Journal of Physical Sciences, Vol. 29, 2024, 1-11
ISSN: 2350-0352 (print), www.vidyasagar.ac.in/publication/journal
Published on 31 December 2024
DOI: http://dx.doi.org/10.62424/jps.2024.29.00.01

γ-Separation Axioms on Fuzzy Soft T_2 Space

R. Islam[*] and M. S. Hossain[1]

[*]Department of Mathematics, Pabna University of Science and Technology
Pabna-6600, Bangladesh
Email: rafiq.math@pust.ac.bd
[1]Department of Mathematics, University of Rajshahi, Rajshahi, Bangladesh
Email: sahadat@ru.ac.bd
[*]Corresponding author: rafiq.math@pust.ac.bd

Received 1 August 2024; accepted 20 December 2024

ABSTRACT

In this article we introduce the four new inferences of fuzzy soft T_2 spaces by using the concept of fuzzy soft topological spaces. After that we present several new theories and some implications of such spaces. Finally, we observe that, all these notions preserve some soft invariance properties as 'Soft hereditary' and 'Soft topological' property.

***Keywords*:** Soft sets, fuzzy soft sets, soft topology, fuzzy soft topology, fuzzy soft open sets, fuzzy soft mapping, and image of fuzzy soft mapping.

AMS Mathematics Subject Classification (2010): 06D72, 54A40

1. Introduction

For formal modeling, reasoning, and computing, the majority of the mathematical tools currently in use are crisp, deterministic, and precise. However, in practical applications, issues in the fields of economics, engineering, environmental research, social science, medicine, and so forth do not necessarily require precise data. The different kinds of uncertainty that these difficulties bring make it impossible for us to apply the conventional classical approaches with any degree of effectiveness. Some theories were provided in order to overcome these inconstancies, such as the theory of fuzzy sets [18, 21], intuitionistic fuzzy sets [1, 5, 15], and rough sets [13], which are mathematical instruments for handling uncertainty. But all these theories have their inherent difficulties, as pointed out by Russian mathematician Dmtri Molodstov in 1999 [12].

Consequently, the mathematician [12], in order to overcome the existing difficulty of defining properly the membership function of a fuzzy set, proposed the soft sets as a new mathematical tool for dealing with uncertainty in a parametric manner in that year. After the essence of the notion of soft sets, a number of scholars improved this concept. Next, Maji *et al.* [7-9] presented the use of soft sets in decision-making problems that are based on the reduction of parameters to keep the optimal choice objects. Furthermore, soft sets are a subclass of special information systems, as demonstrated by Pei and Miao [14]. Sabir and Naz [11] also explored the topological nature of soft sets. Then they examined the ideas of soft open sets, soft closed sets, soft closure, soft interior points, and soft neighborhoods of a point, soft separation axioms, and their fundamental features. They also defined the soft topological spaces, which are expressed over an initial universe with a specified set of parameters. Later, Roy and Samanta [16] gave the definition of fuzzy soft topology over the original universe set. Additional research was conducted by Varol and Aygunn [2] and Cetkin and Aygun [3], and others. In addition, Sabir Hussain and Bashir Ahmad [17] redefined and investigated a number of features of the soft T_i, i = 0, 1, 2, soft regular, soft T_3, soft normal, and soft T_4 axioms using the soft points defined by I. Zorlutuna [22]. Additionally, they talked about the soft topological and soft hereditary aspects of soft invariance. In this work, we describe fuzzy soft T_2 spaces in four novel ways, propose several theories, and examine their topological and hereditary features, among other properties.

All over this article, X and Y will be nonempty sets, $\emptyset$ be the empty set, and E will be the set of all parameters. F_E will be the soft set, f_A will be the fuzzy soft set, $\widetilde{T}$ and τ will be the soft topology and fuzzy soft topology, respectively. The remaining portions of this work are organized as follows: Section 2 presents a quick overview of some of the fundamental definitions of fuzzy sets, soft sets, fuzzy soft sets, soft topology, fuzzy soft topology, fuzzy soft mapping, and the image of fuzzy soft mapping. In Section 3, we establish four ideas of fuzzy soft T_2 spaces, show some implications among them, and present a number of new theories related to fuzzy soft T_2 spaces. The ideas of “Good extension," “hereditary” property, and its related theorems are given in Section 4. Finally, Section 5 represents the conclusion of this paper.

2. Preliminaries

We call off some basic definitions and known results of soft set, fuzzy soft sets, and some operations on fuzzy soft sets, soft topology, fuzzy soft topology and fuzzy soft mapping and so on.

.

Definition 2.1. [21] Let X be a non-empty set and $I = [0, 1]$. A fuzzy set in X is a function $u: X \to I$ which assigns to each element $x \in X$, a degree of membership $u(x) \in I$.

Definition 2.2. [19] A pair (F, E) denoted by F_E is called a soft set over X, where F is a mapping given by $F: E \longrightarrow P(X)$. We denote the family of all soft sets over X by $SS(X, E)$.

Definition 2.3. [15] A soft set (F, E) over X is called a null soft set and denoted by $\widetilde{\emptyset}$, if $F(e) = \emptyset$ for every $e \in E$.

Definition 2.4. [19] A soft set (F, E) over X is called an absolute soft set and denoted by $\widetilde{X}$, if $F(e) = X$ for every $e \in E$.

Definition 2.5. [19] Let X be an initial universal set, and $A \subseteq E$. Let $\tilde{\mathcal{T}}$ a subfamily of the family of all soft sets $S(X)$. We say that the family $\tilde{\mathcal{T}}$ is a soft topology on X if the following axioms are holds:

(i) $\widetilde{\emptyset_A}, \widetilde{X_A} \in \tilde{\mathcal{T}}$.
(ii) If $F_A, G_A \in \tilde{\mathcal{T}}$ then $F_A \cap G_A \in \tilde{\mathcal{T}}$.
(iii) If $G_{iA} \in \tilde{\mathcal{T}}$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} G_{iA} \in \tilde{\mathcal{T}}$.

