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Effect of Magnetic Field on Pulsatile Blood Flow Through an Inclined Circular Tube with Periodic Body Acceleration

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ABSTRACT

A mathematical model has been developed for studying the characteristics of blood flow in a rigid inclined circular tube with periodic body acceleration under the influence of a uniform magnetic field. The blood is supposed to be couple stress fluid. Finite Hankel and Laplace transforms are used to obtain the analytical expressions for velocity profile, flow rate and acceleration of blood. The flow velocity deviates with various parameters such as Hartmann number, gravitational parameter, body acceleration, inclination angle, time etc. and this deviation of flow velocity can be regulated by a proper use of magnetic field.

Key words and phrases: *Pulsatile motion, couple stress fluid, body acceleration.*

1. Introduction

In our daily life we often face some external body acceleration, such as travelling in high velocity vehicles, aircrafts etc. Again in various sports it needs a high acceleration suddenly. These type of situations undoubtedly effects the normal flow of blood which lead to headache, vomiting tendency, loss of vision, abnormality in pulse rate etc. So we have to maintain such type of body accelerations to avoid these types of health hazards.

Various mathematical models have been investigated by several researchers to explore the behaviour of blood flow under the influence of external acceleration. Sud et. al [1,2] studied the characteristics of blood flow accompanied with body accelerations.

Sud and Sekhon [3,4,5] took into account various types of body accelerations and studied different characteristics of blood flow according to the nature of accelerations. Chaturani and Palanisamy [6,7,8] studied the flow characteristics of blood under external body acceleration assuming blood as a Newtonian fluid, Casson fluid and power law fluid respectively. Chaturani and Upadhya [9] studied the gravity flow of fluid with couple stress along an inclined plane with application to blood flow. A good number of researchers [10,11,12,13] investigated the nature of blood flow in an inclined channel or surface . El-Shehawey et. al [14] studied the nature of unsteady flow of blood in the presence of magnetic field through a circular pipe taking blood as an electrically conducting, visco-elastic, non-Newtonian fluid. Recently, Rathor et. al [15], have studied the pulsatile flow of blood through rigid inclined circular tubes under the influence of periodic body acceleration .

In the present paper , it is proposed to develop a mathematical model to study the characteristics of inclined pulsatile flow of blood through an inclined circular tube under the influence of periodic body acceleration in presence of transverse magnetic field . Here we consider blood as a couple stress fluid. The axial blood flow velocity is determined using finite Hankel and Laplace transforms. Also the effect of magnetic field, gravitational parameter, inclination angle, body acceleration, time etc. on axial blood flow, flow rate and acceleration of blood has been discussed graphically.

2. Mathematical Formulation of the Problem

Let us consider a one-dimensional pulsatile blood flow through a uniform straight and inclined rigid circular tube in presence of transverse magnetic field. Let us take the flow of blood as axially symmetric, pulsatile and fully developed. Taking blood as couple stress fluid we may write the basic equation in cylindrical polar co-ordinates as

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\rho G+\mu\nabla^2 u-\eta\nabla^2\left(\nabla^2 u\right)+\rho g\sin\theta-\sigma B_0^2 u \tag{1}$$

where $\nabla^2\equiv\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right)$.

Here u=u(r,t) is the axial velocity, ρ is the density of blood, μ is the co-efficient of viscosity of blood, η is the couple stress parameter, g is the acceleration due to gravity, B_0 is the transverse component of magnetic field and σ is the electrical conductivity of the medium. The pressure gradient $\frac{\partial p}{\partial z}$ is taken as

$$-\frac{\partial p}{\partial z}=A_0+A_1\cos\omega t,\qquad t\geq 0 \tag{2}$$

in which A_0 is the steady state part of pressure gradient, A_1 is the amplitude of the oscillatory part, $\omega = 2\pi f$, f being the heart pulse frequency. Again the body acceleration G is given by

$$G = a_0 \cos\left(\omega_1 t + \phi\right), \quad t \geq 0 \tag{3}$$

in which a_0 is the amplitude of body acceleration, ϕ is the phase difference, $\omega_1 = 2\pi f_1$ and f_1 is the body acceleration frequency.

We can write equation (1) in non-dimensional form , by substituting

$$u' = \frac{u}{\omega R}, \quad r' = \frac{r}{R}, \quad t' = t\omega, \quad A_0' = \frac{R}{\mu\omega} A_0, \quad A_1' = \frac{R}{\mu\omega} A_1, \quad a_0' = \frac{\rho R}{\mu\omega} a_0, \quad z' = \frac{z}{R},$$

$$g' = \frac{\rho R}{\mu\omega} g$$

as (dropping the primes)

$$\alpha^2 \alpha'^2 \frac{\partial u}{\partial t} = \alpha'^2 A_0 + \alpha'^2 A_1 \cos t + \alpha'^2 a_0 \cos(bt + \phi) + \alpha'^2 \nabla^2 u - \nabla^2\left(\nabla^2 u\right) + \alpha'^2 g \sin\theta - M^2 \alpha'^2 u \tag{4}$$

where $\alpha = R\sqrt{\frac{\omega\rho}{\mu}}$ is the Womersley's parameter , $\alpha' = R\sqrt{\frac{\mu}{\eta}}$ is the couple stress parameter , $M = B_0 R\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number , R is the radius of the tube and $b = \frac{\omega_1}{\omega}$.

