 PG/IVS/MTM/497(A)/24(Pr.) (New)

M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

(*Optimization Method*)

(Using MATLAB/LINGO)

[Practical]

PAPER – MTM-497(A)(New)

Full Marks : 25

Time : 2 hours

Answer any **one** question from **both** Groups

The figures in the right hand margin indicate marks

Questions will be given on lottery basis :

GROUP – A

Answer any **one** question : 6 × 1

1. Write the code in LINGO and then to solve the following LPP using simplex method.

$$\text{Max } Z = 1200x_1 + 1300x_2 + 1300x_3 + 1200x_4 + 899x_5 + 999x_6 + 899x_7 + 1099x_8$$

Subject to $12x_1 + 13x_2 + 13x_3 + 12x_4 + 9x_5 + 10x_6 + 9x_7 + 11x_8 \leq 39$

$$x_1 + x_2 + x_3 + x_4 + 0x_5 + x_6 + x_7 + x_8 \leq 3$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + 0x_6 + x_7 + x_8 \leq 3$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + 0x_7 + x_8 \leq 3$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + 0x_8 \leq 3$$
$0 \leq x_i \leq 1$ for 1, 2, ..., 8

2. Write the code in LINGO and then to find the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

3. Write the code in LINGO and then to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines' parts using lathes, milling machines and grinding machines. The machining times available per week on different machines and the profit on machine part are given below. The machining times required on different machines for each part are not known precisely (as they vary from worker to worker) but are known to follow normal distribution with mean and standard deviations as indicated in the following table.

	Machining time required per unit (minutes)				
Types of Machine	Part-I		Part-II		Maximum time available per week (minutes)
	Mean	Standard deviation	Mean	Standard deviation	

Lathes	$\bar{a}_{11} = 10$	$\sigma_{a11} = 6$	$\bar{a}_{12} = 4$	$\sigma_{a12} = 4$	$b_1 = 2500$
Milling machines	$\bar{a}_{21} = 4$	$\sigma_{a21} = 6$	$\bar{a}_{22} = 10$	$\sigma_{a22} = 7$	$b_2 = 2000$
Grinding machines	$\bar{a}_{31} = 1$	$\sigma_{a31} = 2$	$\bar{a}_{32} = 1.5$	$\sigma_{a31} = 3$	$b_3 = 450$
Profit per unit (Rs)	$c_1 = 50$		$c_2 = 100$		

Determine the number of machine Parts I and II to be manufactured per week to maximize the profit without exceeding the available machining times more than once in 100 weeks.

4. Write the code in LINGO and then to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x) = 7x_1x_2^{-1} + 7x_2x_3^{-2} + 5x_1^{-3}x_2x_3 + x_1x_2x_3$$

$$x_1, x_2, x_3 > 0.$$

5. Write the code in LINGO and then to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } Z = 18x_1 + 3x_2 - 0.001x_1^2 - 0.005x_2^2 - 100$$

$$\text{Subject to, } \quad 2x_1 + 3x_2 \leq 2500$$

$$x_1 + 2x_2 \leq 1500$$

$$x_1, x_2 \geq 0.$$

6. Write the code in LINGO and then to solve the following Goal Programming problem.

$$\text{Min } \; Z = P_1 (2d_2^- + 3d_3^-) + P_2 d_1^-$$

$$\text{Subject to } \quad 20x_1 + 10x_2 \leq 60$$

$$10x_1 + 10x_2 \leq 40$$

$$30x_1 + 60x_2 + d_1^- - d_1^+ = 600$$

$$x_1 + d_2^- - d_2^+ = 4$$

$$x_2 + d_3^- - d_3^+ = 4$$

$$x_1, x_2, d_i^-, d_i^+ \geq 0, \; i = 1, 2, 3.$$

7. Write the code in LINGO and then solve the following fuzzy LPP using simplex method.

Max $Z = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{1}x_3$

Subject to, $2x_1 + 5x_2 - x_3 \leq 5$

$x_1 + x_2 + 2x_3 = \tilde{6}$

$2x_1 - x_2 + 3x_3 = 7$

$x_1, x_2 \geq 0.$

Here, $\tilde{2}$, $\tilde{3}$, and $\tilde{1}$, $\tilde{6}$ are symmetric triangular fuzzy numbers with spread 2.

