M.Sc. 4th Semester Examination, 2024

PHYSICS

PAPER – PHS-401.1 & 401.2

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Notations used have their usual meaning

PAPER – PHS-401.1

(*Quantum Field Theory*-II)

GROUP – A

1. Answer any *two* of the following : 2 x 2

(Turn Over)

(*a*) The normal ordered Hamiltonian for the free real scalar theory is given by

$:H: = \int \frac{d^3p}{(2\pi)^3} E_p \left(a_p^\dagger a_p\right)$. Show that

$$:H:|p\rangle = E_p\,|p\rangle .$$

(*b*) If $Ca_pC^{-1} = b_p$, where C is the charge conjugation operator and (a_p, b_p) are the annihilation operators for a complex scalar field. Show that the free complex scalar field theory is invariant under charge conjugation.

(*c*) The solution for the free Dirac equation are written as $u^s(p)e^{-ip.x}$ and $v^s(p)e^{-ip.x}$. Show that

$$\bar{v}^r(p)\,v^s(p) = -2m\delta^{rs} \text{ and } \bar{u}^r(p)\,v^s(p) = 0.$$

(*d*) Determine if $\bar{\psi}\psi$ is invariant under the transformation $\psi(x) \to e^{i\alpha\gamma^5}\,\psi(x)$ (α is a constant) where ψ is the Dirac field.

Hence find the condition under which a free Dirac theory is invariant under this transformation.

2. Answer any *two* of the following : 4 × 2

(*a*) For the following theory with the Lagrangian.

$$L = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + \frac{1}{2}(\partial_{\mu}\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$-g\bar{\psi}\psi\phi - g_5 i\bar{\psi}\gamma^5\psi\phi - \frac{\lambda}{4!}\phi^4.$$

where ψ is a Dirac fermion and ϕ a real scalar field, identify all the interaction terms and the symmetries of the theory. Find the dimensions of the coupling constants in natural units. How should the Lagrangian be modified so that the Dirac formion also couples to the electromagnetic field A_{μ} and the theory is gauge invariant ?

(*b*) Given that the Dirac field transforms as

$\psi \to \Lambda_{\frac{1}{2}} \psi$, where $\Lambda_{\frac{1}{2}} = \exp\left[-\frac{i}{2} S^{\mu\nu} \omega_{\mu\nu}\right]$.

Show that $\bar{\psi}\, \gamma^{\mu} \psi$ transforms as a vector under Lorentz transformation.

[Given :

$$S^{\mu\nu} = \frac{i}{4}\left[\gamma^{\mu}\, \gamma^{\nu}\right] and \left[\gamma^{\mu}\, S^{\rho\sigma}\right] = \left[J^{\rho\sigma}\right]^{\mu}_{\nu}\, \gamma^{\nu}$$

where $J^{\rho\sigma}$ are Lorentz generators corresponding to a vector representation].

(*c*) For a real scalar field theory, using the commutation relations for the creation and annihilation operators and the mode expansion for the real scalar field.

(*i*) Show that

$$[\phi(\vec{x},0), \pi(\vec{y},0)] = i\delta^{3}(\vec{x} - \vec{y})$$

(*ii*) Evaluate $[\phi(x), \phi(y)]$ as an integral over the three momenta and show that the it vanishes for $x_0 = y_0$.

(*d*) The normal ordered conserved charge corresponding to the global transformation $\phi(x) \to e^{i\alpha}\phi(x)$ for the free complex scalar theory is given by

$$:Q: = \int \frac{d^3p}{(2\pi)^3}\left[a_p^\dagger a_p - b_p^\dagger b_p\right].$$

Compute the commutator $[\phi(x), :Q:]$ and show that

$$e^{-i\alpha :Q:}\, \phi(x) e^{i\alpha :Q:} = e^{i\alpha}\, \phi(x),$$

where α is a constant.

3. Answer any *one* of the following : 8 × 1

(*a*) (*i*) For a free real scalar field theory having mass m, compute the energy momentum tensor $T^{\mu\nu}$ using the

Noether's prescripting and show that $\partial_\mu T^{\mu\nu} = 0$. Compute the momentum operator $P^i = \int d^3x T^{0i}$ in terms of the creation and annihilation operators $\left(a_p, a_p^\dagger\right)$.

(*ii*) Given that the Dirac field transforms as $\psi \to \Lambda_{\frac{1}{2}}\psi$, where

$$\Lambda_{\frac{1}{2}} = \exp\left[-\frac{i}{2}S^{\rho\sigma}\omega_{\rho\sigma}\right].$$

Evaluate the transformation matrix $\Lambda_{\frac{1}{2}}$ for rotations about the z-axis. Hence show that the Dirac field changes sign upon rotation by an angle, 2π. 5 + 3

(*b*) (*i*) Consider the free Dirac theory

$$S = \int d^4x \ \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi.$$

Compute the Hamiltonian in terms of the field ψ, Next evaluate the normal ordered Hamiltonian $: H :$ for the free Dirac theory in terms of the creation and annihilation operators $\left(a_p^s, b_p^s, a_p^{s\dagger}, b_p^{s\dagger}\right)$.