3. Mathematical Analysis

We take the boundary conditions of the problem as [15]

$$\text{u and } \nabla^2 u \text{ both are finite at r=0,} \tag{5}$$

$$\text{u=0 and } \nabla^2 u = 0 \text{ at r=1.} \tag{6}$$

Taking finite Hankel transform of (4) and using boundary conditions (5) and (6) we obtain

$$u^*\left(\lambda_n, t\right) = \sum_{n=1}^{\infty} \frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n} \; \frac{A_0 + A_1 + a_0 \cos\left(bt + \phi\right) + g\sin\theta}{\lambda_n^2\left(\lambda_n^2 + \alpha'^2\right) + M^2 \alpha'^2}$$

where $u^*\left(\lambda_n, t\right) = \int_0^1 r u\left(r, t\right) J_0\left(r\lambda_n\right) dr$ and λ_n are the roots of the equation $J_0\left(\lambda_n\right) = 0$.

Initially at t=0

$$u^{*}\left(\lambda_n,0\right)=\sum_{n=1}^{\infty}\frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n}\;\frac{A_0+A_1+a_0\cos\phi+g\sin\theta}{\lambda_n^2\left(\lambda_n^2+\alpha'^2\right)+M^2\alpha'^2}\;. \tag{7}$$

To solve the problem we employ the Laplace transform in equation (1) and with the help of (5) and (6) we obtain

$$\alpha^2\alpha'^2\left\{s\bar{u}-u\left(r,0\right)\right\}=\alpha'^2\frac{A_0}{s}+\alpha'^2\frac{A_1 s}{s^2+1}+\alpha'^2 a_0\frac{s\cos\phi-b\sin\phi}{s^2+b^2}+\alpha'^2\nabla^2\bar{u}-\nabla^2\left(\nabla^2\bar{u}\right)$$
$$+\alpha'^2\frac{g\sin\theta}{s}-M^2\alpha'^2\bar{u}$$

i.e.,

$$ms\bar{u}-mu\left(r,0\right)=\alpha'^2\frac{A_0}{s}+\alpha'^2\frac{A_1 s}{s^2+1}+\alpha'^2 a_0\frac{s\cos\phi-b\sin\phi}{s^2+b^2}+\alpha'^2\nabla^2\bar{u}-\alpha'^2\nabla^2\left(\nabla^2\bar{u}\right)$$
$$+\alpha'^2\frac{g\sin\theta}{s}-M^2\alpha'^2\bar{u} \tag{8}$$

where $\bar{u}\left(r,s\right)=\int_0^{\infty}e^{-st}u\left(r,t\right)dt$ and m=$\alpha^2\alpha'^2$

Now taking finite Hankel transform of (8) and using (7) we get

$$ms\bar{u}^{*}\left(\lambda_n,s\right)-m\frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n}\sum_{n=1}^{\infty}\frac{A_0+A_1+a_0\cos\phi+g\sin\theta}{\lambda_n^2\left(\lambda_n^2+\alpha'^2\right)+M^2\alpha'^2}=$$
$$\frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n}\left[\frac{A_0}{s}+\frac{A_1 s}{s^2+1}+\frac{a_0\left(s\cos\phi-b\sin\phi\right)}{s^2+b^2}+\frac{g\sin\theta}{s}\right]$$
$$-\left\{\lambda_n^2\left(\lambda_n^2+\alpha'^2\right)+M^2\alpha'^2\right\}\bar{u}^{*}$$

where $\bar{u}^{*}\left(\lambda_n,s\right)=\int_0^{1}ru\left(r,s\right)J_0\left(r\lambda_n\right)dr$

or,

$$\left\{sm+\beta\right\}\bar{u}^{*}\left(\lambda_n,s\right)=\frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n}\left[\frac{A_0}{s}+\frac{A_1 s}{s^2+1}+\frac{a_0\left(s\cos\phi-b\sin\phi\right)}{s^2+b^2}+\frac{g\sin\theta}{s}+m\sum_{n=1}^{\infty}\frac{\gamma}{\beta}\right]$$

[Taking $\beta=\lambda_n^2\left(\lambda_n^2+\alpha'^2\right)+M^2\alpha'^2$ and $\gamma=A_0+A_1+a_0\cos\phi+g\sin\theta$]

i.e.,

$$\bar{u}^{*}\left(\lambda_n,s\right)=\frac{J_1\left(\lambda_n\right)\alpha'^2}{\lambda_n}\left[\frac{A_0}{s}+\frac{A_1 s}{s^2+1}+\frac{a_0\left(s\cos\phi-b\sin\phi\right)}{s^2+b^2}+\frac{g\sin\theta}{s}+m\sum_{n=1}^{\infty}\frac{\gamma}{\beta}\right]\frac{1}{sm+\beta}\;. \tag{9}$$