Then the triple $(\widetilde{X_A}, \tilde{\mathcal{T}}, A)$ is called soft topological space (STS, for short) and the members of $\tilde{\mathcal{T}}$ are called soft open sets (SOS for short). A soft set F_A is called soft closed set (SCS, for short) if and only if its complement is soft open set. That is $F_A^C \in \tilde{\mathcal{T}}$.

Definition 2.6. [11] A soft topological space $(F_A, \tilde{\mathcal{T}}, A)$ is called soft T_2 (ST_2) space if for each $x_1, x_2 \in X$, with $x_1 \neq x_2$, there exist soft open sets (SOSs for short) $F_A, F_B \in \tilde{\mathcal{T}}$ such that $x_1 \in F_A, x_2 \in F_B$ and $F_A \cap F_B = \widetilde{\emptyset}$.

Definition 2.7. [2] A fuzzy soft set f_A on the universe X is a mapping from the parameter set E to I^X, i. e., $f_A: E \longrightarrow I^X$, where $f_A(e) \neq 0_x$ if $e \in A \subseteq E$ and $f_A(e) = 0_x$ if $e \notin A$, where 0_x is empty fuzzy set on X.

From now on, we will use $\mathcal{F}(X, E)$ instead of the family of all fuzzy soft sets over X. Obviously, a classical soft set F_A over a universe X can be seen as a fuzzy soft set by using the characteristic function of the set $F_A(e)$:

$$f_A(e)(a) = \chi_{F_A(e)}(a) = \begin{cases} 1, & \text{if } a \in F_A(e); \\ 0, & \text{otherwise.} \end{cases}$$

Definition 2.8. [6] Two fuzzy soft sets f_A and g_B on X we say that f_A is called fuzzy soft subset of g_B and write $f_A \subseteq g_B$ if $f_A(e) \leq g_B(e)$, for every $e \in E$.

Definition 2.9. [6] Two fuzzy soft sets f_A and g_B on X are called equal if $f_A \subseteq g_B$ and $g_B \subseteq f_A$.

Definition 2.10. [6] Let $f_A, g_B \in (\widetilde{X, E})$. Then the union of f_A and g_B is also a fuzzy soft set h_C, defined by $h_C(e) = f_A(e) \vee g_B(e)$ for all $e \in E$, where $C = A \cup B$. Here we write $h_C = f_A \cup g_B$.

Definition 2.11. [6] Let $f_A, g_B \in (\widetilde{X, E})$. Then the intersection of f_A and g_B is also a fuzzy soft set h_C, defined by $h_C(e) = f_A(e) \wedge g_B(e)$ for all $e \in E$, where $C = A \cap B$. Here we write $h_C = f_A \tilde{\cap} g_B$.

Definition 2.12. [6] A fuzzy soft sets f_E on X is called a null fuzzy soft set denoted by 0_E if $f_E(E) = 0_x$ for each $e \in E$.

Definition 2.13. [6] A fuzzy soft sets f_E on X is called an absolute fuzzy soft set denoted by 1_E if $f_E(E) = 1_x$ for each $e \in E$.

Definition 2.14. [6] Let $f_A \in (\widetilde{X, E})$. Then the complement of f_A is denoted by f_A^c and is defined by $f_A^c(e) = 1 - f_A(e)$ for each $e \in E$.

Definition 2.15. [10] A fuzzy soft set g_A is said to be a fuzzy soft point, denoted by e_{gA}, if for the element $e \in A, g(e) \neq \tilde{\Phi}$ and $g(e') = \tilde{\Phi}, \forall\, e' \in A - \{e\}$.

Definition 2.16. [10] A fuzzy soft point e_{gA}, is said to be in a fuzzy soft set h_A, denoted by $e_{gA} \tilde{\in} h_A$ if for the element $e \in A, g(e) \leq h(e)$.

Definition 2.17. [10] Let f_A be a fuzzy soft set, $\mathcal{FS}(f_A)$ be the set of all fuzzy soft subsets of f_A and τ be a subfamily of $\mathcal{FS}(f_A)$. Then τ is called a fuzzy soft topology on f_A if the following conditions are satisfied:

(i) Φ_A, f_A belong to τ;
(ii) $f_{1A}, f_{2A} \in \tau \Longrightarrow f_{1A} \tilde{\cap}\ f_{2A} \in \tau$;
(iii) For any index set I, $f_{iA} \in \tau$, for any $i \in I$, then $\cup \{f_{iA}, i \in I\} \in \tau$.

Then the pair $(f_A, \tau)$is called fuzzy soft topological space (FSTS, for short) and the members of τ are called the fuzzy soft open sets (FSOS, for short). A fuzzy soft open set (FSOS) g_A is called a fuzzy soft closed set (FSCS, for short) if $g_A^c \in \tau$, where g_A^c is a complement of g_A.

Definition 2.18. [10] A fuzzy soft topological space (f_A, τ) is said to be a fuzzy soft T_2 space if for any pair of distinct fuzzy soft points e_{gA}, e_{hA} of $f_A, \exists$ two fuzzy soft open sets $f_{1A}$and f_{2A} such that $e_{gA} \tilde{\in} f_{1A}, e_{gA} \tilde{\notin} f_{2A}$ and $e_{hA} \tilde{\in} f_{2A}, e_{hA} \tilde{\notin} f_{1A}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$.

