M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

(Lab : OR methods using MATLAB and LINGO)

(Practical)

PAPER – MTM-498 A (New)

Full Marks : 25

Time : $1\frac{1}{2}$ hours

Solve **one** problem using MATLAB and **another** problem with LINGO

The questions are to be distributed by using a **Lottery**

(Marks distribution:MATLAB 9; LINGO 6; Viva & Lab Note Book: 5; Filed Visit: 5)

1. Let
p = shortage cost per unit short per unit time short,
S = inventory level just after a batch of Q units is added to inventory,

Q-S = shortage in inventory just before a batch of Q units is added.

Production or ordering cost per cycle (OC)= K + cQ.

Holding cost per cycle $(HC)=\frac{hS}{2}\frac{S}{a}=\frac{hS^2}{2a}$.

Shortage cost per cycle $(SC)=\frac{p(Q-S)}{2}\frac{Q-S}{a}=\frac{p(Q-S)^2}{2a}$.

Total cost per cycle $TC = OC + HC + SC$

Total cost per unit time $ATC=\frac{TC}{t}, t=\frac{Q}{a}$.

Write LINGO/MATLAB code to find the optimum values of Q, S, t, when $K = 12000$, $h = 0.30$, $a = 8000$, $p = 1.10$, such that the total cost per unit time is minimum.

2. A contractor has to supply 120 bearings per day to an automobile manufacturer. The manufacturer can produce 260 bearings per day in a production run. The cost of holding a bearing in stock for one year is Rs. 2, and the set-up cost of a production run is Rs. 180.

Write a program in MATLAB/LINGO to find the following.

(*i*) The lot size (Q) that minimize the cost

(*ii*) The total optimal cost.

(*iii*) Cycle time (t)

3. Solve the following non-linear programming problem by dynamic programming method using LINGO/MATLAB :

Min $z = y_1^2 + y_1 y_2 + y_1 y_3 - y_4^2$

Subject to $y_1 + y_2 + y_3 + y_4 \geq 30;$

$2y_1 + 3y_2 + y_3 + y_4 \leq 50;$

$y_1, y_2, y_3, y_4, >= 0;$

4. Assume the following notations and formula :

λ : Arrival rate

μ : Service rate

P_o: Probability of no customer in the system $= \dfrac{\mu - \lambda}{\mu}$

L_s: Expected (average) number of units in the system $= \dfrac{\lambda}{\mu - \lambda}$

L_q : Expected (average) queue length $= \dfrac{\lambda^2}{\mu(\mu - \lambda)}$

W_q: Mean (or expected) waiting time in the queue (excluding service time) $= \dfrac{\lambda}{\mu(\mu - \lambda)}$

W_s: Expected waiting time in the system (including service time) $= \dfrac{1}{\mu - \lambda}$

In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Write a code in MATLAB/LINGO to find the following.

(*i*) The average number of trains in the system

(*ii*) The average number of trains in the queue

(*iii*) Mean (or expected) waiting time in the queue (excluding service time)

(*iv*) Expected waiting time in the system (including service time)

5. The probability of n customer in the queue of (M/M/1:c/FCFS/∞) model is

$$p_n = \rho^n p_o = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \text{for } 0 \le n \le N \text{ and } \rho \ne 1 \\ \dfrac{1}{N+1}, & \text{for } 0 \le n \le N \text{ and } \rho = 1 \end{cases}$$

$$\rho = \lambda/\mu$$

Expected line length, i.e. the average number of customers in the system

$$L_s = \frac{\rho[1-(N+1)\rho^N + N\rho^{N+1}]}{(1-\rho)(1-\rho^{N+1})}, \qquad \rho \ne 1$$

Average number of customers in the queue

$$L_q = L_s - 1 + p_o = \frac{\rho^2[1-N\rho^{N-1}+(N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})}, \quad \rho \ne 1$$

Waiting time in the queue

$$W_q = \frac{\rho^2[1-N\rho^{N-1}+(N-1)\rho^N]}{\lambda(1-\rho)(1-\rho^N)}$$

Waiting time in the system

$$W_s = W_q + \frac{1}{\mu}$$

Effective arrival rate $\lambda_{eff} = \lambda(1 - p_N)$

Patients arrive at a clinic according to a Poisson distribution at the rate of 50 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.