GROUP – B

Answer any **one** question : 9 × 1

8. Write the code in MATLAB and then to find Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$A = \begin{bmatrix} 8 & 0 \\ 30 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 30 \\ 0 & 2 \end{bmatrix}$$

9. Write the code in MATLAB and then to solve the following Geometric Programming Problem.

Minimize $f(x) = 5x_1 x_2^{-1} x_3^{2} + x_1^{-2} x_2^{-1} + 10x_2^{2} + 2x_1^{-1} x_2 x_3^{-2}$

$x_1, x_2, x_3 > 0.$

10. Write the code in LINGO to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines' parts using lathes, milling machines and grinding machines. The machining times required on different machines for each part and the profit on machine part are given below. If the machining times available on different machines are probabilistic (normally distributed) with parameters as given in the following table, find the number of machine parts I and II to be manufactured per week to maximize the profit. The constraint have to be satisfied with a probability of at least 0.99.

Type of Machine	Machining time required per piece (minutes)		Maximum time available per week (minutes)	
	Part-I	Part-II	Mean	Standard deviation
Lathes	$a_{11} = 10$	$a_{12} = 5$	$b_1 = 2500$	$\sigma_{b1} = 500$
Milling Machines	$a_{21} = 4$	$a_{22} = 10$	$b_2 = 2000$	$\sigma_{b2} = 400$
Grinding Machines	$a_{31} = 1$	$a_{32} = 1.5$	$b_3 = 450$	$\sigma_{b3} = 50$
Profit per unit (Rs.)	$c_1 = 50$	$c_2 = 100$		

11. Write the code in LINGO to solve the following QPP using Wolfe's modified simplex method.

Max $Z = 2x_1 + x_2 - x_1^2$

Subject to, $2x_1 + 3x_2 \le 6$

$2x_1 + x_2 \le 4$

$x_1, x_2 \ge 0.$

12. Write the code in MATLAB and then to solve the following fuzzy LPP.

$$\text{Max } Z = \tilde{3}x_1 + \tilde{4}x_2$$

$$\text{Subject to,} \quad x_1 + x_2 \leq 10$$

$$2x_1 + 3x_2 \leq \widetilde{18}$$

$$x_1 \leq 8$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0.$$

Here, $\tilde{3}$, $\tilde{4}$ and $\widetilde{18}$ are symmetric triangular fuzzy numbers with spread 1.5.

13. Write the code in MATLAB and then to the following Stochastic Programming Problem.

A manufacturing firm produces two machines parts using lathes, milling machines and grinding machines. The machining times available per week on different machines and the machining times required on different machines for each part are given below. Assuming that the profit per unit of each of the machine parts I and II is a normally distributed random variable, find the number of machine parts to be manufactured per week to maximize the profit. The mean value and standard deviation of profit are Rs. 50 and 20 per unit for part I and Rs. 100 and 50 per unit for part II.

Type of Machine	Machining time required per piece (minutes)		Maximum time available per week (minutes)
	Part-I	Part-II	
Lathes	$a_{11} = 10$	$a_{12} = 5$	$b_1 = 2500$
Milling Machines	$a_{21} = 4$	$a_{22} = 10$	$b_2 = 2000$
Grinding Machines	$a_{31} = 1$	$a_{32} = 1.5$	$b_3 = 450$

14. Write the code in MATLAB and then to solve the following goal programming problem :

$$\text{Min } Z = P_1 u_1 + P_2(u_2 + O_2) + P_3 u_3$$

Subject to
$$8x_1 + 6x_2 + u_1 - O_1 = 36$$
$$x_1 - 2x_2 + u_2 - O_2 = 0$$
$$2x_1 + 4x_2 + u_3 = 20$$
$$3x_1 + 4x_2 + u_4 = 24$$
$$x_1, x_2, u_1, u_2, u_3, u_4, O_1, O_2 \geq 0.$$

15. Write the code in MATLAB and then to solve the following LPP.

$$\text{Max } Z = 61x_1 + 209x_2 + 324x_3 + 33x_4 + 276x_5 + 285x_6 + 250x_7 + 100x_8 + 12x_9 + 282x_{10}$$

Subject to constraints

$$16x_1 + 25x_2 + 22x_3 + 4x_4 + 9x_5 + 8x_6 + 11x_7 + 29x_8 + 20x_9 + 22x_{10} \leq 18$$

$$5x_1 + 22x_2 + 15x_3 + 30x_4 + 24x_5 + 15x_6 + 14x_7 + 28x_8 + 31x_9 + 25x_{10} \le 53$$

$$22x_1 + 17x_2 + 9x_3 + 32x_4 + 26x_5 + 20x_6 + 16x_7 + 16x_8 + 26x_9 + 24x_{10} \le 50$$

$$32x_1 + 30x_2 + 10x_3 + 30x_4 + 7x_5 + 29x_6 + 15x_7 + x_8 + 2x_9 + 23x_{10} \le 4$$

$$14x_1 + 9x_2 + 32x_3 + 22x_4 + 30x_5 + 18x_6 + 18x_7 + 32x_8 + 15x_9 + x_{10} \le 40$$

$$12x_1 + 4x_2 + 30x_3 + 11x_4 + 23x_5 + 29x_6 + 8x_7 + 2x_8 + 23x_{10} \le 31$$

$$22x_1 + 23x_2 + 26x_3 + 13x_4 + 6x_5 + 13x_6 + 32x_7 + 11x_8 + 8x_9 + 5x_{10} \le 39$$