Definition 2.19. [2] Let $(f_A,\ \tau_1)$ and $(g_B,\ \tau_2)$ be two fuzzy soft topological spaces (FSTS's), on the two universal sets X and Y respectively. Then a fuzzy soft mapping $(\varphi, \psi): (f_A,\ \tau_1) \longrightarrow (g_A,\ \tau_2)$ is called

(i) Fuzzy soft continuous if $(\varphi, \psi)^{-1}(g_B) \in \tau_1, \forall\ g_B \in \tau_2$
(ii) Fuzzy soft open if $(\varphi, \psi)(f_A) \in \tau_2,\ \forall\ f_A \in \tau_1$
(iii) Fuzzy soft closed if $(\varphi, \psi)(f_A)$ is a fuzzy soft closed of τ_2 for each fuzzy soft closed set f_A of τ_1
(iv) Fuzzy soft homeomorphism if (φ, ψ) is bijective, continuous and open.

Definition 2.20. [6] Let $\varphi: X \to Y$ and $\psi: E \to F$ be two mappings, where E and F are parameter sets for the crisp sets X and Y, respectively. Then (φ, ψ) is called a fuzzy soft mapping from $(\widetilde{X, E})$ into $(\widetilde{Y, F})$ and denoted by $(\varphi, \psi): (\widetilde{X, E}) \longrightarrow (\widetilde{Y, F})$.

Definition 2.21. [6] Let f_A and $g_B$be two fuzzy soft sets over X and Y, respectively and (φ, ψ) be a fuzzy soft mapping from $(\widetilde{X, E})$ into $(\widetilde{Y, F})$.

(i) The image of f_A under the fuzzy soft mapping (φ, ψ), denoted by $(\varphi, \psi)(f_A)$ and is defined as, $(\varphi, \psi)(f_A)k(y) =$
$$\begin{cases} \vee\varphi(x) = y \vee \psi(e) = k f_A(e)(x), \text{if } \varphi^{-1}(y) \neq \emptyset, \psi^{-1}(k) \neq \emptyset; \\ 0, \qquad\qquad\qquad \text{otherwise} \end{cases}$$
for all $k \in F$, for all $y \in Y$.

(ii) The image of g_B under the fuzzy soft mapping (φ, ψ), denoted by $(\varphi, \psi)^{-1}(g_B)$ and is defined as, $(\varphi, \psi)^{-1}(g_B)(e)(x) = g_B\big(\psi(e)\big)\big(\varphi(x)\big)$, for all $e \in E$, for all $x \in X$.

Proposition 2.22. [4] Let $\left(\widetilde{X_A},\ \tilde{\mathcal{T}},\ A\right)$ be a soft topological space over X. Then the collection $\tilde{\mathcal{T}}_e = \{F(e) | (F, E) \in \tilde{\mathcal{T}}\}$ for each $e \in E$, defines a topology on X.

3. Definitions and properties of fuzzy soft T_2 spaces

Before we mentioned the definition of fuzzy soft T_2 space, and now in this section we introduce four ideas of fuzzy soft T_2 spaces, establish some implications among them and develop several new theories on fuzzy soft T_2 spaces. We denote the grade of membership and the grade of non-membership of any point in fuzzy soft set is $\tilde{1}$ and $\tilde{0}$ respectively. Here $\tilde{\gamma}$ means that the grade of membership of any point in fuzzy soft set lies between 0 and 1.

Definition 3.1. A fuzzy soft topological space (FSTS) $(f_A,\ \tau)$ is called

(a) FST_2(i) if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A$, $\exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_2) = \tilde{1}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$.
(b) FST_2(ii) if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A$, $\exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{\gamma}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_2) = \tilde{\gamma}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$ as $0 < \tilde{\gamma} < 1$.
(c) FST_2(iii) if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A$, $\exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) > f_{2A}(e)(x_1)$, and $f_{2A}(e)(x_2) > f_{1A}(e)(x_2)$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$.
(d) FST_2(iv) if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A$, $\exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1)$ and $f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2)$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$.

Theorem 3.1. Let $(f_A,\ \tau)$ be a fuzzy soft topological space. Then the above four notions of it form the following implications.

$$\begin{tikzcd}
\mathrm{FST}_2(\mathrm{iv}) \arrow[r, shift right, "/" marking] \arrow[d, shift right, "/" marking] & \mathrm{FST}_2(\mathrm{i}) \arrow[l, shift right] \arrow[r, shift left] \arrow[d, shift left] & \mathrm{FST}_2(\mathrm{iii}) \arrow[l, shift left, "/" marking] \arrow[d, shift right] \\
\mathrm{FST}_2(\mathrm{iii}) \arrow[u, shift right] \arrow[r, shift left, "/" marking] & \mathrm{FST}_2(\mathrm{ii}) \arrow[u, shift left, "/" marking] \arrow[l, shift left] \arrow[r, shift right] & \mathrm{FST}_2(\mathrm{iv}) \arrow[l, shift right, "/" marking] \arrow[u, shift right, "/" marking]
\end{tikzcd}$$

Figure 1: The implications of four notions are shown by two quadrilaterals

Proof: Let $(f_A,\ \tau)$ be a FST_2(i). Then by definitions if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A$, $\exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_2) = \tilde{1}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) = \tilde{\gamma},\ f_{1A}(e)(x_2) = \tilde{0} \\ f_{2A}(e)(x_2) = \tilde{\gamma}, f_{2A}(e)(x_1) = \tilde{0} \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \quad \text{as } 0 < \tilde{\gamma} < 1 \qquad (1)$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) > f_{1A}(e)(x_2) \\ f_{2A}(e)(x_2) > f_{2A}(e)(x_1) \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \qquad (2)$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1) \\ f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2) \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \qquad (3)$$

Hence from (1), (2) and (3) we see that $\text{FST}_2(\text{i}) \Longrightarrow \text{FST}_2(\text{ii}) \Longrightarrow \text{FST}_2(\text{iii}) \Longrightarrow \text{FST}_2(\text{iv})$.