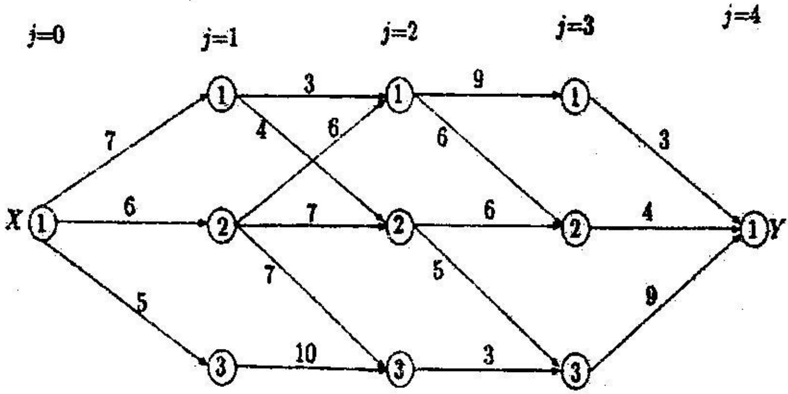
Write a MATLAB/LINGO code to answer the following :

(*i*) Find the effective arrival rate at the clinic.

(*ii*) What is the average number of customers in the queue ?

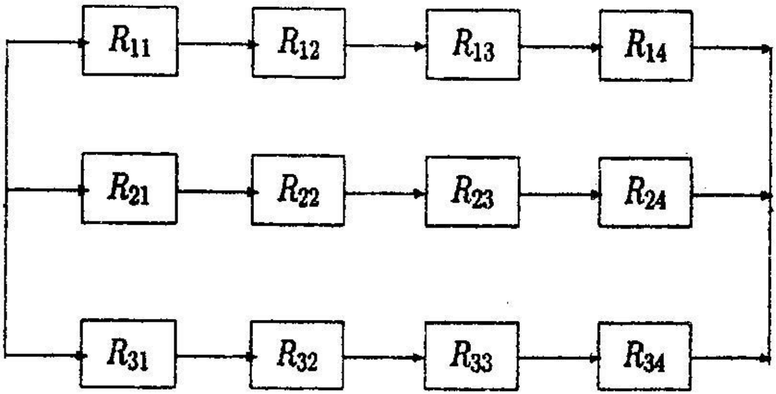
(*iii*) What is the expected *waiting time until* a patient is discharged from the clinic ?

6. Write MATLAB/LINGO code to find the shortest path using dynamic programming method and the corresponding distance from the vertex X to the vertex Y along the edges joining various vertices lying between X and Y shown in the following figure. The numbers associated with the edges represent edge weights.



7. Write MATLAB/LINGO code to estimate the probability to obtain 8 or more heads, if a coin is tossed 10 times, using the Monte Carlo simulation technique.

8. Consider the following diagram of a system of components.



The reliability of the component R_{ij} is given by

$$R_{ij}(t) = e^{-ij\lambda t + 1}.$$

Write MATLAB/LINGO code to calculate the reliability of the system for 100 hours for $\lambda = 0.005$.

9. The system is comprised of 500 transistors, 10,500 resistors, and 500 capacitors connected in series. The failure rates for these components are as follows :

transistors have a failure rate of 0.7×10^{-7} per hour,

resistors have a failure rate of 0.1×10^{-6} per hour, and

capacitors have a failure rate of 0.4×10^{-6} per hour.

Write MATLAB/LINGO code to find the failure rate of the system and its reliability over a duration of 100 hours.

PG/IVS/MTM/498(B)/24(Pr.)
(New)

M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

(*Lab : Dynamical Oceanology*)

(Practical)

PAPER – MTM-498 (B) (New)

Full Marks : 25

Time : 2 hours

Answer any **one** question from both groups on **lottery** basis

The figures in the right hand margin indicate marks

GROUP – A

Answer any **one** questions : 10 × 1

1. Given 15 instantaneous data measurements points for 5 min, plot and discuss the distribution of normalized longitudinal, transverse and vertical mean velocities.

2. Given 15 instantaneous data measurements points for 5 min, plot and discuss the distribution of normalized longitudinal, transverse and vertical turbulence intensities.

3. Given 15 instantaneous data measurements points for 5 min, plot and discuss the distribution of normalized longitudinal, transverse and vertical normal stresses.

4. Given 15 instantaneous data measurements points for 5 min, plot and discuss the distribution of normalized longitudinal, transverse and vertical Reynolds shear stresses.

GROUP – B

Answer any **one** questions : 5 × 1

5. Using Matlab programming, find out the distribution of probability density function for the given 5 instantaneous data measurements points for 3 min at the bottom wall region.