$0 \le X \le U$ where $U^T = (0.125, 0.133, 0.04, 0.133, 0.571, 0.138, 0.266, 0.62, 0.21, 0.153)$

16. Laboratory Note Book. 5

17. Viva. 5

M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

[Practical]

PAPER – MTM-495 B(Old)

Full Marks : 25

Time : 2 hours

Answer any **one** question from each Group

The figures in the right hand margin indicate marks

Questions will be given on lottery basis :

GROUP – A

Answer any **one** question : 6 x 1

(*Turn Over*)

1. Write the code in LINGO to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x) = 7x_1x_2^{-1} + 7x_2x_3^{-2} + 5x_1^{-3}x_2x_3 + x_1x_2x_3$$

2. Write the code in LINGO to solve the following problem on Inventory.

An engineering factory consumes 5000 units of a component per year. The ordering, receiving and handling cost are Rs. 300 per order while trucking cost is Rs.1200 per order, internet cost Rs. 0.06 per unit per year, Deterioration and obsolesce cost Rs. 0.004 per year and storage cost Rs. 1000 per year for 5000 units. Calculate the economic order quantity and minimum average cost.

3. Write the code in LINGO to find the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

4. Write the code in LINGO to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines' parts using lathes. milling machines and grinding machines. The machining times available per week on different machines and the profit on machine part are given below. The machining times require on different machines for each part are not known precisely (as they vary from worker to worker) but are known to follow normal distribution with mean and standard deviations as indicated in the following table.

Type of Machine	Machining time required per unit (minutes)				Maximum time available per week (minutes)
	Part-I		Part-II		
	Mean	Standard deviation	Mean	Standard deviation	
Lathes	$\bar{a}_{11} = 10$	$\sigma_{a11} = 6$	$\bar{a}_{12} = 4$	$\sigma_{a12} = 4$	$b_1 = 2500$
Milling machines	$\bar{a}_{21} = 4$	$\sigma_{a21} = 6$	$\bar{a}_{22} = 10$	$\sigma_{a22} = 7$	$b_2 = 2000$
Grinding machines	$\bar{a}_{31} = 1$	$\sigma_{a31} = 2$	$\bar{a}_{32} = 1.5$	$\sigma_{a31} = 3$	$b_3 = 450$
Profit per unit (Rs)	$c_1 = 50$		$c_2 = 100$		

Determine the number of machine Parts I and II to be manufactured per week to maximize the profit without exceeding the available machining times more than once in 100 weeks.

5. Write the code in LINGO to solve the following LPP using simplex method.

Max $Z = 2x_1 + 3x_2 - 3x_3$

Subject to
$$2x_1 + 5x_2 + x_3 \leq 5$$
$$x_1 + x_2 + 2x_3 \leq 6$$
$$2x_1 - x_2 + 3x_3 \leq 7$$
$$x_1 , x_2 \geq 0.$$

6. Write the code in LINGO to solve the following problem on Inventory.

The demand for an item is deterministic and constant over time and is equal to 600 units per year. The unit cost of the item is Rs. 50.00 while the cost of placing and order is Rs. 100.00 The inventory carrying cost is 20% of the item and the shortage cost per month is Rs. 1. Find the optimal ordering quantity. If shortages are not allowed, what *would be the loss of the* company ?

7. Write the code in LINGO to solve the following Geometric Programming Problem.

Minimize $f(x) = 5x_1x_2^{-1} + 2x_1^{-1}x_2 + 5x_1 + x_2^{-1}$.

8. Write the code in LINGO to solve the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$A = \begin{bmatrix} 8 & 0 \\ 30 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 30 \\ 0 & 2 \end{bmatrix}$$

9. Write the code in LINGO to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines' parts using lathes. milling machines and grinding machines. The machining times available per week on different machines and the machining times required on different machines for each part are given below. Assuming that the profit per unit of each of the machine parts I and II is a normally distributed random variable, find the number of machine parts to be manufactured per week to maximize the profit. The mean value and standard deviation of profit are Rs. 50 and 20 per unit part I and Rs. 100 and 50 per unit for part II.

