $\chi_1 = 0$ in equation (10).

Data used for solving equation (16) and (8) with the help of equation (15) and (9) given as follows:

$$\chi_o = 0, \chi_1 = 0, u = 2ms^{-1}, L = 1000m.$$

$$\chi^* = 40\mu g, v_2 = 0, Q = 24.461\mu g s^{-1} \qquad [1, p50]$$

$$v_1 = v_{11} = 8ms^{-1}$$

$$v_1 = v_{12} = 10ms^{-1}$$

Method of iteration has been applied satisfying the conditions in equation (13) on convergence of order 10^{-4}.

Table 1

t	f(v_{11} t/L)	f(v_{12} t/L)	f’(v_{11} t/L)	f’(v_{12} t/L)
0	0	0	0	0
10	0.115	0.123	0.301	0.407
20	0.291	0.400	0.386	0.434
25	0.373	0.477	0.476	0.488
30	0.468	0.594	0.534	0.548
40	0.586	0.673	0.634	0.651
50	0.663	0.720	0.702	0.710
60	0.712	0.754	0.743	0.755
70	0.750	0.783	0.777	0.780
80	0.777	0.805	0.803	0.806
90	0.799	0.850	0.823	0.826
100	0.821		0.840	
110	0.837		0.855	

Table 2

t	χ_{11}(t)	χ_{12}(t)	χ'_{11}(t)	χ'_{12}(t)
0	0	0	0	0
10	19483	19300	15600	14500
20	31200	30100	30040	29150
25	34200	33040	32010	31990
30	35150	34500	34150	33150
40	36410	35700	35800	34150
50	37150	36000	36400	35500
60	38150	37100	37700	36000
70	38500	37910	38100	37600
80	39150	38200	38520	37900
90	39320	38600	38950	38150
100	39400		39050	
110	39500		39150	

2. Result and discussion

In Table 1, $f\left(\frac{v_{11}t}{L}\right)$ and $f\left(\frac{v_{12}t}{L}\right)$ designate the reduction of pollutant for 90 vehicles with velocities 8ms^{-1} and 10ms^{-1} ; $f^{/}\left(\frac{v_{11}t}{L}\right)$ and $f^{/}\left(\frac{v_{12}t}{L}\right)$specify the reduction for 100 vehicles. χ

In Table 2, $\chi_{11}(t)$, $\chi_{12}(t)$, $\chi'_{11}(t)$, $\chi'_{12}(t)$ are designated in the same manner with the respective velocities and respective number of moving vehicles as maintained in the first case.

Here it reveals form Table 1, as the velocity of the vehicles increase form 8ms^{-1} and 10ms^{-1}, the emission of the pollutant in the atmosphere after reduction decreases as given in Table 2, which in consequence clarifies the aspect of increasing reduction with the increase of velocity of same number of moving vehicles either 90 or 100 as considered in Table 1. The case of increasing reduction with increasing velocity of vehicles has clearly been mentioned in [1, p 51-59] where catalytic convertor has been used with more inclusion of secondary air and greater increase of throttle opening to convert the harmful gases into less harmful gases from the human health standpoint through chemical reduction and oxidation while the gases pass over a bed of active material either deposited on supports or unsupported [1, p51].

It is to note further from Table 2, that either with the increase number of moving vehicles having the same velocity or with the same number of vehicles having increasing velocities, the emission after reduction in the atmosphere decreases in corresponding times. All those cases may sometimes not fit from the thinking of practical standpoint. But as we are considering optimal control problem such that the integrated concentration of pollutant over the whole air-shed does not at any time during the process cross the level of human health standard, better to be such more below that level, there must have some feasible means of constrained reduction, as specified earlier, which increase so as to maintain our conditional aspect.

However, as the vehicles stop at the end of the path due to red traffic signal, the vehicles may be practically expected to retard its motion at a certain distance from the signal by stopping the engine of the vehicle, so as to maintain the possibity of no emission, from times 110s and 90s(say) for velocities 8ms^{-1} and 10ms^{-1} respectively, that is, at a distance of 120m and 100m from the end respectively.

3. Conclusion

This paper considers an optimal control problem on reduction of pollutants from the moving vehicles with a view to achieve to maintain the integrated concentration of the pollutants below the air quality standard, particularly, with basic concept not to cross the human health standard of some pollutants harmful to human health. This study will primary specify an approximate solution of the stated problem, that is, to find out reduction to achieve our goal. But one can well be known with fact that in different atmospheric situations, specially in rainy season and in cold weather, the inversion layer changes which should be considered in the study. Besides, most important experimental study in the "Indian Context" was made in [1] about a long time past which draws the interest of the authors to reduce pollutants by means of catalytic converter. But after long days passed, it may be expected that some more advancement in the concept of computerized techniques with modern engineering knowledge will find out a way in future giving some feasible technical devices in manufacturing vehicles to ply on the congested streets in cities.

Further, the cost of reduction of pollutants by technical devices is one of the vital thing to be considered from vehicle manufacturing concerns and this study tries to obtain an approximate concept on this aspect as stated earlier. The pollution concentration is a burning problem of the day, this study possibly give a primary concept in the maximum number of moving vehicles on the streets, or average number feasible and viable vehicles on the human standpoint not to cross the human air quality standard in later future to come.

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