6. Using Matlab programming, find out the distribution of probability density function for the given 5 instantaneous data measurements points for 3 min at the intermediate region.

7. Using Matlab programming, find out the distribution of probability density function for the given 5 instantaneous data measurements points for 3 min at the free surface region.

8. Using Matlab programming, find out the distribution of joint probability density function for the given 5 instantaneous data measurements points for 3 min at the bottom wall region.

9. Using Matlab programming, find out the distribution of joint probability density function for the given 5 instantaneous data measurements points for 3 min at the intermediate region.

10. Using Matlab programming, find out the distribution of joint probability density function for the given 5 instantaneous data measurements points for 3 min at the free surface region.

Notebook + Viva = [2+3] = [5]

Field excursion & report = [5]

**PG/IVS/MTM/498(C)/24/(Pr.)
(New)**

PG 4th Semester Examination, 2024

MATHEMATICS

Lab : *Semi-Analytical and Computational Methods* (Using MATLAB and Mathematica)

(Practical)

PAPER—MTM-498(C)(New)

Full Marks : 25

Time : 2 hours

The figures in the right hand margin indicate marks

Symbols/Notations have their usual meaning

Select **two** questions by **lottery** : **one** from MATLAB and **other one** from Mathematica $8\frac{1}{2}\times 2$

(Turn Over)

1. Consider the following one-dimensional heat-conduction equation.

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$$

An insulated rod initially has a temperature of $T(x, 0) = 0°C$, and at $t = 0$ hot reservoirs ($T = 100°C$) are brought into contact with the two ends, A and B. Write a Mathematica/ MATLAB code using Forward-Time-Central -Space (FTCS) scheme to find the numerical solution of subsequent temperature $T(x, t)$ of any point in the rod with time-step 0.1. Take the appropriate space-step so that the solution converges.

2. For the above problem, write a Mathematica/ MATLAB code using Crank-Nicolson Method to find the numerical solution of subsequent temperature $T(x, t)$ of any point in the rod with time-step 0.1. Take the appropriate space-step so that the solution converges.

3. Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100°C and 500°C, respectively. The one-dimensional problem is governed by

$$\frac{d}{dx}\left(K\frac{dT}{dx}\right)=0$$

Write a Mathematica/MATLAB code to calculate the steady state temperature distribution in the rod. Thermal conductivity k equals 1000 W/m.K, cross-sectional area A is $10 \times 10^{-3}\text{m}^2$. Use Finite Volume Method (FVM) and explicit scheme.

4. Consider the following one-dimensional Burger's equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$

with the initial condition $u(x, 0) = 2x, t > 0$. Write a Mathematica/MATLAB Code to find the numerical solution of the above problem using the Variational Iteration Method (VIM) with linear operator $L(u) = \frac{\partial u}{\partial t}$. Show the solution graphically.

5. Consider the following hyperbolic non-linear problem

$$\frac{\partial u}{\partial t} = u\frac{\partial u}{\partial x}, 0 \leq x \leq 1, 0 \leq t \leq 1.$$

$$u(x, 0) = \frac{x}{10}$$

Write a Mathematica/MATLAB Code to find the 5th order approximate solution of the above problem using the Adomian Decomposition Method (ADM) with linear operator $L(u) = \frac{\partial u}{\partial t}$.

6. Consider the following non-linear problem

$$f''(x)+f'^2(x)=0$$
$$f(0)=1,\ f'(0)=2$$

Write a Mathematica/MATLAB Code to find the 5th order approximate solution of the above problem using the Homotopy Perturbation Method (HPM) with linear operator $L(f)=f''$. Exact solution of this problem is $f(x) = 1 + \ln(2x + 1)$. Show the comparison between the numerical solution and exact solution graphically.

7. Consider the following non-linear problem

$$f''(x)+f'^2(x)=0$$
$$f(0)=1,\ f'(0)=2$$

Write a Mathematica/MATLAB Code to find the 5th order approximate solution of the above problem using the Homotopy Analysis Method (HAM) with linear operators $L(f)=f''+f'$ and $L(f)=f''$. Show the comparison between these two solutions graphically.

8. Consider the second order non-linear differential equation with an exponential non-linearity as $u'' = e^u, u(0) = 0, u'(0) = 0.$ Write a Mathematica/MATLAB Code using the Adomain Decomposition Method, find the 2nd order approximated solution and show it graphically.

[Viva and Notebook] 4 + 4