Again suppose that $(f_A,\ \tau)$ be a $\text{FST}_2(\text{i})$. Then by definitions if for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A, \exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_2) = \tilde{1}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) > f_{2A}(e)(x_1) \\ f_{2A}(e)(x_2) > f_{1A}(e)(x_2) \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \qquad (4)$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1) \\ f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2) \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \qquad (5)$$

Hence from (4) and (5) we see that $\text{FST}_2(\text{i}) \Longrightarrow \text{FST}_2(\text{iii})$, and $\text{FST}_2(\text{i}) \Longrightarrow \text{FST}_2(\text{iv})$.

And finally let $(f_A,\ \tau)$ be a $\text{FST}_2(\text{ii})$. Then from (1) for any pair of $x_1, x_2 \in X$, with $x_1 \neq x_2$, and for all $e \in A, \exists$ FSOS's $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{\gamma},\ f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_2) = \tilde{\gamma}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$ as $0 < \tilde{\gamma} < 1$.

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1) \\ f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2) \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \qquad (6)$$

From (6) we see that $\text{FST}_2(\text{ii}) \Longrightarrow \text{FST}_2(\text{iv})$.

None of the reverse implications is true in general which can be seen in the following counter examples:

Example 3.1.1. Let $X = \{x_1,\ x_2\}, E = \{e_1, e_2, e_3, e_4, e_5\}$ a set of parameters, $A = \{e_1, e_2\} \subset E$, and τ be a fuzzy soft topology on a universal set X generated by $\tau = \{\tilde{0}, \tilde{1}, f_{1A}, f_{2A}\}$ where $f_{1A} = \left\{e_1 = \left\{\frac{0}{x_1}, \frac{0.5}{x_2}\right\}, e_2 = \left\{\frac{0}{x_1}, \frac{0.7}{x_2}\right\}\right\}, f_{2A} = \left\{e_1 = \left\{\frac{0.5}{x_1}, \frac{0}{x_2}\right\}, e_2 = \left\{\frac{0.7}{x_1}, \frac{0}{x_2}\right\}\right\}$. Here $f_{1A}(e_1)(x_1) = 0, f_{1A}(e_1)(x_2) = 0.5,$ $f_{1A}(e_2)(x_1) = 0, f_{1A}(e_2)(x_2) = 0.7$ and $f_{2A}(e_1)(x_1) = 0.5, f_{2A}(e_1)(x_2) = 0, f_{2A}(e_2)(x_1) = 0.7, f_{2A}(e_2)(x_2) = 0$. Therefore, we have for all $e \in A, f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1)$, $f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2)$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$. Hence we observe that $(f_A,\ \tau)$ is $\text{FST}_2(\text{iv})$ but not $\text{FST}_2(\text{i}), \text{FST}_2(\text{ii})$, and $\text{FST}_2(\text{iii})$. Therefore $\text{FST}_2(\text{iv}) \nRightarrow \text{FST}_2(\text{i}), \text{FST}_2(\text{iv}) \nRightarrow \text{FST}_2(\text{ii})$, and $\text{FST}_2(\text{iv}) \nRightarrow \text{FST}_2(\text{iii})$.

Example 3.1.2. Let $X = \{x_1,\ x_2\}, E = \{e_1, e_2, e_3, e_4, e_5\}$ a set of parameters, $A = \{e_1, e_2\} \subset E$, and τ be a fuzzy soft topology on a universal set X generated by $\tau =$

$\{\tilde{0}, \tilde{1}, f_{1A}, f_{2A}\}$ where $f_{1A} = \left\{e_1 = \left\{\frac{0.7}{x_1}, \frac{0}{x_2}\right\}, e_2 = \left\{\frac{0.7}{x_1}, \frac{0}{x_2}\right\}\right\}, f_{2A} = \left\{e_1 = \left\{\frac{0}{x_1}, \frac{0.5}{x_2}\right\}, e_2 = \left\{\frac{0}{x_1}, \frac{0.5}{x_2}\right\}\right\}$. Here we have for all $e \in A$, $f_{1A}(e)(x_1) > f_{2A}(e)(x_1)$, $f_{2A}(e)(x_2) > f_{1A}(e)(x_2)$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$. Hence we see that (f_A, τ) is FST_2(iii) but not FST_2(i), and FST_2(ii). Henceforth $\mathrm{FST}_2(\mathrm{iii}) \nRightarrow \mathrm{FST}_2(\mathrm{i})$, and $\mathrm{FST}_2(\mathrm{iii}) \nRightarrow \mathrm{FST}_2(\mathrm{ii})$. Finally if we consider $f_{1A} = \left\{e_1 = \left\{\frac{\tilde{\gamma}}{x_1}, \frac{\tilde{0}}{x_2}\right\}, e_2 = \left\{\frac{\tilde{\gamma}}{x_1}, \frac{\tilde{0}}{x_2}\right\}\right\}$ and $f_{2A} = \left\{e_1 = \left\{\frac{\tilde{0}}{x_1}, \frac{\tilde{\gamma}}{x_2}\right\}, e_2 = \left\{\frac{\tilde{0}}{x_1}, \frac{\tilde{\gamma}}{x_2}\right\}\right\}$ as $0 < \tilde{\gamma} < 1$ then we have (f_A, τ) is FST_2(ii) but not FST_2(i).