For simplicity of calculations, we rearrange (9) as

$$\bar{u}^{*}(\lambda_n,s)=\frac{J_1(\lambda_n)\alpha'^2}{\lambda_n}\Big[\frac{A_0}{\beta}\left(\frac{1}{s}-\frac{1}{s+h}\right)+\frac{A_1\beta}{\beta^2+m^2}\left(-\frac{1}{s+h}+\frac{s}{s^2+1}+\frac{m}{\beta(s^2+1)}\right)$$

$$+\frac{a_0\beta\cos\phi}{\beta^2+m^2b^2}\left(-\frac{1}{s+h}+\frac{s}{s^2+b^2}+\frac{mb^2}{\beta(s^2+b^2)}\right)$$

$$-\frac{a_0bm\sin\phi}{\beta^2+m^2b^2}\left(\frac{1}{s+h}-\frac{s}{s^2+b^2}+\frac{\beta}{m(s^2+b^2)}\right)$$

$$+\frac{g\sin\theta}{\beta}\left(\frac{1}{s}-\frac{1}{s+h}\right)+m\sum_{n=1}^{\infty}\frac{\gamma}{\beta}\left(\frac{1}{m}\cdot\frac{1}{s+h}\right)\Big] \tag{10}$$

where h= $\dfrac{\beta}{m}$.

Taking inverse Laplace transform of (10) we get

$$u^{*}(\lambda_n,t)=\frac{J_1(\lambda_n)\alpha'^2}{\lambda_n}$$

$$\Big[\left\{\frac{A_0}{\beta}+\frac{A_1(\beta\cos t+m\sin t)}{\beta^2+m^2}+\frac{a_0\{\beta\cos(bt+\phi)+bm\sin(bt+\phi)\}}{\beta^2+m^2b^2}+\frac{g\sin\theta}{\beta}\right\}$$

$$-e^{-ht}\left\{\frac{A_0}{\beta}+\frac{A_1\beta}{\beta^2+m^2}+\frac{a_0(\beta\cos\phi+bm\sin\phi)}{\beta^2+m^2b^2}+\frac{g\sin\theta}{\beta}-\sum_{n=1}^{\infty}\frac{\gamma}{\beta}\right\}\Big]. \tag{11}$$

Also taking finite Hankel inversion of (11) we obtain the required solution for blood velocity as

$$u(r,t)=2\sum_{n=1}^{\infty}u^{*}(\lambda_n,t)\frac{J_0(r\lambda_n)}{J_1^2(\lambda_n)}$$

$$=2\sum_{n=1}^{\infty}\frac{J_0(r\lambda_n)\alpha'^2}{\lambda_nJ_1(\lambda_n)}$$

$$\Big[\left\{\frac{A_0}{\beta}+\frac{A_1(\beta\cos t+m\sin t)}{\beta^2+m^2}+\frac{a_0\{\beta\cos(bt+\phi)+bm\sin(bt+\phi)\}}{\beta^2+m^2b^2}+\frac{g\sin\theta}{\beta}\right\}$$

$$-e^{-ht}\left\{\frac{A_0}{\beta}+\frac{A_1\beta}{\beta^2+m^2}+\frac{a_0(\beta\cos\phi+bm\sin\phi)}{\beta^2+m^2b^2}+\frac{g\sin\theta}{\beta}-\frac{\gamma}{\beta}\right\}\Big]. \tag{12}$$

The flow rate Q is given by

$$Q(r,t)=2\pi\int_0^1 urdr$$

$$= 4\pi \sum_{n=1}^{\infty} \frac{\alpha'^2}{\lambda_n^2}$$

$$\Bigg[\left\{ \frac{A_0}{\beta} + \frac{A_1(\beta\cos t + m\sin t)}{\beta^2 + m^2} + \frac{a_0\{\beta\cos(bt+\phi) + bm\sin(bt+\phi)\}}{\beta^2 + m^2b^2} + \frac{g\sin\theta}{\beta} \right\}$$

$$-e^{-ht}\left\{ \frac{A_0}{\beta} + \frac{A_1\beta}{\beta^2+m^2} + \frac{a_0(\beta\cos\phi + bm\sin\phi)}{\beta^2+m^2b^2} + \frac{g\sin\theta}{\beta} - \frac{\gamma}{\beta} \right\} \Bigg] \tag{13}$$

and the fluid acceleration F is

$$F(r,t) = \frac{\partial u}{\partial t}$$

$$= 2\sum_{n=1}^{\infty} \frac{J_0(r\lambda_n)\alpha'^2}{\lambda_n J_1(\lambda_n)} \Bigg[\left\{ \frac{A_1(m\cos t - \beta\sin t)}{\beta^2+m^2} + \frac{a_0 b\{bm\cos(bt+\phi) - \beta\sin(bt+\phi)\}}{\beta^2+m^2b^2} \right\}$$

$$+he^{-ht}\left\{ \frac{A_0}{\beta} + \frac{A_1\beta}{\beta^2+m^2} + \frac{a_0(\beta\cos\phi + bm\sin\phi)}{\beta^2+m^2b^2} + \frac{g\sin\theta}{\beta} - \frac{\gamma}{\beta} \right\} \Bigg]. \tag{14}$$

4. Numerical Results and Discussions

The expression for velocity profile u(r,t) obtained in equation (12) has been depicted in figures 1-9 by plotting r versus u in presence / absence of M, for different values of gravitational parameter g, inclination angle θ, amplitude of body acceleration a_0, steady state part of pressure gradient A_0, amplitude of the oscillatory part of pressure gradient A_1, Womersley's parameter α, couple stress parameter α', time t and Hartmann number M. Figure - 10 is constructed for r versus fluid acceleration F and Fig - 11 is constructed for t versus flow rate Q.