(*ii*) The action for a complex scalar field coupled to the electromagnetic field is given by,

$$S = \int d^4x \left[|D_{\mu}\phi(x)|^2 - m^2 |\phi|^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right]$$

where the covariant derivative, $D_{\mu} = \partial_{\mu} + ieA_{\mu}$. Show that the action is invariant uder the transformation $\phi(x) \rightarrow e^{i\alpha} \phi(x)$ and

$$A_{\mu} \rightarrow A_{\mu} - \frac{1}{e} \partial_{\mu} \alpha(x).$$

Find the equation of motion for $A_{\mu}(x)$. 5 + 3

PAPER — PHS-401.2

(*Quantum Field Theory*-II)

GROUP—A

5. Answer any *two* of the following : 2 × 2

(*a*) Using the $SU(3)$ Lie-algebra show that the generators t_a are traceless.

(*b*) In the $K^0 + \bar{K}^0$ system the CP even/odd states are $K_{L/S} = \frac{1}{\sqrt{2}}(K^0 \pm \bar{K}^0)$. Giving reasons identify the state which decays into two pions and the one into three pions.

(*c*) Given that the fermion field $\psi(t, x)$ transforms under Parity (P) as $\eta_a \gamma^0 \psi(t, -x)$, determine the transformation of $\bar{\psi}\gamma^5\psi(t, x)$ under P.

(*d*) You are given that the flavor wave function for Δ^{++} is uuu. Using the ladder operators $I_{\pm}$, determine the (Flavor $\otimes$ Spin) wave funtion for Δ^{0} with spin $S_z = 3/2$.

(*e*) If a particle X decays strongly as $X \rightarrow \pi^{+} + \pi^{+}$. What can you conclude about the Isospin and G-Parity of X ?

6. Answer any *two* of the following : 4×2

(*a*) The three center states in $3 \otimes \overline{3}$ representation for $SU(3)$ color are $[r\overline{r} - g\overline{g}]$, $[g\overline{g} - b\overline{b}]$ and $[b\overline{b} - r\overline{r}]$. Determine the linear combination of these states which is the singlet state. You must show all necessary computations needed for arriving at your result.

(*b*) By writing down the $SU(3)$ group algebra, explain what is meant by a conjugate representation. Write down an explicit matrix corresponding to $U_+ = \frac{1}{2}(\lambda_6 + i\lambda_7)$ for the conjugate representation and show that $U_+\bar{d} = -\bar{s}$.

(*c*) Consider the process $\pi^- + d \rightarrow n + n$ where d denotes the deuteron (bound state of a proton and a neutron) in the ground state ($s = 1$, $l = 0$) and n the neutron. Explain how the intrinsic parity of π^- can be determined from the above process.

(*d*) Write down the expression for the G-parity operator and find the values of G for the pions $\pi^{\pm}$.

6. Answer any *one* of the following : 8×1

(*a*) (*i*) Draw the $I_3 - Y$ diagrams corresponding to 3 and the $\bar{3}$ (conjugate) representations of $SU(3)$ labelling the 3 states as (u, d, s). Find all the $l = 0$ bound states in the $3 \otimes \bar{3}$ and identify the particles with spin zero.

(*ii*) For the SU(2) isospin doublet, considering the qqq bound state (baryon) identify the Mixed-Symmetric and the Mixed-Anti-symmetric flavor parts and hence write down the (Falvor $\otimes$ Spin) wave function for the proton with $s_z = \frac{1}{2}$. 4

(*b*) (*i*) Consider a complex scalar field theory with a continuous Global symmetry defined by the Lagrangian.

$$L = \left|\partial_\mu \phi\right|^2 - \mu^2 |\phi|^2 - \frac{\lambda}{4}(|\phi|^2)^2$$

For the case of spontaneous symmetry breaking $\mu^2 < 0, \lambda > 0$, by considering fluctuations about the minimum of the potential express the Lagrangian in terms of the fluctuation fields. What are the masses of the fields after spontaneous symmetry breaking ?

(*ii*) Using isospin invariance find the ratio of the decay rates in the $I = 3/2$ channel for the following.

$\pi^+ + p \rightarrow \pi^+ + p : \pi^- + p \rightarrow \pi^- + p :$

$\pi^- + p \rightarrow \pi^0 + n.$

5 + 3

[Internal Assessment — 10 Marks]