Theorem 3.2. Let (f_A, τ, A) be a fuzzy soft topological space (FSTS) over a universal set X and $e_{x_1}, e_{x_2} \in f_A$ such that $e_{x_1} \neq e_{x_2}$ as $x_1 \neq x_2$ for every pair $x_1, x_2 \in X$. If there exist FSOS's (f_{1A}, A) and (f_{2A}, A) such that $e_{x_1} \in (f_{1A}, A)$ and $e_{x_2} \in (f_{1A}, A)^c$ or $e_{x_2} \in (f_{2A}, A)$ and $e_{x_1} \in (f_{2A}, A)^c$ then

(a) (f_A, τ, A) is FST_2 space
(b) $(F_A, \tilde{\mathcal{T}}, A)$ is ST_2 space
(c) $(X, \mathcal{T}_e)$ is T_2 space

Proof: Firstly we prove (a). It is clear that $e_{x_1} \in (f_{1A}, A) \Rightarrow f_{1A}(e)(x_1) = \tilde{1}$ and $e_{x_2} \in (f_{1A}, A)^c = (f_{1A}^c, A) \Rightarrow e_{x_2} \notin (f_{1A}, A)$ which implies that $f_{1A}(e)(x_2) = \tilde{0}$. Again $e_{x_2} \in (f_{2A}, A) \Rightarrow f_{2A}(e)(x_2) = \tilde{1}$ and $e_{x_1} \in (f_{2A}, A)^c = (f_{2A}^c, A) \Rightarrow e_{x_1} \notin (f_{2A}, A)$ which implies that $f_{2A}(e)(x_1) = \tilde{0}$. Thus for every pair of $x_1, x_2 \in X$ with $x_1 \neq x_2$ then there exist two FSOS's (f_{1A}, A) and (f_{2A}, A) such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$ and $f_{2A}(e)(x_2) = \tilde{1}, f_{2A}(e)(x_1) = \tilde{0}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$. These prove that (f_A, τ, A) is FST_2 space.

Secondly we prove (b). For it, define a characteristic function 1_{F_A} such that

$$f_A(e)(x) = 1_{F_A}(e) = \begin{cases} 1 \text{ if } x \in F_A(e) \\ 0, \text{otherwise} \end{cases}$$

Let $f_A = (f_{1A}, f_{2A})$ and $F_A = (F_{1A}, F_{2A})$. Now for any $x_1, x_2 \in X$ with $x_1 \neq x_2$ we have

$f_{1A}(e)(x_1) = \tilde{1}$ implies $e_{x_1} \in (F_{1A}, A)$ and $f_{1A}(e)(x_2) = \tilde{0}$ implies $e_{x_2} \notin (F_{1A}, A)$. Again $f_{2A}(e)(x_2) = \tilde{1}$ implies $e_{x_2} \in (F_{2A}, A)$ and $f_{2A}(e)(x_1) = \tilde{0}$ implies $e_{x_1} \notin (F_{2A}, A)$. Therefore $e_{x_1} \in (F_{1A}, A), e_{x_2} \notin (F_{1A}, A)$ and $e_{x_2} \in (F_{2A}, A), e_{x_1} \notin (F_{2A}, A)$ and $F_{1A} \tilde{\cap} F_{2A} = \tilde{\emptyset}$. Hence $(F_A, \tilde{\mathcal{T}}, A)$ is ST_2 space. Finally to prove (c), for any $e \in A$, $(X, \mathcal{T}_e)$ is a topological space on X (see proposition 2.22) and $e_{x_1} \in (F_{1A}, A), e_{x_2} \in (F_{1A}, A)^c$ and $e_{x_2} \in (F_{2A}, A), e_{x_1} \in (F_{2A}, A)^c$. So that $x_1 \in F_{1A}(e), x_2 \notin F_{1A}(e)$ and $x_2 \in F_{2A}(e), x_1 \notin F_{2A}(e)$ and $F_{1A} \cap F_{2A} = \emptyset$. Thus $(X, \mathcal{T}_e)$ is T_2 space.

4. Good extension, hereditary and topological property

We discussed some fuzzy soft invariance properties namely 'Good Extension', 'Hereditary' and 'Soft Topological' property in this section:

Definition 4.1. Let $(F_A, \tilde{\mathcal{T}}, A)$ be a soft topological space and $\tau = \{1_{F_A} : F_A \in \tilde{\mathcal{T}}\}$, and $1_{F_A} = f_{1A}$. Then $(f_A,\ \tau)$ is the corresponding fuzzy soft topological space of $(F_A, \tilde{\mathcal{T}}, A)$. Let P be a property of soft topological spaces and FP be its fuzzy soft topological analogue. Then FP is called a 'Good extension' of P if the statement $(F_A, \tilde{\mathcal{T}}, A)$ has P if and only if $(f_A,\ \tau)$ has FP holds good for every soft topological space $(F_A, \tilde{\mathcal{T}}, A)$.

Theorem 4.1. Let $(F_A, \tilde{\mathcal{T}}, A)$ be a soft T_2 space and $(f_A,\ \tau)$ be $FST_2(j)$ spaces where $\mathrm{j} = \mathrm{i, ii, iii, iv}$. Then $(F_A, \tilde{\mathcal{T}}, A)$ will be $FST_2(j)$ spaces if and only if $FST_2(j)$ will be also a soft T_2 space.