Figure - 1 shows that u increases with increasing g, but influence of M reduces the effect of variation in g. Figure - 2 shows that u increases with increasing θ, but influence of M reduces the effect of variation in θ. Similarly figures - 3,4,5,6,7 show that u increases with increasing $a_0, A_0, A_1, \alpha, \alpha'$ respectively; but the presence of M reduces the effect of variation of those parameters. On the other hand, from figure - 8 we observe that u decreases with increasing t and presence of M reduces the effect of variation of t. In figures 9 and 10 different values of M are taken to visualise the effect of M on blood velocity u(r,t) and acceleration F of blood respectively, retaining all other parameters unaltered . In fig - 9 we observe that the transverse magnetic field reduces the value of u(r,t) . Fig - 10 shows that there occurs a retardation in the flow velocity u(r,t) and the amount of retardation is reduced by the influence of M. In fig-11 for different values of

M we observe that the flow rate Q depends largely on time t and the transverse magnetic field reduces the amount of flow rate Q.

5. Conclusions

It is clear from the above discussions that the transverse magnetic field effects largely on the axial flow velocity of blood. Again from the figures 1-9 the amount of deviation in u(r, t) due to M is shown. So, by taking appropriate values of M we may regulate the axial flow velocity.

In case of arthritis, gout etc. patients are often advised to take protective pads or tractions. By applying proper magnetic field attached with those instruments we may enhance their activities. Again, in the case of magnetotherapy , by maintaining a proper magnetic field, the influence of magnetic instruments on blood flow velocity may be regulated .

In our daily life we often use several kinds of electromagnetic instruments, such as cellular phones, transistors, television, computers etc. which have some magnetic field effect. This magnetic field effects the blood flow velocity leading to various kinds of health hazards such as headache, vomiting tendency, partial loss of vision etc.

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REFERENCES

1. V.K.Sud , H.E.Vongierke , I.Kalpes and H.L.Oestreicher : *Blood flow under the influence of externally applied periodic accelerations in large and small arteries.* Med. and Biol. Eng. and Comput., 21(1983)446-452.
2. V.K.Sud , H.E.Vongierke , I.Kalpes and H.L.Oestreicher : *Analysis of blood flow under time dependent acceleration.* Med. and Biol. Eng. and Comput., 23 (1985) 69-73.
3 V.K.Sud and G.S.Sekhon : *Blood flow subject to a single cycle of body acceleratrion*. Bull. Math. Biol., 46(1984)937-949.
4. V.K.Sud and G.S.Sekhon : *Analysis of blood flow through a model of human arterial system under periodic body acceleration*. J. Biomech., 19(1986)929-941.
5. V.K.Sud and G.S.Sekhon : *Arterial flow under periodic body acceleration.* Bull. Math. Biol., 47(1985)35-52.
6. P.Chaturani and V.Palanisamy : *Pulsatile flow of blood with periodic body acceleration*. Int. J. Engg. Sci., 29(1991)113-121.
7. P.Chaturani and V.Palanisamy : *Casson fluid model for pulsatile flow of blood under periodic body acceleration*. Biorheology, 27(1990)619-630.

8. P.Chaturani and V.Palanisamy : *Pulsatile flow of power law fluid model for blood flow under periodic body acceleration*. Biorheology, 27(1990)747-758.
9. P.Chaturani and V.S.Upadhya : *Gravity flow of fluid with couple stress along an inclined plane with application to blood flow* . Biorheology, 14(1977) 237-246.
10. V.P.Rathod and G.Thippeswamy : *Gravity flow of pulsatile blood through closed rectangular inclined channel with micro-organisms*. The Mathematics Education, 33, No.1(1999).
11. B.Leonid , P.Laure , F.Dias and T.J.Bridges : *Linear pulse structure and signalling in a film flow on an inclined plane.* J. Fluid Mech., 396(1999)37-71.
12. T.S.Chen and K.L.Tzuoo : *Vortex instability of free convection flow over horizontal and inclined surface*. J. Heat Transfer, 104(1982)637-643.
13. G.Astarita , G.Mariucxi and G.Palumbo : *Non-Newtonian gravity flow on inclined surface. Ind.* Engg. Chem. Fundam., 3(1964)333.
14 E.F.El-Shehawey , E.M.E.Elbarbary , N.A.S.Afifi and M.Elshahed : *MHD flow of an elastico-viscous fluid under periodic body acceleration.* Int. J. Math. and Math. Sci., (2000)795-799.
15. V.P.Rathod , N.Begum and I.Sheeba Rani : *Inclined pulsatile flow of blood with periodic body acceleration.* Acta Ciencia Indica, 32M, No. 4 , 1509(2006).

