 PG/IVS/MTM/405(A, B & C)/24
(New)

M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

PAPER—MTM-405A/MTM-405B/MTM-405C

Full Marks : 25

Time : 1 hour

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

PAPER : MTM-405A

(*Operational Research Modelling*-II)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) Define joint, marginal and conditional entropies.

(*b*) Define reliability. Distinguish reliability from probability.

(*c*) What do you mean by memoryless channel and channel matrix ?

(*d*) Explain the Shannon-Fano encoding procedure.

2. Answer any *two* questions : 4×2

(*a*) A particle attached to the lower end of a vertical spring whose other end is fixed is oscillating about its equilibrium position. If x denotes the particle's displacement from the equilibrium position, the governing differential

equation for this motion is $\ddot{x} = -\omega^2 x$. If the particle is at its maximum displacement $x = a$ at time $t = 0$ and at this instant of time, a force u per mass is applied to the particle in order to bring the particle to rest when its displacement is zero, find such a force u.

(*b*) A word consists of three letters with respective probabilities $\frac{5}{12}, \frac{1}{2}$ and $\frac{1}{12}$. Find the average amount of information associated with the transmission of letters.

(*c*) How many identical components each of which is 90 % reliable over 50 hours be used to obtain a, 99.99% parallel redundancy system over 50 hours ? If we want to obtain the same system reliability over 100 hours, how many components should be added ?

(*d*) Use Shannon's encoding procedure to find the code for the alphabets A, B, C, D using the following information.

Alphabet	A	B	C	D
Probability	0.4	0.2	0.3	0.1

3. Answer any *one* question : 8×1

(*a*) Let a car be driven from a stationary position on a horizontal way to a stationary position in a garage moving a total distance '*a*'. The available control for the driver is the accelerator and the break. Find the minimum time to bring the car to the stationary position at a distance *a* and the optimal control to be applied to the car.

(*b*) A transmitter and receiver have information consisting of three letters. The joint probabilities for communication are given below.

$P(x_i, y_j)$	y_1	y_2	y_3
x_1	0.25	0.28	0.05
x_2	0.06	0.12	0.02
x_3	0.04	0.08	0.10

Determine the entropies $H(X)$, $H(Y)$ and $H(X/Y)$ for this channel.

[Internal Assessment — 5 Marks]

PAPER : MTM-405B

(*Dynamical Oceanology* : *Coastal Processes*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) What is the reason for wave decay ?

(*b*) Write a short note on 'wave shoaling'.

(*c*) Define the terms 'salinity' and sigma-t' for sea water.

(*d*) Write down the basic physical laws used in oceanography.

2. Answer any *two* questions : 4×2

(*a*) Write down the wave breaking criteria ? State the condition of wave breaking for shallow and deep water waves.

(*b*) Classify the forces and motions in oceanography and write down its physical significance.

(*c*) What is the reason for tsunamis and explain its effects. Also, write down the equation of conservation of mass and total travel time from the epicenter to the desired location.

(*d*) Determine the amout of damping that will occur after a wave propagates a distance l in water of constant depth h.

3. Answer any *one* question : 8×1

(*a*) Derive the horizontal velocity expression using the governing equations of water waves in a viscous fluid for laminar boundary layer. Hence, derive the largest term of mean rate of energy dissipation per unit time. Also, plot the normalized velocity profiles for various phase positions in a laminar boundary layer. $5 + 2 + 1$

(*b*) (*i*) Explain, the terms 'wave refraction' and 'wave reflection'. $2 + 2$

(*ii*) A wave has 3 m height and 7 seconds period in deep water. It travels towards shore over parallel bed contours. If its crest line makes and angle of 300 with the bed contour of 10 m before refraction, calculate the

wave height after crossing this contour line. 4

Data :

d/L_0	d/L	n
1.1300	0.1655	0.7621
0.1310	0.1674	0.7606

[Internal Assessment — 5 Marks]

PAPER : MTM-405C

(Computational and Semi-Analytical Methods)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

   (*a*) Discuss the boundary condition for pressure correction at lower boundary of the computational domain when velocity components are specified at the boundary.

(*b*) What is the Semi-analytical method? Write some of the semi-analytical methods.

(*c*) Write the formula for Adomain polynomials and hence express the first three polynomials.

(*d*) For the following non-linear equation, derive the variation iteration formula

$$L(u) + N(u) - g(x) = 0.$$

Where symbols have their usual meaning.

2. Answer any *two* questions : 4×2

(*a*) Write the steps for Semi Implicit Method for Pressure Linked Equations (SIMPLE) and hence draw its flow chart.

(*b*) Consider the second order non-linear differential equation with an exponential non-linearity as $u'' = e^u$, $u(0) = 0$, $u'(0) = 0$. Using the Adomain Decomposition Method, find the 2nd order approximated solution.

(*c*) (*i*) Compare between Homotopy Analysis Method (HAM) and Homotopy Pertubation Method (HPM). 2

(*ii*) What are approaches available for finding the value of controlling parameter h of HAM ? 2

(*d*) Consider the following two-dimentional Burger's equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

with the initial conditions

$$u(x,y,0)=x+y,\ t>0$$

$$v(x,y,0)=x-y,\ t>0$$

With the linear operator $L(u)=\frac{\partial u}{\partial t}$ and $L(v)=\frac{\partial v}{\partial t}$, write the Variation Iteration scheme for the above problem.

3. Answer any *one* question : 8×1

(*a*) (*i*) Let the discretised equation for x- and y-momentum equations are, respectively,

$$a_j^u u_{j-1,k}^{n+1} + b_j^u u_{j,k}^{n+1} + c_j^u u_{j+1,k}^{n+1} = d_j^u - \Delta y\left(p_{j+1,k}^{n+1} - p_{j,k}^{n+1}\right) \text{ and}$$

$$a_j^v v_{j-1,k}^{n+1} + b_j^v v_{j,k}^{n+1} + c_j^u v_{j+1,k}^{n+1} = d_j^v - \Delta x\left(p_{j,k+1}^{n+1} - p_{j,k}^{n+1}\right)$$

Using these, derive the Pressure Poisson equation. Symbols have their usual meaning. 6

(*ii*) For the non-linear differential equation $N[f(x)] = 0$, write the homotopy equation for the Homotopy Analysis Method (HAM) by defining appropriate parameters/operator/ functions. 2

(*b*) Consider the Lotka-Volterra equation for prey-predator of two species as

$$u'(t) = u(t)(2 - v(t)) \text{ and}$$

$$v'(t) = v(t)(u(t) - 1)$$

With the initial conditions $u(0) = 16$ & $v(0) = 10$, find the first order Homotopy Pertubation Method (HPM) solution for the above equation with the linear operator $L(u) = u'$ and $L(v) = v'$.

[Internal Assessment — 5 Marks]