Proof: Suppose that $(F_A, \tilde{\mathcal{T}}, A)$ be a soft T_2 (ST_2) space. We prove that $(F_A, \tilde{\mathcal{T}}, A)$ is $\mathrm{FST_2(j)}$ spaces. Since $(F_A, \tilde{\mathcal{T}}, A)$ is soft T_2 space, thence if for each $x_1, x_2 \in X, x_1 \neq x_2$, and for all $e \in A$, there exist two soft open sets (SOS's) $F_{1A}, F_{2A} \in \tilde{\mathcal{T}}$ such that $x_1 \in F_{1A}, x_2 \notin F_{1A}$ and $x_1 \notin F_{2A}, x_2 \in F_{2A}$ and $F_{1A} \tilde{\cap} F_{2A} = \tilde{\emptyset}$.Then by a characteristics function 1_{F_A} we have

$$\Rightarrow \begin{cases} 1_{F_A}(e)(x_1) = \tilde{1},\ 1_{F_A}(e)(x_2) = \tilde{0} \text{ and} \\ \quad 1_{F_A}(e)(x_1) = \tilde{0}, 1_{F_A}(e)(x_2) = \tilde{1} \end{cases}$$

Let $1_{F_A} = (f_{1A}, f_{2A})$. Therefore

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0} \\ f_{2A}(e)(x_1) = \tilde{0}, f_{2A}(e)(x_2) = \tilde{1} \text{ and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \tag{1}$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) = \tilde{\gamma}, f_{1A}(e)(x_2) = \tilde{0} \\ f_{2A}(e)(x_1) = \tilde{0}, f_{2A}(e)(x_2) = \tilde{\gamma} \qquad \text{as } 0 < \tilde{\gamma} < 1 \\ \text{and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \tag{2}$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) > f_{1A}(e)(x_2) \\ f_{2A}(e)(x_2) > f_{2A}(e)(x_1) \\ \quad \text{and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \tag{3}$$

$$\Rightarrow \begin{cases} f_{1A}(e)(x_1) \neq f_{2A}(e)(x_1) \\ f_{2A}(e)(x_2) \neq f_{1A}(e)(x_2) \\ \text{and } f_{1A} \tilde{\cap} f_{2A} = \tilde{0} \end{cases} \tag{4}$$

Hence from (1), (2), (3), and (4) we see that ST_2 space is $\mathrm{FST_2(i), FST_2(ii), FST_2(iii)}$ and $\mathrm{FST_2(iv)}$ spaces. That implies soft T_2 space is $\mathrm{FST_2(j)}$ spaces where $\mathrm{j = i, ii, iii, iv}$.

Conversely we assume that $(f_A,\ \tau)$ is $\mathrm{FST_2(j)}$ spaces. We prove that $(f_A,\ \tau)$ is a soft T_2 space, and for this it will be proved only for j=i. Since $(f_A,\ \tau)$ is a $\mathrm{FST_2(i)}$. Then by definitions for all $x_1, x_2 \in X, x_1 \neq x_2$, and for all $e \in A$, there exist two fuzzy soft open sets (FSOS's) $f_{1A}, f_{2A} \in \tau$ such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_1) = \tilde{0}, f_{2A}(e)(x_2) = \tilde{1}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$.

$$\Rightarrow \begin{cases} f_{1A}^{-1}(e)(\tilde{1}) = \{x_1\}, f_{1A}^{-1}(e)(\tilde{0}) = \{x_2\} \\ f_{2A}^{-1}(e)(\tilde{0}) = \{x_1\}, f_{2A}^{-1}(e)(\tilde{1}) = \{x_2\} \\ \text{and } f_{1A}^{-1} \tilde{\cap}\ f_{2A}^{-1} = (f_{1A} \tilde{\cap} f_{2A})^{-1} = \tilde{0} \end{cases}$$

Let $f_{1A}^{-1}(\tilde{1}) = F_{1A}$ and $f_{2A}^{-1}(\tilde{1}) = F_{2A}$. Therefore $F_{1A}(e) = \{x_1\}$ and $F_{2A}(e) = \{x_2\}$. Therefore, if for each $x_1, x_2 \in X, x_1 \neq x_2$, and for all $e \in A$, there exist two soft open sets (SOS's) $F_{1A}, F_{2A} \in \tilde{\mathcal{T}}$ such that $x_1 \in F_{1A}, x_2 \notin F_{1A}$ and $x_1 \notin F_{2A}, x_2 \in F_{2A}$ and $F_{1A} \tilde{\cap} F_{2A} = \tilde{\emptyset}$. Hence $\text{FST}_2(\text{i})$ is ST_2. In similar manner, $\text{FST}_2(\text{ii}), \text{FST}_2(\text{iii})$ and $\text{FST}_2(\text{iv})$ imply ST_2 space.

Definition 4.2. Let $(f_A,\ \tau)$ be a fuzzy soft topological space (FSTS) and $g_A \subset f_A$. Then the fuzzy soft topology $\tau_{g_A} = \{g_A \tilde{\cap} h_A | h_A \in \tau\}$ is called fuzzy soft subspace topology and (g_A, τ_{g_A}) is called fuzzy soft subspace of $(f_A,\ \tau)$. A fuzzy soft topological property *'P'* is called hereditary if each subspace of a fuzzy soft topological space with property *'P'* also has property *'P'*.

Theorem 4.2. Let $(f_A,\ \tau)$ be a fuzzy soft topological space (FSTS) and (g_A, τ_{g_A}) be a subspace of its. Then $(f_A,\ \tau)$ is $\text{FST}_2(\text{j})$ implies $\left(g_A, \tau_{g_A}\right)$ is also $\text{FST}_2(\text{j})$ where $\text{j} = \text{i}, \text{ii}, \text{iii}, \text{iv}$.

