PG/IVS/MTM/404 (A&B)
/24(New)

M. Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

(*Non-Linear Optimization Or Dynamical Meteorology-II*)

PAPER – MTM-404A or MTM-404B (New)

Full Marks : 25

Time : 1 hour

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – MTM-404A

(*Non-Linear Optimization*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) What are the advantages and limitations for a geometric programming problem ?

(*b*) Let X^0 be an open set in R^n, let θ and g be defined on X^0. Find the conditions under which a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(*c*) Define Nash equilibrium mixed strategies and outcome in connection with bi-matrix game.

(*d*) State Dorn's duality theorem in connection with duality in quadratic programming.

2. Answer any *two* questions : 4×2

(*a*) Minimize the following using geometric programming

$$f(x) = 80x_1x_2 + 40x_2x_3 + 20x_1x_3 + \frac{80}{x_1x_2x_3},$$
$$x_1, x_2, x_3 > 0.$$

(*b*) Define multi-objective non-linear programming problem. Discuss the $\in$-constraint method for multi-objective non-linear programming problem. State the related theorem for Pareto optimal solution of multi-objective non-linear programming problem.

(*c*) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP.

(*d*) Define the following :

(*i*) Minimization problem;

(*ii*) Local minimization problem;

(*iii*) Kuhn-Tucker stationary point problem;

(*iv*) Fritz-John stationary point problem.

3. Answer any *one* question : 8 × 1

(*a*) (*i*) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when a_{ij} is a random variable :

$$\text{Minimize} \quad F(x) = \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ij}\, x_j \le b_i$$

$$x_j \ge 0,\ i,\ j = 1, 2, \ldots, n.$$ 6

(*ii*) Prove that all strategically equivalent bimatrix game have the Nash equilibria. 2

(*b*) (*i*) Let X be an open set in R^n and θ and g be differential and convex on X and let $\bar{x}$ solve the minimization problem and let g satisfy the Kuhn-Tucker constraint qualification. Show that there exisis a $\overline{u} \in R^m$ such that $(\overline{x}, \overline{u})$ solves the dual maximization problem and $\theta(\overline{x}) = \psi(\overline{x}, \overline{u})$. 6

(*ii*) Define the following terms : 2

The (primal) quadratic minimization problem (QMP).

The quadratic dual (maximization) problem (QDP).

[Internal Assessment — 05 Marks]

PAPER — MTM-404B

(*Dynamical Meteorology-II*)

[*Marks* : 20]

1. Answer any *two* questions : 2×2

(*a*) What is the concept of surface of discontinuity in the atmosphere ?

(*b*) Define the absolute and relative vorticity of an air parcel in the atmosphere.

(*c*) What is the concept of divergence of an air parcel in the atmosphere ?

(*d*) What is the concept of coriolis force in the atmosphere ?

2. Answer any *two* questions : 4×2

(*a*) Derive the angle between the frontal surface and the earth surface in the atmosphere.

(*b*) Write down the momentum equation of an air parcel in the atmosphere in vector form.

(*c*) Find the pressure tendency at a place on the earth surface in the presence of geostrophic wind field.

(*d*) Derive the expression of the pressure gradient force in the atmosphere.

3. Answer any *one* question : 8×1

(*a*) Find the rate of change of circulation of an air parcel in the atmosphere and interpret each term.

(*b*) Show that the sum of kinetic energy, potential energy and enthalpy of an air parcel in the atmosphere remains constant when the flow is steady, adiabatic and frictionless.

[Internal Assessment — 05 Marks]