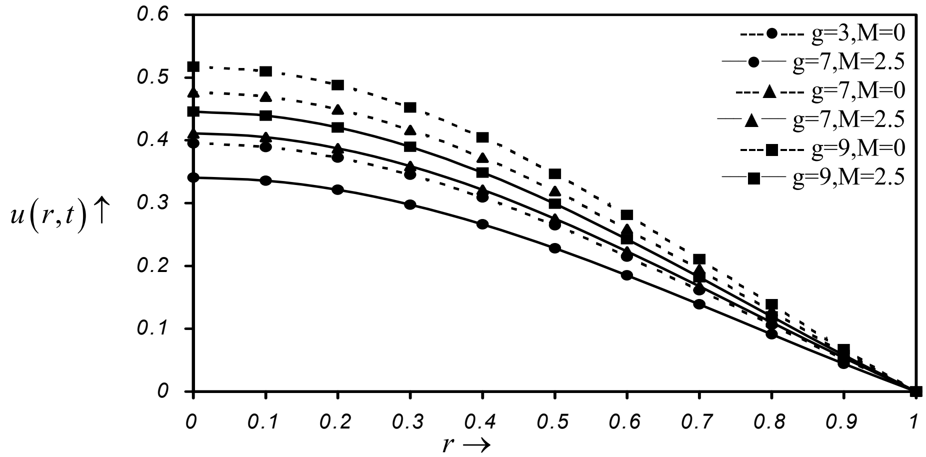

Fig. :1 Variation of velocity profile for different values of g ;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$, $\alpha = 1$, $\overline{\alpha} = 1$

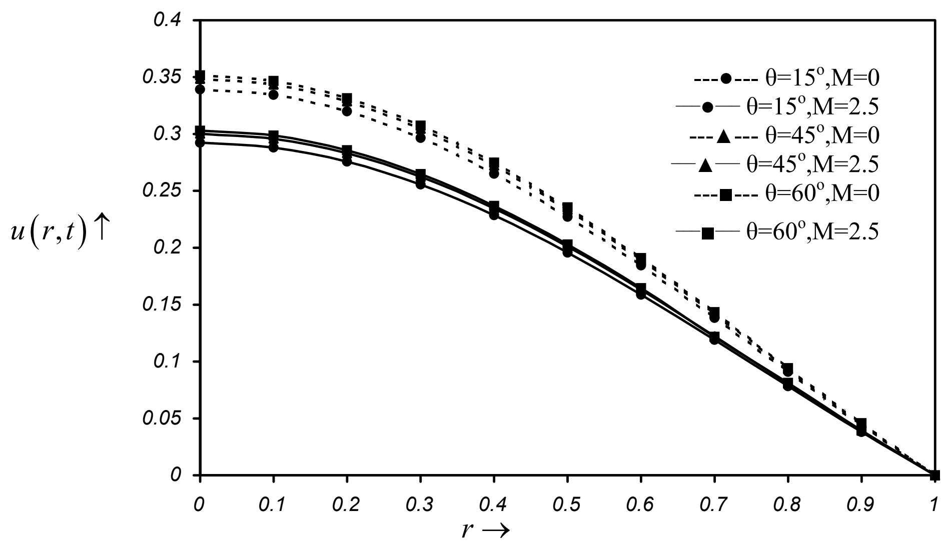

Fig. :2 Variation of velocity profile for different values of θ ;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$,g=0.5, $\alpha = 1$, $\overline{\alpha} = 1$

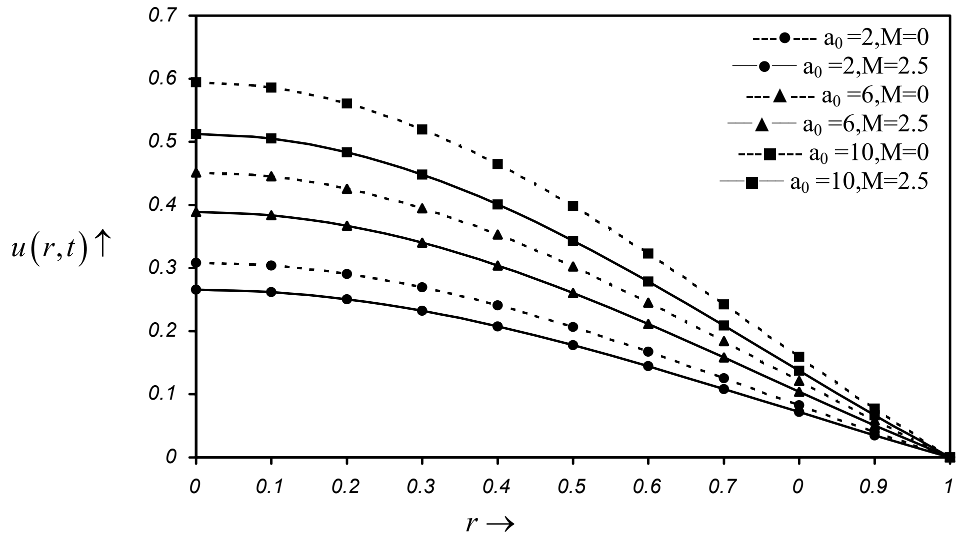

Fig. :3 Variation of velocity profile for different values of a_0;
taking A_0=2.0,A_1=4.0 ,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$,g=0.5, $\alpha = 1$, $\overline{\alpha} = 1$

