PG/IVS/MTM/405(B)/24(Old)

M.Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

(*Operational Research Modelling*-II)

PAPER—MTM-405(B)

Full Marks : 25

Time : 1 hour

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *two* questions : 2×2

(*a*) Define joint, marginal and conditional entropies.

(*b*) Define reliability. Distinguish reliability from probability.

(*c*) What do you mean by memoryless channel and channel matrix ?

(*d*) Explain the Shannon-Fano encoding procedure.

2. Answer any *two* questions : 4×2

(*a*) A particle attached to the lower end of a vertical spring whose other end is fixed is oscillating about its equilibrium position. If x denotes the particle's displacement from the equilibrium position, the governing differential equation for this motion is $\ddot{x} = -\omega^2 x$. If the particle is at its maximum displacement $x = a$ at time $t = 0$ and at this

instant of time, a force u per mass is applied to the particle in order to bring the particle to rest when its displacement is zero, find such a force u.

(*b*) A word consists of three letters with respective probabilities $\frac{5}{12}, \frac{1}{2}$ and $\frac{1}{12}$. Find the average amount of information associated with the transmission of letters.

(*c*) How many identical components each of which is 90 % reliable over 50 hours be used to obtain a, 99.99% parallel redundancy system over 50 hours ? If we want to obtain the same system reliability over 100 hours, how many components should be added ?

(*d*) Use Shannon's encoding procedure to find the code for the alphabets A, B, C, D using the following information.

Alphabet	A	B	C	D
Probability	0.4	0.2	0.3	0.1

3. Answer any *one* question : 8×1

(*a*) Let a car be driven from a stationary position on a horizontal way to a stationary position in a garage moving a total distance *a*. The available control for the driver is the accelerator and the break. Find the minimum time to bring the car to the stationary position at a distance *a* and the optimal control to be applied to the car.

(*b*) A transmitter and receiver have information consisting of three letters. The joint probabilities for communication are given below.

$P(x_i, y_j)$	y_1	y_2	y_3
x_1	0.25	0.28	0.05
x_2	0.06	0.12	0.02
x_3	0.04	0.08	0.10

Determine the entropies $H(X)$, $H(Y)$ and $H(X/Y)$ for this channel.

[Internal Assessment — 5 Marks]