Proof: We prove this theorem only for $\text{j} = \text{i}$. Suppose that $(f_A,\ \tau)$ is $\text{FST}_2(\text{i})$, it will be shown that (g_A, τ_{g_A}) is $\text{FST}_2(\text{i})$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$, and for all $e \in A$ such that $g_{1A}(e)(x_1) = \tilde{1}, g_{1A}(e)(x_2) = \tilde{0}$ and $g_{2A}(e)(x_1) = \tilde{0}, g_{2A}(e)(x_2) = \tilde{1}$ and $g_{1A} \tilde{\cap} g_{2A}$=$\tilde{0}$. Again since $(f_A,\ \tau)$ is FST_2, thence by definitions for all $x_1, x_2 \in X, x_1 \neq x_2$, and for all $e \in A, \exists$ FSOS's $h_{1A}, h_{2A} \in \tau$ such that $h_{1A}(e)(x_1) = \tilde{1}, h_{1A}(e)(x_2) = \tilde{0}$, and $h_{2A}(e)(x_1) = \tilde{0}, h_{2A}(e)(x_2) = \tilde{1}$ and $h_{1A} \tilde{\cap} h_{2A} = \tilde{0}$. Since $g_{1A}(e)(x_1) = \tilde{1}$ and $h_{1A}(e)(x_1) = \tilde{1}$, thence $(g_{1A} \tilde{\cap} h_{1A})(e)(x_1) = \tilde{1}$. Similarly $(g_{1A} \tilde{\cap} h_{1A})(e)(x_2) = \tilde{0}$, and $(g_{2A} \tilde{\cap} h_{2A})(e)(x_1) = \tilde{0}, (g_{2A} \tilde{\cap} h_{2A})(e)(x_2) = \tilde{1}$. Also $(g_{1A} \tilde{\cap} h_{1A}) \tilde{\cap} (g_{2A} \tilde{\cap} h_{2A}) = (g_{1A} \tilde{\cap} g_{2A}) \tilde{\cap} (h_{1A} \tilde{\cap} h_{2A}) = \tilde{0} \tilde{\cap} \tilde{0} = \tilde{0}$. Hence (g_A, τ_{g_A}) is $\text{FST}_2(\text{i})$. For $\text{j} =$ ii, iii, and iv can be proved in similar way.

Theorem 4.3. Let $(f_A,\ \tau_1)$ and $(g_B,\ \tau_2)$ be two fuzzy soft topological spaces (FSTS's), on the two universal sets X and Y respectively, $(\varphi, \psi): (f_A,\ \tau_1) \longrightarrow (g_A,\ \tau_2)$ be fuzzy soft one-one, onto, and continuous map. Then these spaces are maintained in the succeeding feature.

(a) $(f_A,\ \tau_1)$ is $\text{FST}_2(\text{i}) \Leftrightarrow (g_B,\ \tau_2)$ is $\text{FST}_2(\text{i})$
(b) $(f_A,\ \tau_1)$ is $\text{FST}_2(\text{ii}) \Leftrightarrow (g_B,\ \tau_2)$ is $\text{FST}_2(\text{ii})$
(c) $(f_A,\ \tau_1)$ is $\text{FST}_2(\text{iii}) \Leftrightarrow (g_B,\ \tau_2)$ is $\text{FST}_2(\text{iii})$
(d) $(f_A,\ \tau_1)$ is $\text{FST}_2(\text{iv}) \Leftrightarrow (g_B,\ \tau_2)$ is $\text{FST}_2(\text{iv})$

Proof: We prove only (a). Suppose $(f_A,\ \tau_1)$ is $\text{FST}_2(\text{i})$. We prove that $(g_B,\ \tau_2)$ is also $\text{FST}_2(\text{i})$. Let $y_1, y_2 \in Y$ with $y_1 \neq y_2$. Since (φ, ψ) is onto, then there exist $x_1, x_2 \in X$ with $x_1 \neq x_2$, such that $\varphi(x_1) = y_1, \varphi(x_2) = y_2$and $\psi(e) = k$ for all parameters $e \in A$

and for all $k \in B$. Hence $x_1 \neq x_2$ as φ is one-one. Again since $(f_A,\ \tau_1)$ is $\mathrm{FST}_2(\mathrm{i})$, then we have for all $x_1, x_2 \in X, x_1 \neq x_2$, and for all $e \in A, \exists$ FSOS's $f_{1A}, f_{2A} \in \tau_1$ such that $f_{1A}(e)(x_1) = \tilde{1}, f_{1A}(e)(x_2) = \tilde{0}$, and $f_{2A}(e)(x_1) = \tilde{0}, f_{2A}(e)(x_2) = \tilde{1}$ and $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$. Now there exist FSOS's $(\varphi,\psi)(f_{1A}), (\varphi,\psi)(f_{2A}) \in \tau_2$ such that $(\varphi,\psi)(f_{1A})k(y_1) = \{\vee\varphi(x_1) = y_1 \vee\psi(e) = k f_{1A}(e)(x_1) = \tilde{1}$ as $f_{1A}(e)(x_1) = \tilde{1}$. And $(\varphi,\psi)(f_{1A})k(y_2) = \{\vee\varphi(x_2) = y_2 \vee\psi(e) = k f_{1A}(e)(x_2) = \tilde{0}$ as $f_{1A}(e)(x_2) = \tilde{0}$. Similarly $(\varphi,\psi)(f_{2A})k(y_1) = \tilde{0}$ and $(\varphi,\psi)(f_{2A})k(y_2) = \tilde{1}$ and $(\varphi,\psi)(f_{1A}) \tilde{\cap} (\varphi,\psi)(f_{2A}) = (\varphi,\psi)(f_{1A} \tilde{\cap} f_{2A}) = \tilde{0}$ as $f_{1A} \tilde{\cap} f_{2A} = \tilde{0}$. Hence $(g_B,\ \tau_2)$ is $\mathrm{FST}_2(\mathrm{i})$.