Type of Machine	Machining time required per piece (minutes)		Maximum time available per week (minutes)
	Part-I	Part-II	
Lathes	$a_{11} = 10$	$a_{12} = 5$	$b_1 = 2500$
Milling machines	$a_{21} = 4$	$a_{22} = 10$	$b_2 = 2000$
Grinding machines	$a_{31} = 1$	$a_{32} = 1.5$	$b_3 = 450$

10. Write the code in LINGO to solve the following LPP using Revised Simplex Method :

Max $Z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \le 6$

$x_1 + 4x_2 \le 4$

$x_1 , x_2 \ge 0.$

GROUP–B

Answer any **one** question : 9×1

11. Write the code in MATLAB to solve the following Stochastic Programming Problem.

A manufacturing firm produces two machines' parts using lathes, milling machines and grinding machines. The machining times required on different machines for each part and the profit on machine part are given below. If the machining times available on different machines are probabilistics (normally distributed) with parameters as given in the following table, find the number of machine parts I and II to be manufactured per week to maximize the profit. The constraint have to be satisfied with a probability of at least 0.99.

Type of Machine	Machining time required per piece (minutes)		Maximum time available per week (minutes)	
	Part-I	Part-II	Mean	Standard deviation
Lathes	$a_{11} = 10$	$a_{12} = 5$	$b_1 = 2500$	$\sigma_{b1} = 500$
Milling machines	$a_{21} = 4$	$a_{22} = 10$	$b_2 = 2000$	$\sigma_{b2} = 400$
Grinding machines	$a_{31} = 1$	$a_{32} = 1.5$	$b_3 = 450$	$\sigma_{b3} = 50$
Profit per unit (Rs.)	$c_1 = 50$	$c_2 = 100$		

12. Write the code in MATLAB to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } Z = 18x_1 + 3x_2 - 0.001x_1^2 - 0.005x_2^2 - 100$$

$$\text{Subject to} \quad 2x_1 + 3x_2 \leq 2500$$

$$x_1 + 2x_2 \leq 1500$$

$$x_1 , x_2 \geq 0.$$

13. Write the code in MATLAB to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x) = 2x_1 + 4x_2 + 10x_1^{-1}x_2^{-1}$$

14. Write the code in MATLAB to solve the following problem bo Inventory.

The demand for an item in a company

is 18000 units per year. The company can produce the item at a rate of 3000 per month. The cost of one set-up is Rs. 500 and the holding cost of one unit per month is Rs. 0.15. The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity. Also determine the manufacturing time and the time between setup.

15. Write the code in MATLAB to solve the following Queuing theorem problem.

A telephone exchange has two long distance operators. The telephone company finds that, during the peak load long distance all arrive in a Poisson fashion at an average rate of 15 per hour. The lẻngth of service on this call is approximately exponentially distributed with mean length 5 *minutes.*

(*i*) What is the probability that a subscriber will have to wait for this long-distance call during the peak hours of the day ?

(*ii*) If the subscriber waits and are serviced in turn, what is the expected waiting time.

16. Write the code in MATLAB to solve the following LPP using Revised Simplex method.

Max $Z = 3x_1 + 5x_2$

Subject to $x_1 \leq 4$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

$$x_1 , x_2 \geq 0.$$

17. Write the code in MATLAB to solve the following QPP using Wolfe's modified simplex method.

$$\text{Max } Z = 2x_1 + x_2 - x_1^2$$

$$\text{Subject to} \quad 2x_1 + 3x_2 \le 6$$

$$2x_1 + 3x_2 \le 4$$

$$x_1, x_2 \ge 0.$$

18. Write the code in MATLAB to solve the Nash equilibrium strategy and Nash equilibrium outcome of the following bi-matrix game.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

19. Write the code in MATLAB to solve the following Geometric Programming Problem.

$$\text{Minimize } f(x) = 5x_1 x_2^{-1} x_3^{2} + x_1^{-2} x_2^{-1} + 10x_2^{2} + 2x_1^{-1} x_2 x_3^{-2}$$

20. Write the code in MATLAB to solve the following LPP using simplex method.

Max $Z = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \le 10$

$$2x_1 + 3x_2 \le 18$$

$$x_1 \le 8$$

$$x_2 \le 6$$

$$x_1, x_2 \ge 0.$$

21. Laboratory Note Book + Viva voce —05.

22. Field Visit – 05.