PG/IVS/MTM/404 (B)/24
(Old)

M. Sc. 4th Semester Examination, 2024

APPLIED MATHEMATICS

PAPER – MTM-404(B)(Old)

(*Non-Linear Optimization*)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions : 2×4

(*a*) Define posynomial and polynomial in connection with geometric programming with an example.

(*Turn Over*)

(*b*) Let X^0 be an open set in R^n, let θ and g be defined on X^0. Find the conditions under which a solution $(\bar{x}, \bar{r}_0, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.

(*c*) Define bi-matrix game with an example.

(*d*) State Dorn's duality theorem in connection with duality in quadratic programming.

(*e*) Write the basic difference(s) between Beale's and Wolfe's method for solving quadratic programming problem.

(*f*) Under what condition(s) the Kuhn-Tucker conditions for quadratic programming problem are necessary and sufficient ?

2. Answer any *four* questions : 4 × 4

(*a*) Define the following :

(*i*) Minimization problem;

(*ii*) Local minimization problem;

(*iii*) Kuhn-Tucker stationary point problem;

(*iv*) Fritz-John stationary point problem.

(*b*) State and prove Weak duality theorem in connection with duality in non-linear programming.

(*c*) Minimize the following using geometric programming

$$f(x) = 16x_1x_2x_3 + 4x_1x_2^{-1} + 2x_2x_3^{-2} + 8x_1^{-3}x_2,$$
$$x_1, x_2, x_3 > 0.$$

(*d*) State and prove Motzkin's theorem of alternative.

(*e*) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem :

(*i*) Complete optimal solution

(*ii*) Pareto optimal solution

(*iii*) Local Pareto optimal solution

(*iv*) Weak Pareto optimal solution.

(f) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP.

3. Answer any *two* questions : 8×2

(*a*) (*i*) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when c_j is a random variable :

Minimize $F(x) = \sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} a_{ij} x_j \le b_i$

$x_j \ge 0, i, j = 1, 2, \dots, n.$

(*ii*) Define the following terms :

The (primal) quadratic minimization problem (QMP).

The quadratic dual (maximization) problem (QDP). 6 + 2

(*b*) (*i*) Prove that a pair $\{y^*, z^*\}$ constitutes a mixed strategy Nash equilibrium solution to a bimatrix game (A,B) if and only if, there exists a pair $\{p^*, q^*\}$ such that $\{y^*, z^*, p^*, q^*\}$ is a solution of the following bilinear programming *problem* :

Minimize $[y' Az + y'Bz + p + q]$

Subject to $Az \geq -pl_m$

$B'y \geq -ql_n$

$y \geq 0, z \geq 0, y'l_m = 1, z'l_n = 1$

(*ii*) Give the geometrical interpretations of differentiable convex function and concave function. 5 + 3

(*c*) (*i*) Let X be an open set in R^n and θ and g be differential and convex on X and let $\bar{x}$ solve the minimization problem and let g satisfy the Kuhn-Tucker constraint qualification. Show that there exists a $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ solves the dual maximization problem and $\theta(\bar{x}) = \psi(\bar{x}, \bar{u})$.

(*ii*) Prove that all strategically equivalent bimatrix game have the Nash equilibria.

5 + 3

(*d*) (*i*) Solve the following quadratic problems by using Beale's method :

Maximize $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to the constraints $x_1 + 2x_2 \leq 10$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0.$$

(*ii*) Write short note on complementary slackness principle. 6 + 2

[Internal Assessment — 10 Marks]

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