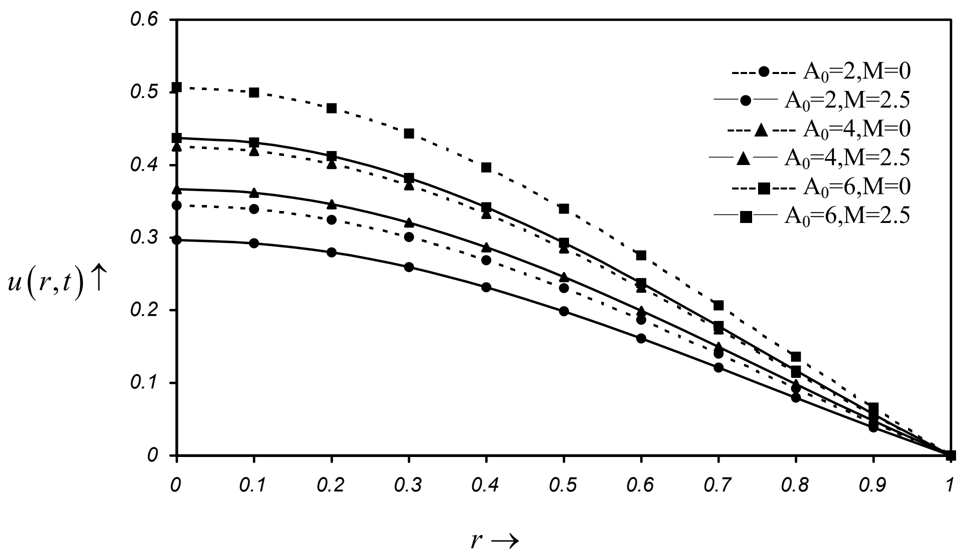

Fig. :4 Variation of velocity profile for different values of A_0;
taking A_1=4.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$,g=0.5, $\alpha = 1$, $\overline{\alpha} = 1$

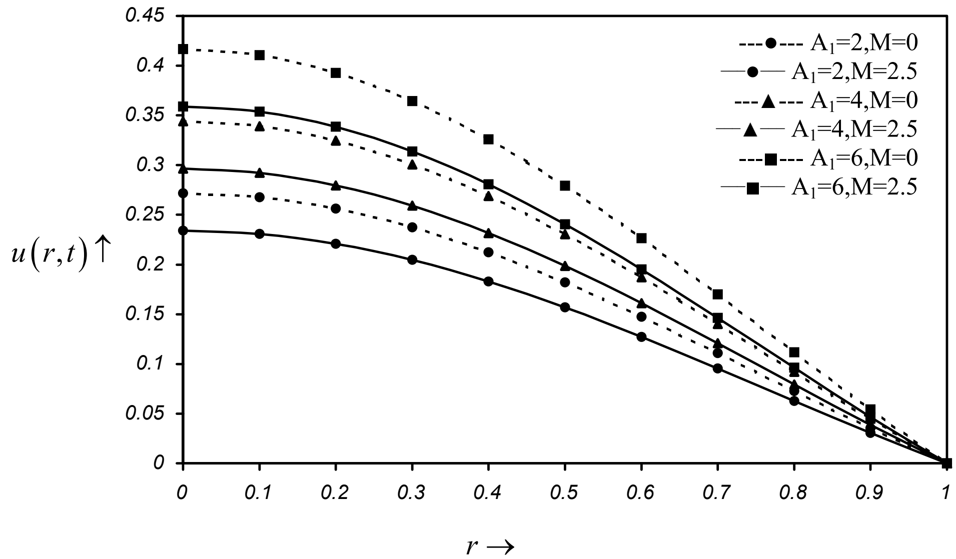

Fig. :5 Variation of velocity profile for different values of A_1;
taking A_0=2.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$,g=0.5, $\alpha = 1, \overline{\alpha} = 1$

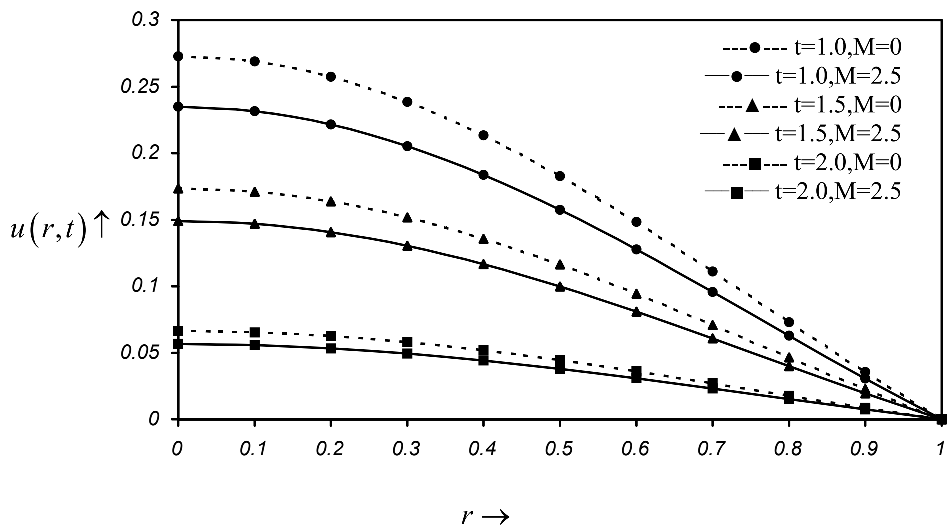

Fig. :6 Variation of velocity profile for different values of t;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5, $\phi = 15^0$, $\theta = 30^0$,g=0.5, $\alpha = 1, \overline{\alpha} = 1$

