16. Let us consider the wave equation

$$u_{tt} = c^2 u_{xx}; \ t > 0, \ 0 < x < 1$$

where the initial conditions are $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$, $0 < x < 1$ and boundary conditions are $u(0,t) = \phi(t)$ and $u(1,t) = \psi(t)$, $t \geq 0$. Describe a finite difference method to solve the above problem.

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2024

M.Sc. 2nd Semester Examination

APPLIED MATHEMATICS

PAPER : MTM-202

(Numerical Analysis)

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.

A. Answer any **four** questions : 2×4=8

1. Compute the order of arithmetic computations to find the inverse of a square non-singular matrix of order *n*.

2. What do you mean by relaxation factor (w) in successive relaxation methods? Write the significance of w for its different values.

3. Compare Newton-Cotes and Gaussian quadrature formulae.

4. Describe minimax polynomial.

5. What are the advantages to approximate a function using orthogonal polynomials?

6. What is the multi-step method to solve an ODE? What is the advantage of this method?

B. Answer any **four** questions : 4×4=16

7. Describe the Newton-Raphson method to solve a pair of non-linear equations $f(x,y)=0$ and $g(x,y)=0$.

8. Find the inverse of the following matrix by partial pivoting

$$\begin{bmatrix} 8 & 1 & -1 \\ 2 & 1 & 9 \\ 1 & -7 & 2 \end{bmatrix}$$

9. Given that $f(0,0)=1$, $f(0,1)=1\cdot414214$, $f(1,0)=1\cdot732051$ and $f(1,1)=2$. Find the Lagrange's bivariate interpolating polynomial and hence find the approximate value of $f(0\cdot25, 0\cdot50)$.

10. Using Milne's predictor-corrector formula, find $y(0\cdot4)$ for the following IVP :

$$\frac{dy}{dx} = x^2 - y;\; y(0)=1$$

with the step length $h = 0\cdot1$.

11. Discuss the stability of Euler's method of the ODE :

$$\frac{dy}{dt} = \lambda y;\; y(0) = y_0.$$

12. Explain a suitable method to solve a system of tri-diagonal linear equations.

C. Answer *any* **two** questions : 8×2=16

13. Find all roots of the equation

$$x^4 - 3x^3 + 5x^2 - 4x + 8 = 0$$

by Braistow's method.

14. Explain 4-point Gauss-Chebyshev quadrature formula. Using it, determine the value of the integration $\int_1^3 \frac{1}{1-x^2}\,dx.$

15. Describe Jacobi's method to find all eigenvalues and eigenvector of a real symmetric matrix.