Conversely, suppose that $(g_B,\ \tau_2)$ is $\mathrm{FST}_2(\mathrm{i})$. We prove that $(f_A,\ \tau_1)$ is $\mathrm{FST}_2(\mathrm{i})$. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. This implies that $\varphi(x_1) \neq \varphi(x_2)$ as φ is one-to-one. Put $\varphi(x_1) = y_1$ and $\varphi(x_2) = y_2$. Then $y_1 \neq y_2$. Since $(g_B,\ \tau_2)$ is $\mathrm{FST}_2(\mathrm{i})$, then there exist FSOSs $g_{1B}, g_{2B} \in \tau_2$ such that $g_{1B}(k)(y_1) = \tilde{1}, g_{1B}(k)(y_2) = \tilde{0}$, and $g_{2B}(k)(y_1) = \tilde{0}, g_{2B}(k)(y_2) = \tilde{1}$ and $g_{1B} \tilde{\cap} g_{2B} = \tilde{0}$. Now there exist FSOSs $(\varphi,\psi)^{-1}(g_{1B})$, $(\varphi,\psi)^{-1}(g_{2B}) \in \tau_1$ such that $(\varphi,\psi)^{-1}(g_{1B})(e)(x_1) = g_{1B}(\psi(e))(\varphi(x_1)) = g_{1B}(k)(y_1) = \tilde{1}$ and $(\varphi,\psi)^{-1}(g_{1B})(e)(x_2) = g_{1B}(\psi(e))(\varphi(x_2)) = g_{1B}(k)(y_2) = \tilde{0}$. Similarly $(\varphi,\psi)^{-1}(g_{2B})(e)(x_1) = \tilde{0}$ and $(\varphi,\psi)^{-1}(g_{2B})(e)(x_2) = \tilde{1}$. And obviously $(\varphi,\psi)^{-1}(g_{1B}) \tilde{\cap} (\varphi,\psi)^{-1}(g_{2B}) = (\varphi,\psi)^{-1}(g_{1B} \tilde{\cap} g_{2B}) = \tilde{0}$ as $g_{1B} \tilde{\cap} g_{2B} = \tilde{0}$. Hence, $(f_A,\ \tau_1)$ is $\mathrm{FST}_2(\mathrm{i})$. In the same way, (b), (c), and (d) can be proved.

Conclusion: In this study, four new conceptions of γ-separation axioms on fuzzy soft T_2 spaces are developed, along with several new theories. Next, the properties of "good extension," "soft hereditary," and "soft topological" are examined. Lastly, in our further work, we will explore the same kinds of ideas and theories for lattice fuzzy soft T_2 spaces.

Acknowledgement. The author thanks the reviewers for their valuable comments and constructive feedback.

REFERENCES

1. K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986) 87-96.
2. Banu Pazar Varol and Halis Aygun, Fuzzy Soft Topology, Hacettepe Journal of Mathematics and Statistics, 41 (2012) 407-419.
3. V. Cetin and H. Aygun, A note on fuzzy soft topological spaces, Proceeding of the 8th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2013) 56-60.
4. O. Gocur and A. Kopuzlu, On Soft Separation Axioms, Ann. Fuzzy Math. Inform. 9 (2015) 817-822.
5. R. Islam, M. S. Hossain, and M. F. Hoque, A study on intuitionistic L-fuzzy T1 spaces, Notes on Intuitionistic Fuzzy Sets, 26 (2020) 33-42.
6. G. Kalpana and C. Kalaivani, Fuzzy Soft Topology, International Journal of Engineering Studies, 9 (2017) 45-56.
7. P. K. Maji and A. R. Roy, An application of soft set in decision making problem, Computers Math. Appl., 44 (2002) 1077-1083.

8. P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math., 9 (2001) 589-602.
9. P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Computers Math. Appl., 45(2003) 555–562.
10. J. Mahanta, P. K. Das, Results on fuzzy soft topological spaces, arXiv: 1203. 0634v1 (2012).
11. Muhammad Shabir and Munazza Naz, On Soft Topological Spaces, Computers and Mathematics with applications, 61 (2011) 1786-1799.
12. D. Molodtsov, Soft set theory-first results, Comput. Math. Appl., 37 (1999) 19-31.
13. Z. Pawlak, Rough sets, Int. J. Comp. Inf. Sci., 11 (1982) 341–356.
14. D. Pie and D. Miao, From soft sets to information systems, Granular Computing, 2 (2005) 617-621.
15. Rafiqul Islam and M. S Hossain, Some Features of Intuitionistic L- R0 Spaces, J. Physical Sci., 22 (2017) 99-106.
16. S. Roy and T. K. Samanta, A note on fuzzy soft topological spaces, Ann. Fuzzy Math. Inform., 3(2011) 305-311.
17. Sabir Hussain and Bashir Ahmad, Soft Separation Axioms in Soft Topological Spaces, Hacettepe Journal of Mathematics and Statistics, 44 (2015) 559-568.
18. S. Sarkar and M. Pal, Multiple Regression of Fuzzy-Valued Variable, J. Physical Sci., 13 (2009) 57-66.
19. O. Tantawy, S. A. El-Sheikh and S. Hamde, Annals of Fuzzy Mathematics and Informatics, x (201y) 1-xx.
20. Tugbahan Simsekler, Saziye Yuksel, Fuzzy Soft Topological Spaces, Ann. Fuzzy Math. Inform., 5(2013) 87-96.
21. L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.
22. I. Zorlutana, N. Akdag, W.K. Min and S. Atmaca, Remarks on Soft Topological Spaces, Annals of Fuzzy Mathematics and Informatics, 3(2012) 171-185.