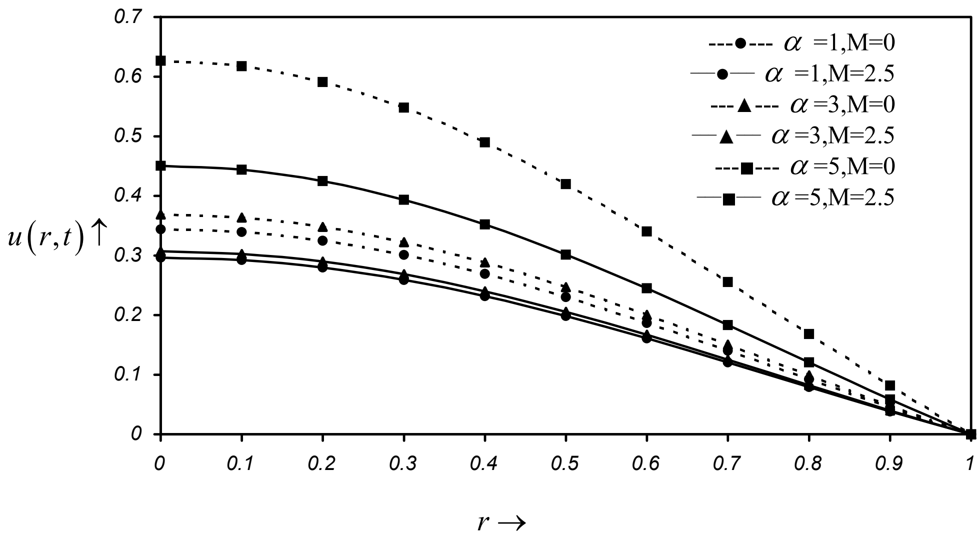

Fig. :7 Variation of velocity profile for different values of α;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$, g=0.5, $\overline{\alpha} = 1$

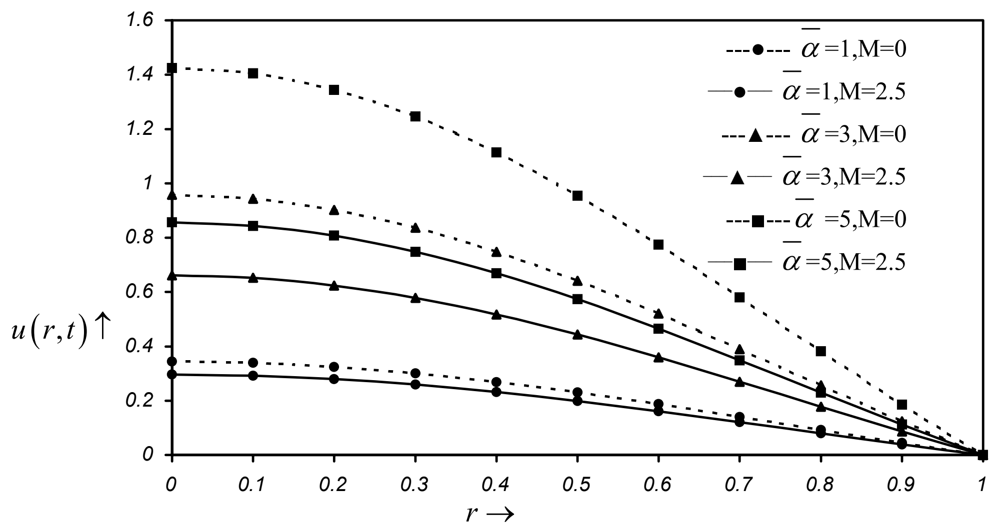

Fig. : 8 Variation of velocity profile for different values of $\overline{\alpha}$;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5, $\phi = 15^0$, $\theta = 30^0$, g=0.5, $\alpha = 1$

