(*b*) Determine the constants λ, μ and γ in order that the liquid motion is possible when its velocity components at a point (x_1, x_2, x_3) are given by $v_1 = \dfrac{x_1 + \lambda r}{r(x_1 + r)}$, $v_2 = \dfrac{x_2 + \mu r}{r(x_1 + r)}$ and $v_3 = \dfrac{x_3 + \gamma r}{r(x_1 + r)}$, where $r^2 = x_1^2 + x_2^2 + x_3^2$. 6

15. (*a*) What is the current function in two-dimensional incompressible fluid motion? 2

(*b*) State and prove Kelvin's Minimum Energy Theorem. 6

16. (*a*) Explain the geometric interpretation of infinitesimal strain tensors. 6

(*b*) Consider the uniform deformation of a square block of side 2 units and initially centered at $X = (0, 0)$. If the deformation is defined by the following mapping :

$$x_1 = 3{\cdot}5 + X_1 + 0{\cdot}5X_2, \quad x_2 = 4 + X_2, \quad x_3 = X_3$$

Sketch the deformation. 2

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2024

M.Sc. 2nd Semester Examination

APPLIED MATHEMATICS

PAPER : MTM-205

(General Theory of Continuum Mechanics)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.

A. Answer any **four** questions : 2×4=8

1. What is the concept of Strain Energy?

2. Prove that the extension of a line element through the center of a strain quadric in the direction of any central radius vector is equal to the inverse of the square of the radius vector.

3. Establish the stress vector and stress tensor relationship.

4. Find the complex potential for a doublet.

5. Find the image of a source with respect to a straight line.

6. Show that Lagrangian and Eulerian strain tensors both are equal for small deformation of a continuum body.

B. Answer any **four** questions : 4×4=16

7. For the deformation defined by the equations $X_1 = \frac{1}{2}\left(x_1^2 + x_2^2\right)$, $X_2 = \tan^{-1}\left(\frac{x_2}{x_1}\right)$, $X_3 = x_3$, $x_1 \neq 0$, find the deformation gradient tensor in spatial and material forms. Hence show that the deformation is isochoric. 4

8. Given the following state of stress at a point in a continuum.

$$(\sigma_{ij}) = \begin{pmatrix} 7 & 0 & 14 \\ 0 & 8 & 0 \\ 14 & 0 & -4 \end{pmatrix}$$

Determine the principal stresses and hence find the stress invariants. 4

9. Given the following Cauchy stress components, $\sigma_{11} = -2x_1^2$, $\sigma_{12} = -7 + 4x_1x_2 + x_3$, $\sigma_{13} = 1 + x_1 - 3x_2$, $\sigma_{22} = 3x_1^2 - 2x_2^2 + 5x_3$, $\sigma_{23} = 0$, $\sigma_{33} = -5 + x_1 + 3x_2 + 3x_3$, determine the body

force components for which the stress field describes a state of equilibrium.

10. Describe the volumetric strain in a continuum body.

11. Derive the stress-strain relation for an isotropic elastic body in terms of Poisson's ratio and Young's modulus.

12. State and prove Kelvin's circulation theorem.

C. Answer *any* **two** questions : 8×2=16

13. *(a)* State and prove the Cauchy's first equation of motion. 5

(b) Given the following Cauchy stress components, $\sigma_{11} = -2x_1^2$, $\sigma_{12} = -7 + 4x_1x_2 + x_3$, $\sigma_{13} = 1 + x_1 - 3x_2$, $\sigma_{22} = 3x_1^2 - 2x_2^2 + 5x_3$, $\sigma_{23} = 0$, $\sigma_{33} = -5 + x_1 + 3x_2 + 3x_3$, determine the normal and shearing components of the stress vector at the point (1, 1, 3) on the plane $x_1 + x_2 + x_3 =$ constant. 3

14. *(a)* Prove that the pressure at a point in the perfect fluid is same in all directions. 2