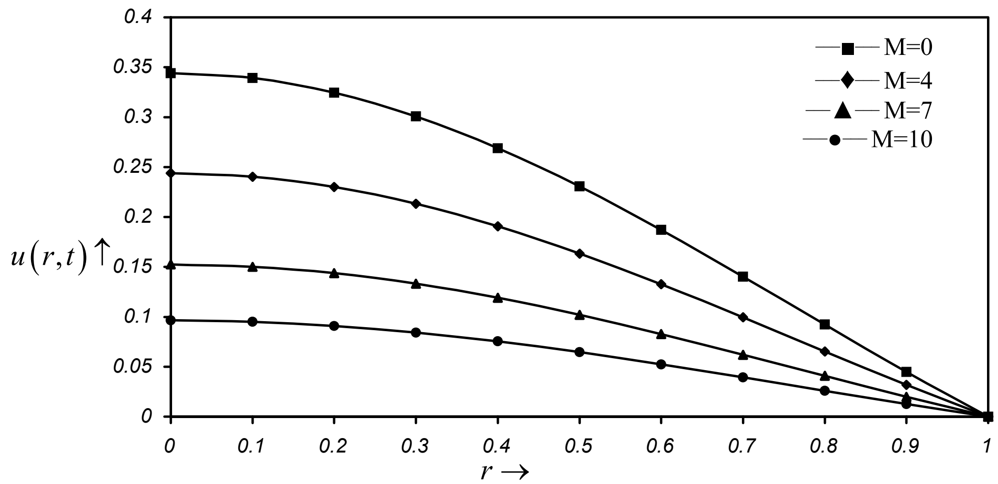

Fig. : 9 Variation of velocity profile for different values of M;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5,$\phi = 15^0$,$\theta = 30^0$,g=0.5, $\alpha = 1, \overline{\alpha} = 1$

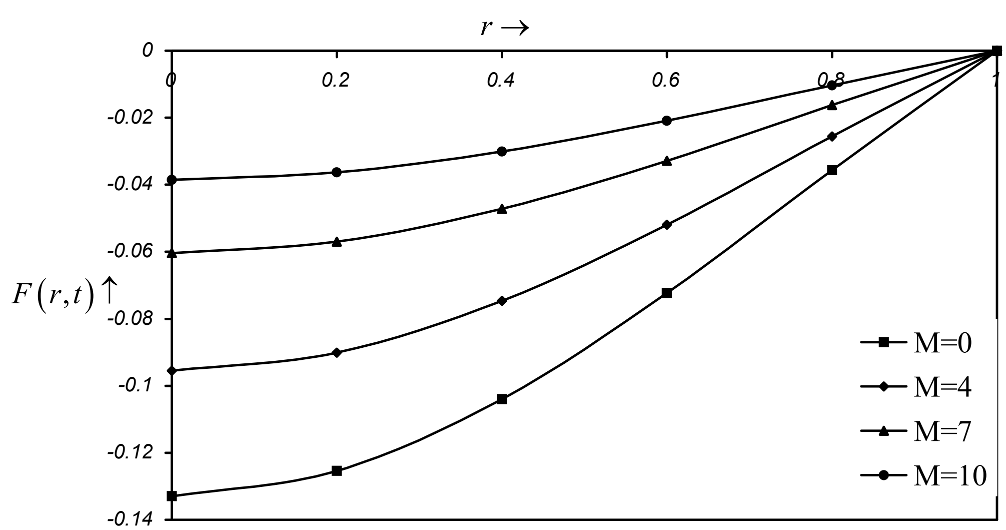

Fig. :10 Variation of acceleration of blood for different values of M;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5,t=0.5,$\phi = 15^0$,$\theta = 30^0$,g=0.5, $\alpha = 1, \overline{\alpha} = 1$

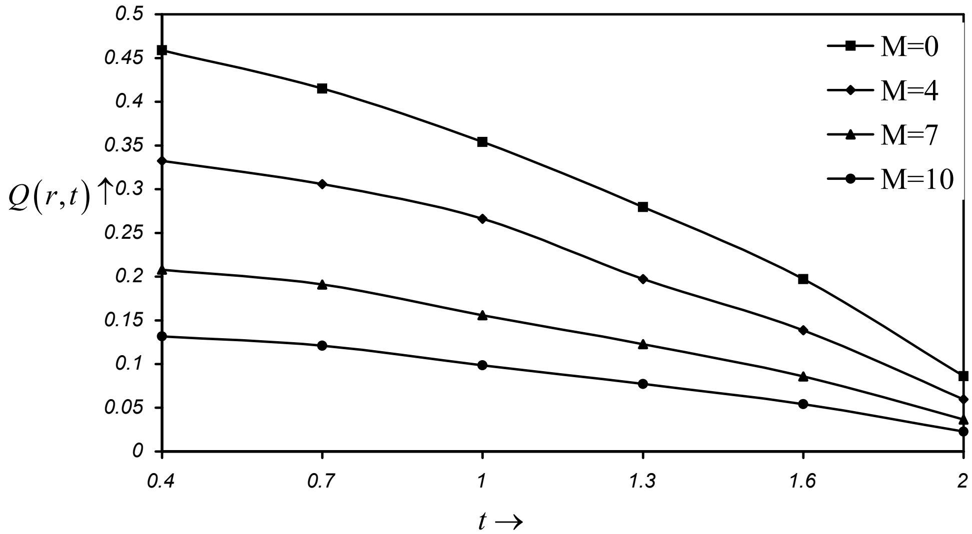

Fig. :11 Variation of flow rate of blood for different values of M;
taking A_0=2.0,A_1=4.0,a_0=3.0,b=0.5, $\phi = 15^0$, $\theta = 30^0$,g=0.5, $\alpha = 1, \overline{\alpha} = 1$